Design of Experiments Example: A Mixture Design with Process Variables

In the following example from Atkinson and Donev (1992), you create a design for an experiment involving both mixture factors and process factors. The design is an 18-run design that is balanced with respect to the levels of a categorical factor. The design enables you to fit a full response surface. You use Design Evaluation plots and results to examine the relative prediction variance of the design.

The response and factors involved in the design are as follows:

- The response is Damping, which measures the electromagnetic damping of an acrylonitrile powder.
- The three mixture ingredients are:
  - CuSO4 (copper sulphate), ranging from 0.2 to 0.8
  - Na2S2O3 (sodium thiosulphate), ranging from 0.2 to 0.8
  - Glyoxal (glyoxal), ranging from 0 to 0.6
- The nonmixture environmental factor of interest is Wavelength (the wavelength of an electromagnetic wave) at three levels denoted L1, L2, and L3.

  Wavelength is a continuous variable. However, the researchers were interested only in predictions at three specific wavelengths. For this reason, you treat Wavelength as a categorical factor with three levels.

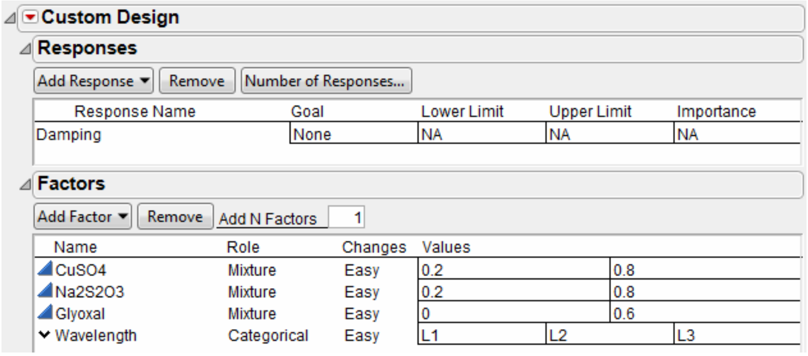
Create the Design

1. Select **DOE > Custom Design**.
2. Double-click Y under Response Name and type Damping.
3. Click **Maximize** under Goal and change it to **None**.

   The goal is set to None because the authors of the study do not mention how much damping is desirable.
4. From the Custom Design red triangle menu, select **Load Factors**.
5. Select **Help > Sample Data Library** and open Design Experiment/Donev Mixture Factors.jmp.

   This loads the three mixture ingredients and the categorical environmental factor. Note that the bounds on the values of the three mixture factors are also loaded.

Figure 1 Responses Outline and Factors Outline



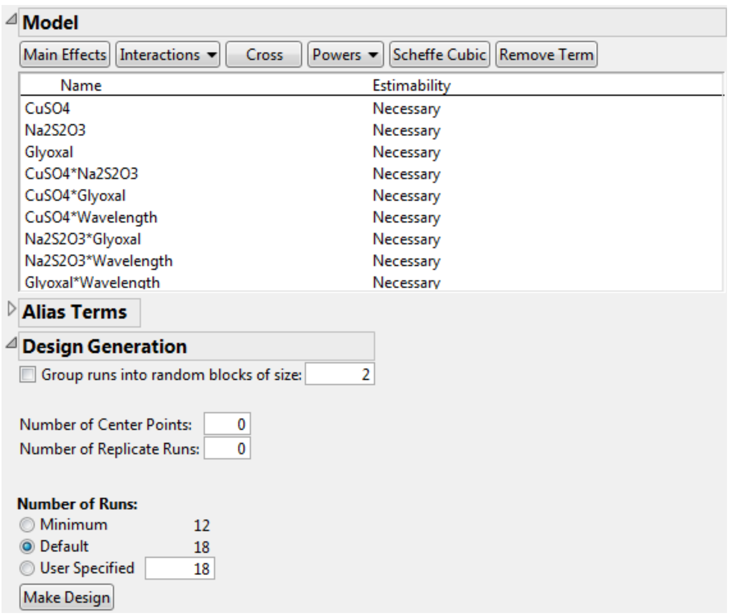
6. Click **Interactions > 2nd**.

   An informational JMP Alert window reminds you that JMP removes the main effect terms for non-mixture factors that interact with all the mixture factors. This means that the main effect of Wavelength is removed, but all two-way interactions of mixture factors with Wavelength are added.

7. Click **OK** to dismiss the message.

   The effects in the Model outline define a response surface model in the mixture ingredients along with the additive effect of the wavelength. See Scheffé (1958).

Figure 2 Model and Design Generation Outlines



8. Leave the default number of runs at 18.

   The choice of 18 runs allows six runs for each of the three levels of the wavelength factor.

   Note: Setting the Random Seed in step 9 and Number of Starts in step 10 reproduces the exact results shown in this example. In constructing a design on your own, these steps are not necessary.

9. (Optional) From the Custom Design red triangle menu, select **Set Random Seed**, type 858576648, and click **OK**.
10. (Optional) From the Custom Design red triangle menu, select **Number of Starts**, type 10, and click **OK**.
11. Click **Make Design**.

Figure 3 Design Outline Showing 18-Run Design

Design

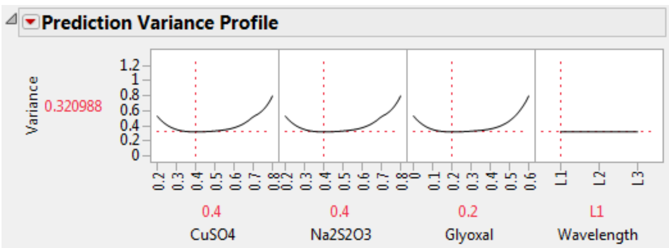
Run	CuSO4	Na2S2O3	Glyoxal	Wavelength
1	0.5	0.5	0	L3
2	0.5	0.2	0.3	L2
3	0.8	0.2	0	L3
4	0.2	0.2	0.6	L2
5	0.5	0.5	0	L2
6	0.2	0.2	0.6	L1
7	0.5	0.2	0.3	L1
8	0.8	0.2	0	L1
9	0.8	0.2	0	L2
10	0.2	0.2	0.6	L3
11	0.5	0.2	0.3	L3
12	0.2	0.8	0	L1
13	0.2	0.5	0.3	L2
14	0.2	0.5	0.3	L1
15	0.2	0.8	0	L2
16	0.2	0.8	0	L3
17	0.2	0.5	0.3	L3
18	0.5	0.5	0	L1

You can check that there are six runs for each level of **Wavelength**.

Evaluate the Design

1. Open the **Design Evaluation** > **Prediction Variance Profile** outline.

Figure 4 Prediction Variance Profile for 18-Run Design



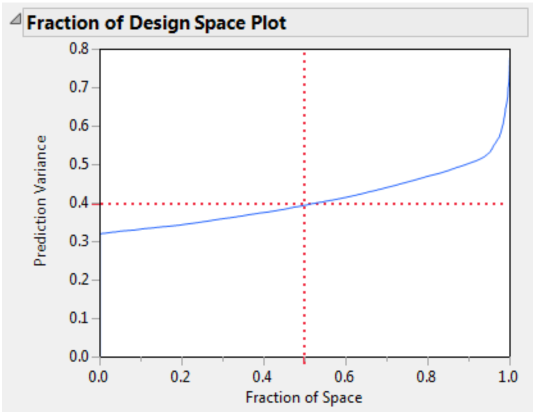
Move the slider for **Wavelength** to verify that the relative prediction variance profiles for the mixture factors do not change across the levels of **Wavelength**. Move the slider for any one of the mixture factors. The sliders for the other two mixture factors adjust to make the mixture ingredients sum to one. Notice that the smallest relative prediction variances occur near the center settings for the mixture factors.

2. From the Prediction Variance Profile red triangle menu, select **Maximize Desirability**.

   Notice that the maximum relative prediction variance over the design space is 0.8 times the error variance.

3. Open the **Fraction of Design Space Plot** outline.

Figure 5 Fraction of Design Space Plot for 18-Run Design



Over the entire design space, the relative prediction variance is below 0.8. The minimum relative prediction variance is about 0.32. As seen in Figure 4, the minimum occurs near the center settings for the mixture factors.

4. Open the **Design Diagnostics** outline.

Figure 6 Design Diagnostics Outline for 18-Run Design

Design Diagnostics

D Optimal Design	
D Efficiency	3.627407
G Efficiency	91.28709
A Efficiency	0.328715
Average Variance of Prediction	0.410395
Design Creation Time (seconds)	0.1

The design is optimal relative to the D-optimality criterion, even though its D-efficiency is very low (3.6%). Because mixture designs are far from orthogonal due to the mixture constraint, they typically have very low D-efficiencies. The Average (relative) Variance of Prediction is 0.410395. This is consistent with the Fraction of Design Space plot in Figure 5.