# *Exploring Reliability with JMP: Competing Risks*

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# Topics

- Reliability Concepts and Terminology
- Life Distribution Fitting
- **Competing Cause Analysis**
- Accelerated Life Testing
- Recurrence Analysis
- Degradation Studies

# Multiple Modes: Series Model

Consider a system made up with *n* components in series. If the *i* th component has reliability $\mathbf{R}_i\mathbf{(t)}$, the system reliability is the product of the individual reliabilities, that is,

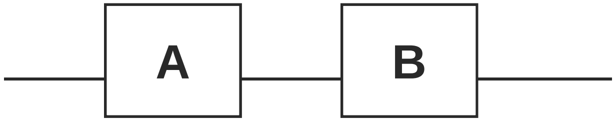
$$R_s(t) \quad R_1 \quad t \quad R_2 \quad t \quad \ldots \quad R_n \quad t$$

The system reliability cannot be greater than the lowest component reliability (weakest link).

The **system failure rate** is the **sum** of the individual **component failure rates**. The system failure rate is higher than the highest individual failure rate.

# Reliability Block Diagram for Components in Series

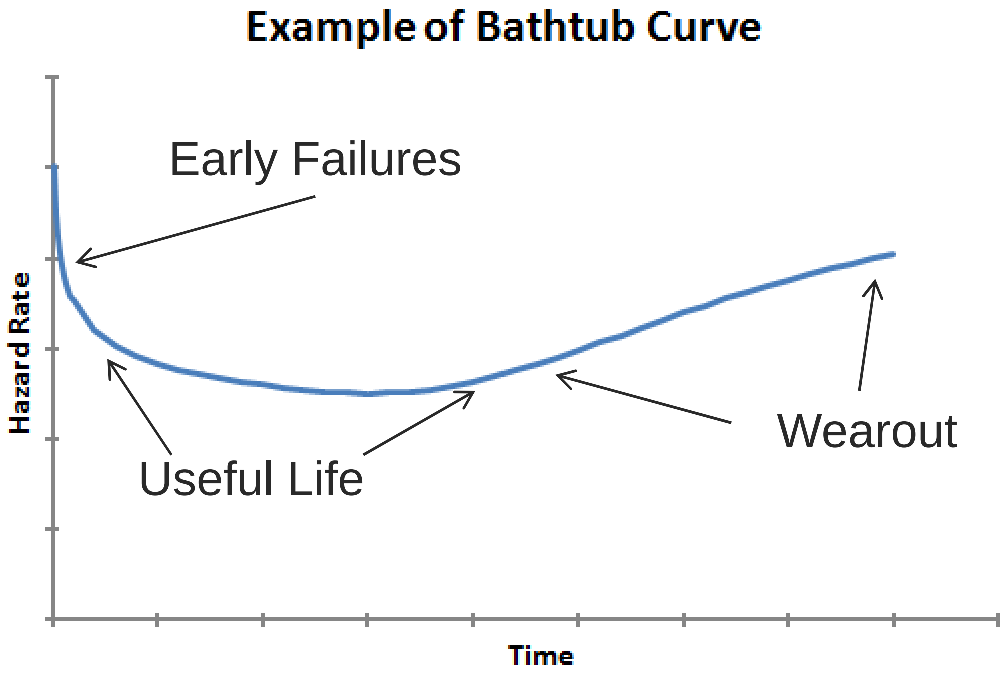
For two components in series:




The first component to fail causes system failure.

# Reliability Terminology

The ***bathtub curve*** is a popular representation for the reliability of products with competing risks failures.



# Product Reliability Issue

Company ABC is becoming aware of an increase in field failures from a mode of failure not previously seen in testing to 15,000 cycles, considered equivalent to three years in the field with normal use.

Production stress testing through 15,000 cycles showed the hazard rate was steadily decreasing. Management was actually considering extending the warranty period to 25,000 cycles before this issue arose.

Some customers were heavy users who were cycling the product at two to three types the expected rate and began experiencing failures after only one year.

# Product Reliability Stress Test

A **sample** of 40 devices will be stressed for 30,000 cycles to see if the new failure mode can be reproduced.

The exact time at which each system fails is recorded. All failure are then sent to failure analysis for identification of the mode of failure.

At the end of the 30,000 cycle test, 28 units had failed and 12 survived.

# Objectives of Reliability Test

- Learn whether the new failure mode can be reproduced.
- Model the distribution of both failure modes.
- Predict the failure fraction after 25,000 cycles in the field.
- Estimate how long it will take to reach 10% failures in the field for the second mode.
- Learn if burn-in can improve product reliability for the existing known mode.

# Analysis of Multiple Failure Modes

- Nonparametric analysis can reveal unusual patterns in the EDF.
- Failures associated with one failure modes will be **censoring times** for other failure modes.
- With information on modes associated with each failure, it is possible to do **separate analysis on each mode**.
- Independence of failure modes is an important assumption.

# JMP Data Table

Two Modes Stress Results

- Two Modes Stress
- Source

Columns (2/0)
- TF
- Mode Type

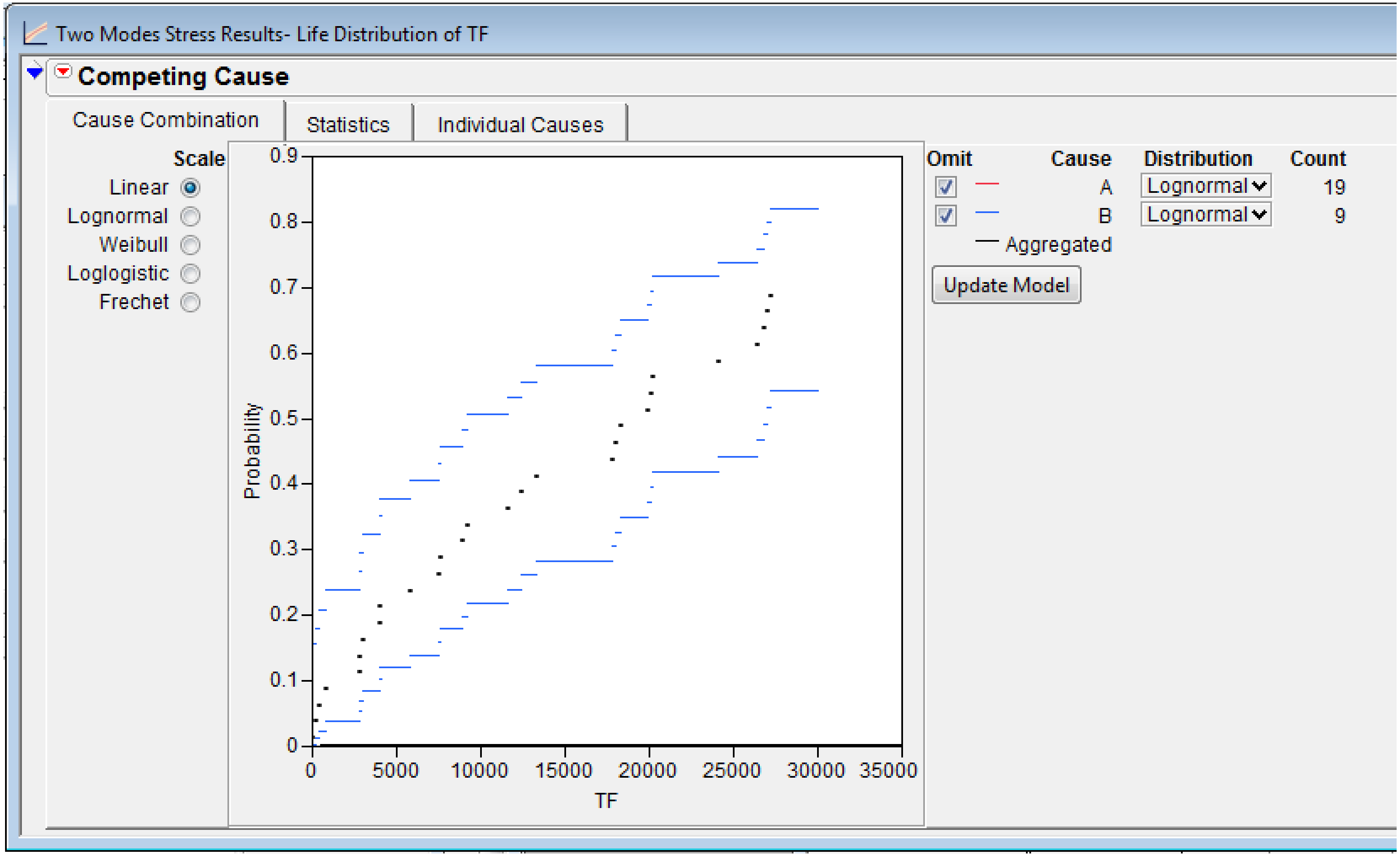
Rows

| | |
|---|---|
| All rows | 40 |
| Selected | 0 |
| Excluded | 0 |
| Hidden | 0 |
| Labelled | 0 |

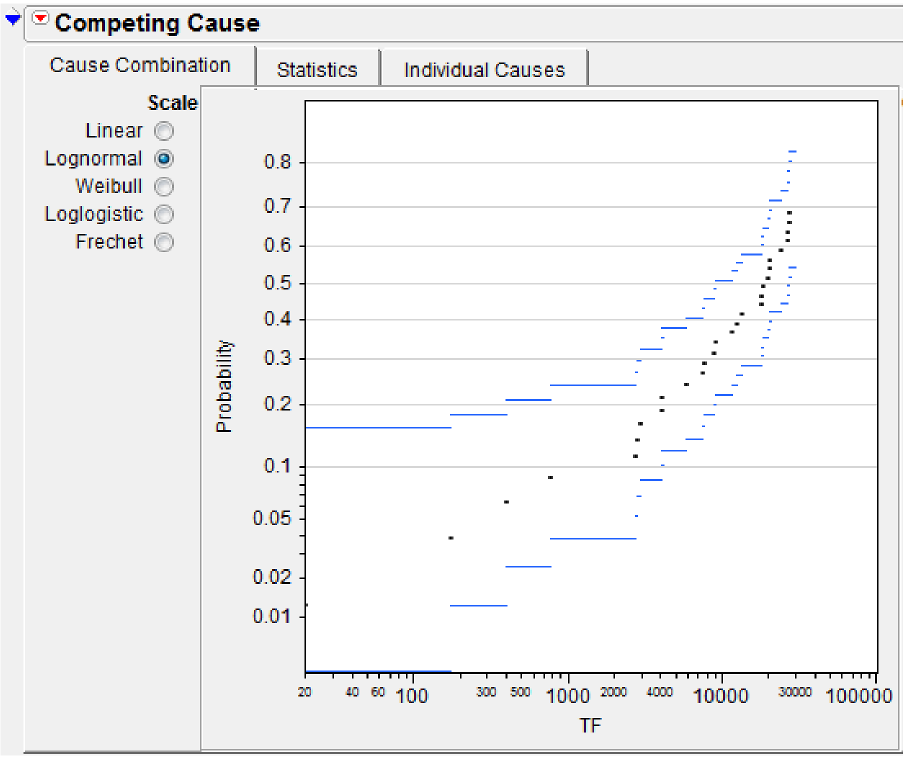
| | TF | Mode Type |
|---|---|---|
| 1 | 20 | A |
| 2 | 173 | A |
| 3 | 404 | A |
| 4 | 777 | A |
| 5 | 2755 | A |
| 6 | 2848 | A |
| 7 | 2954 | A |
| 8 | 4025 | A |
| 9 | 4043 | A |
| 10 | 5808 | A |
| 11 | 7472 | A |
| 12 | 7598 | A |
| 13 | 8948 | A |
| 14 | 9156 | A |
| 15 | 11574 | A |
| 16 | 12438 | A |
| 17 | 13289 | A |
| 18 | 17802 | B |
| 19 | 17976 | A |
| 20 | 18257 | A |
| 21 | 19949 | B |
| 22 | 20118 | B |
| 23 | 20165 | B |
| 24 | 24117 | B |
| 25 | 26389 | B |
| 26 | 26754 | B |
| 27 | 27006 | B |
| 28 | 27190 | B |
| 29 | 30000 | Censored |
| 30 | 30000 | Censored |
| 31 | 30000 | Censored |
| 32 | 30000 | Censored |
| 33 | 30000 | Censored |
| 34 | 30000 | Censored |
| 35 | 30000 | Censored |
| 36 | 30000 | Censored |
| 37 | 30000 | Censored |
| 38 | 30000 | Censored |
| 39 | 30000 | Censored |
| 40 | 30000 | Censored |

We will analyze using JMP's Life Distribution Platform.

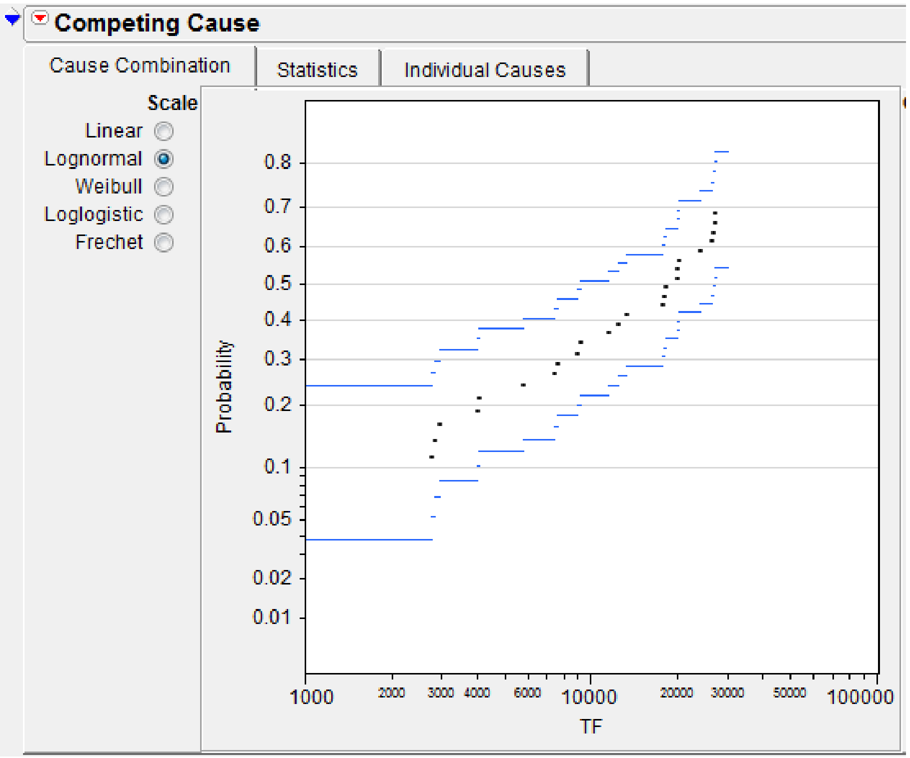
# Nonparametric Plot All Points



# Lognormal Probability Plots

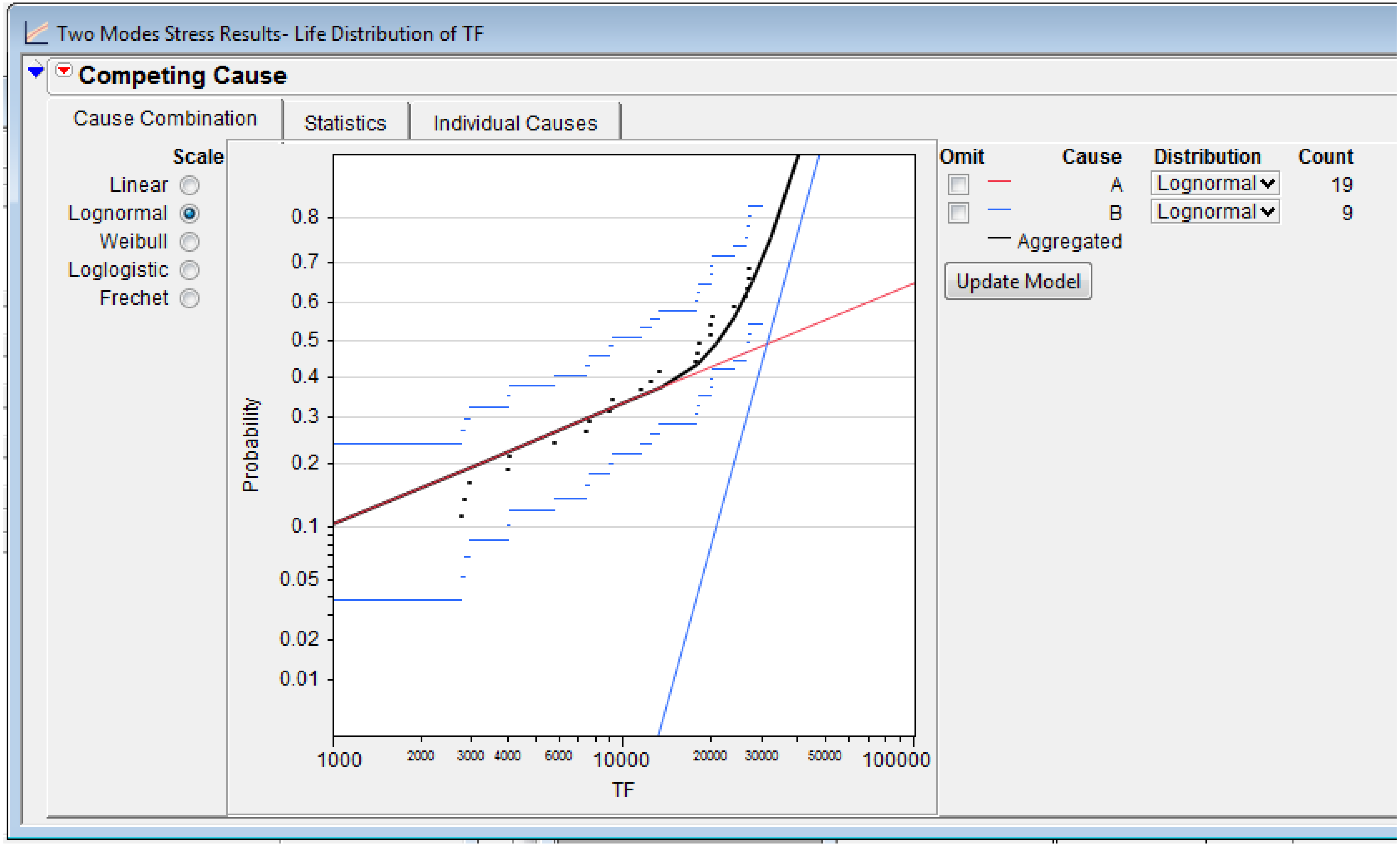



Full Horizontal Scale

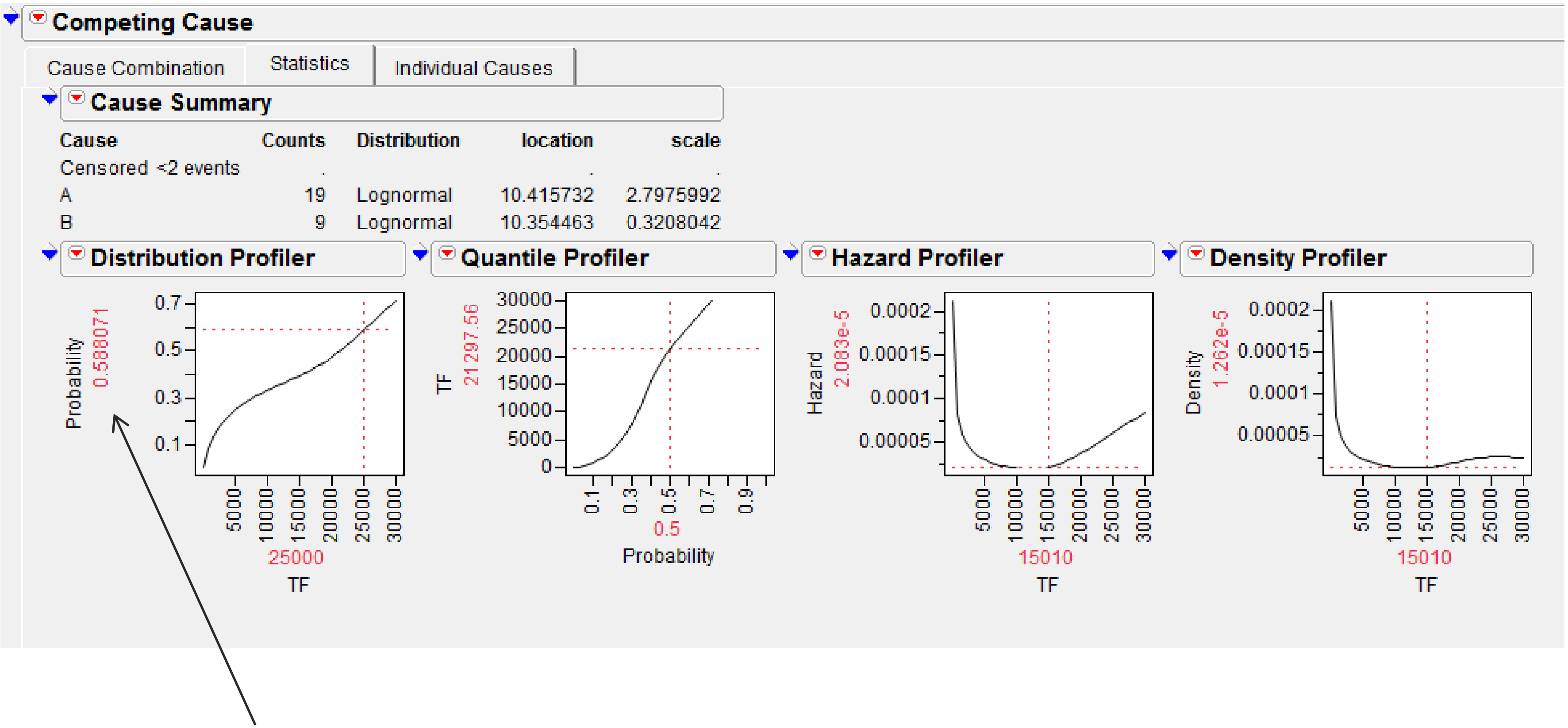



Adjusted Horizontal Scale

# Lognormal Probability Plot with Modes



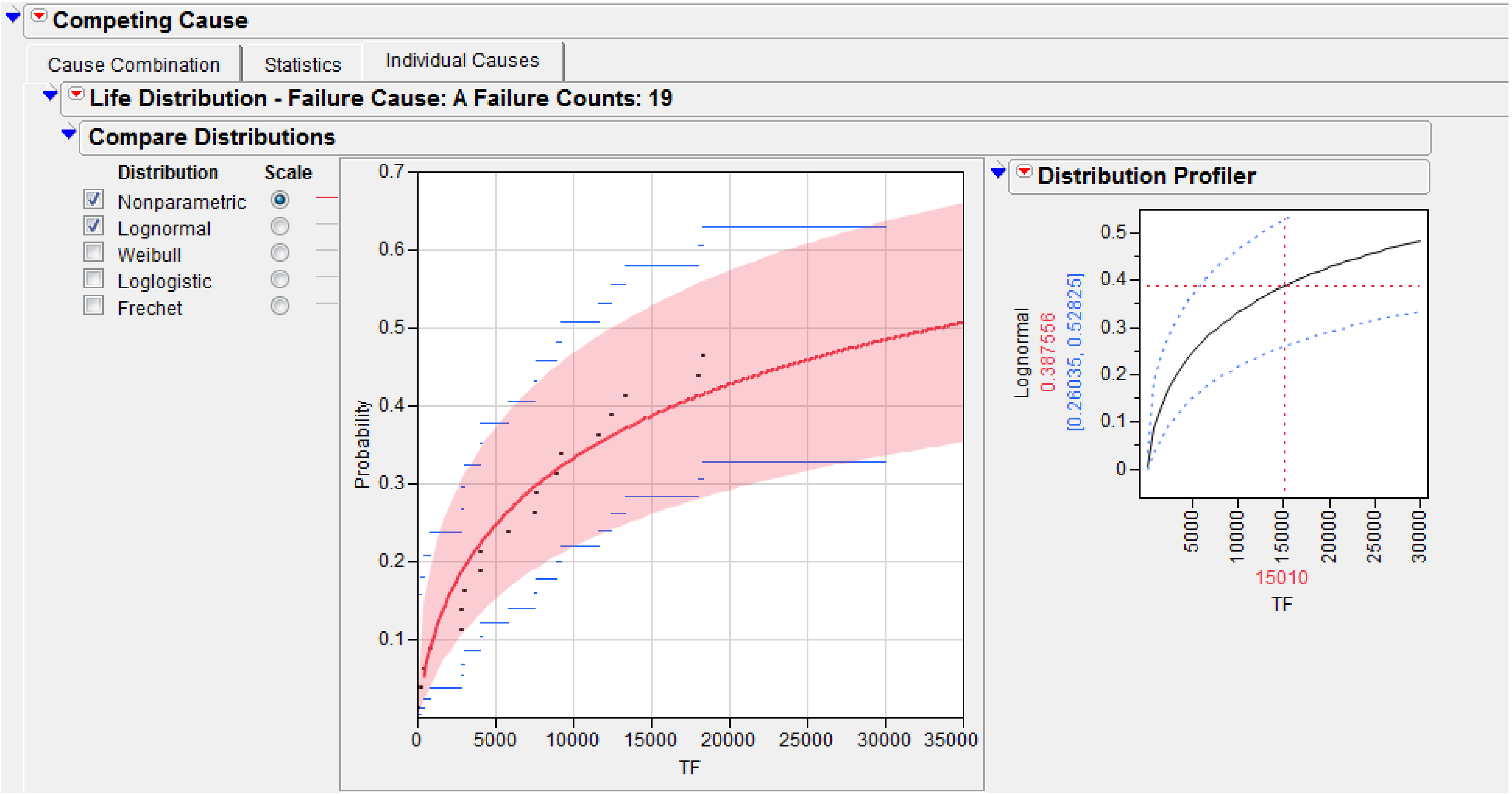
# Statistics and Profiler



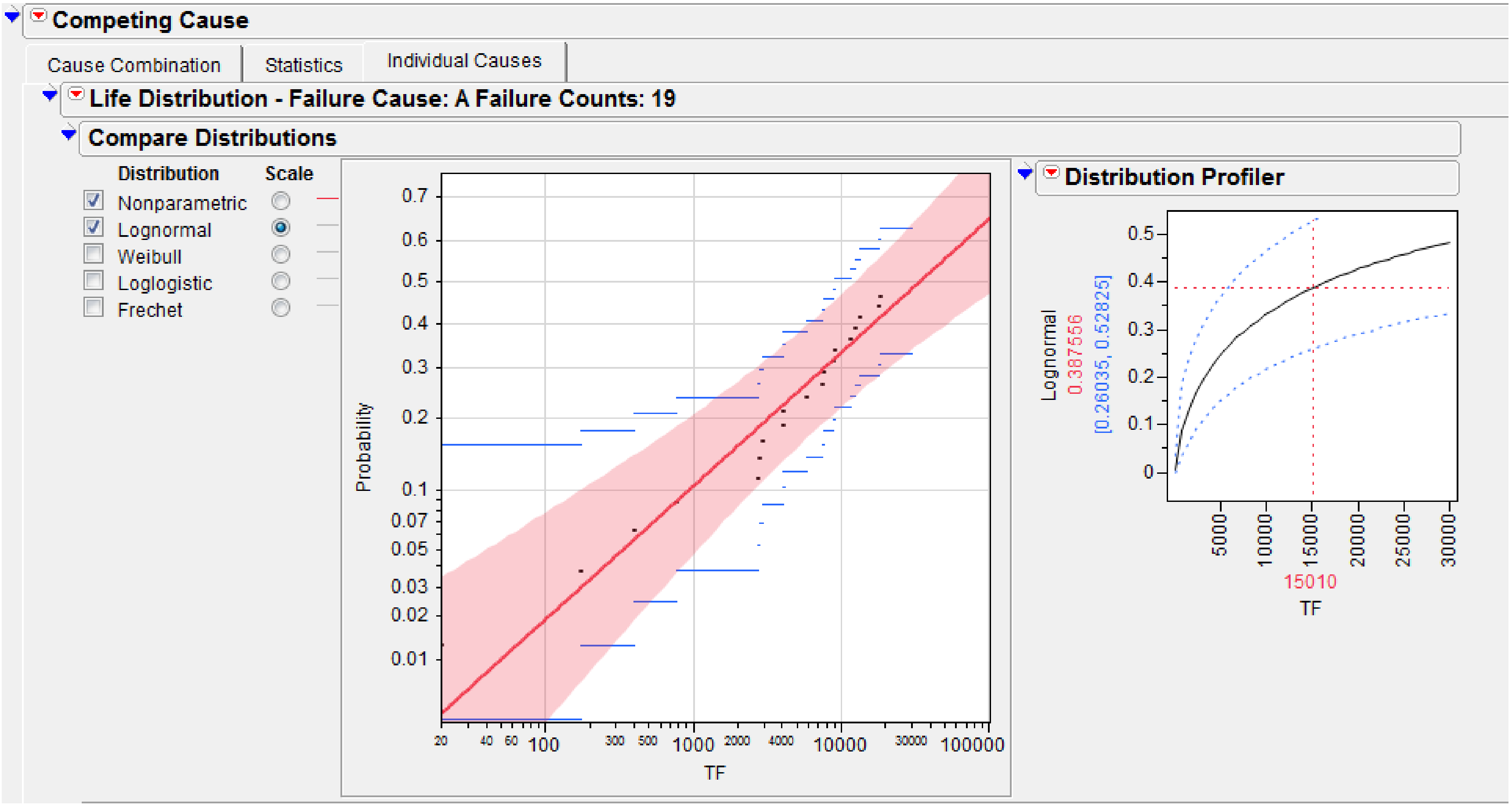

| Cause | Counts | Distribution | location | scale |
|---|---|---|---|---|
| Censored <2 events | . | | . | . |
| A | 19 | Lognormal | 10.415732 | 2.7975992 |
| B | 9 | Lognormal | 10.354463 | 0.3208042 |

Fraction failing at 25000 cycles

# Nonparametric Plot Mode A



# Lognormal Probability Plot Mode A



# Statistics and Profiler Mode A

Statistics

## Model Comparisons

| Distribution | AICc | -2Loglikelihood | BIC |
|---|---|---|---|
| Lognormal | 432.94702 | 428.62269 | 436.00045 |

Burn-In Possibility

## Summary of Data

## Nonparametric Estimate

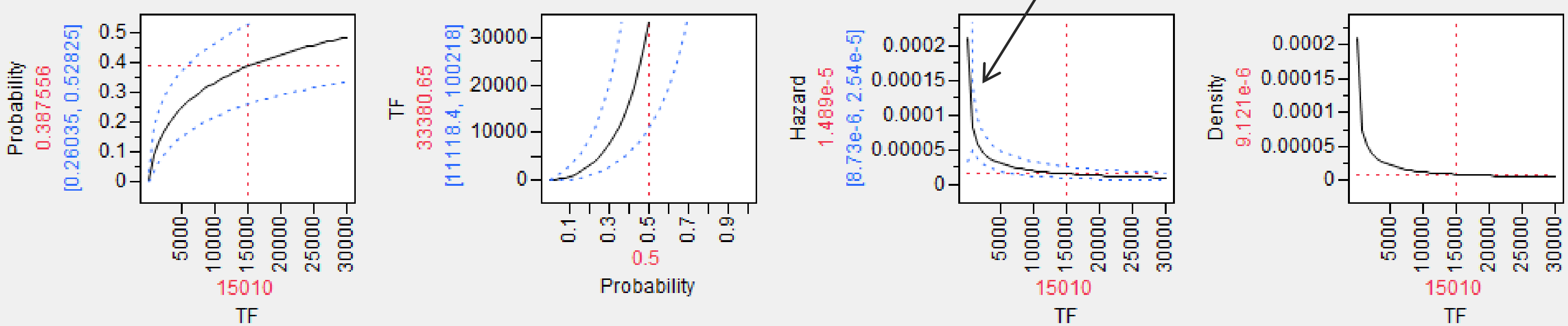
## Parametric Estimate - Lognormal

| Parameter | Estimate | Std Error | Lower 95% | Upper 95% |
|---|---|---|---|---|
| location | 10.415732 | 0.56091492 | 9.3163587 | 11.515105 |
| scale | 2.797599 | 0.50600249 | 1.8058525 | 3.789346 |

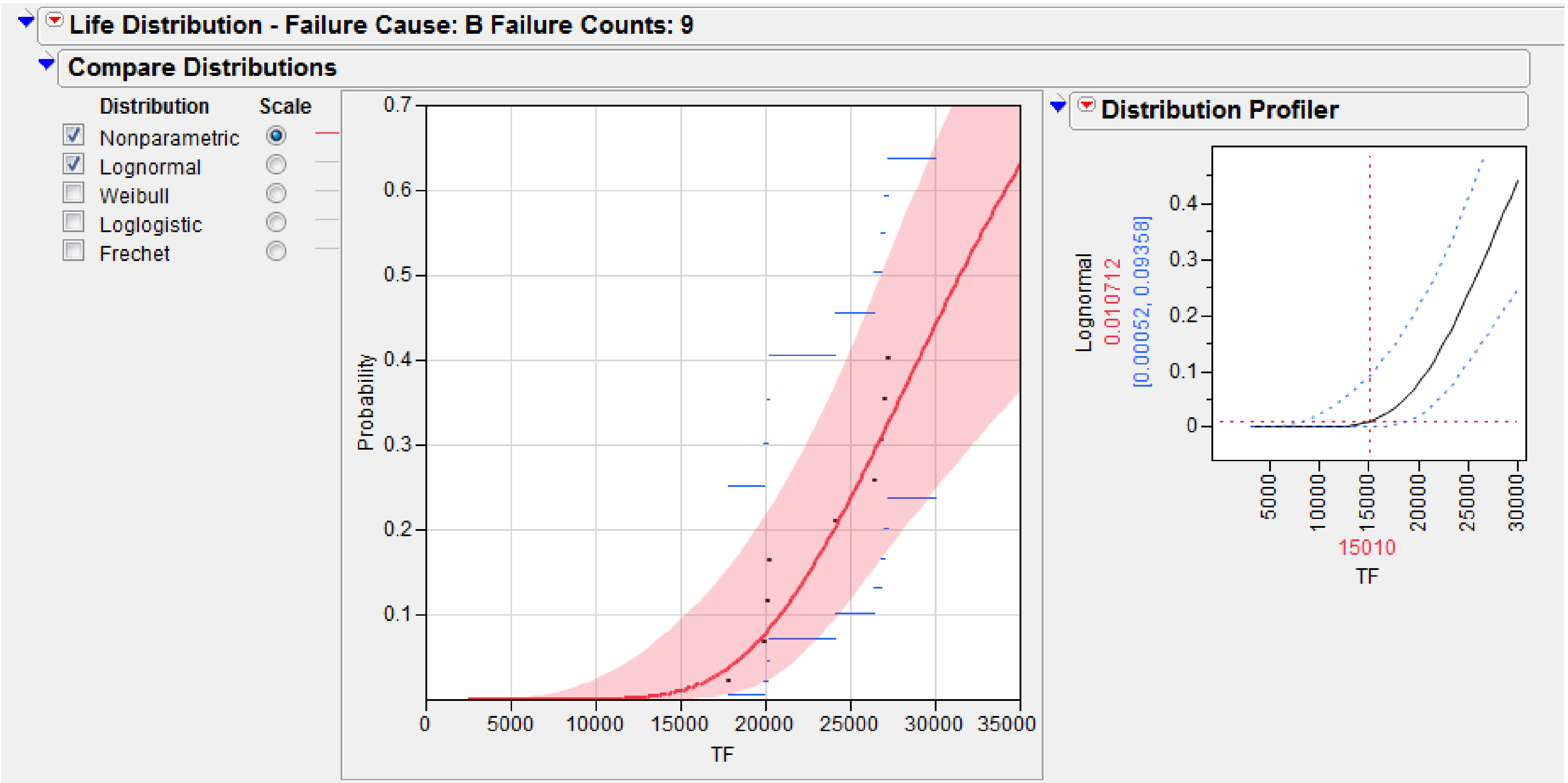
| Criterion | |
|---|---|
| -2 LogLikelihood | 428.62269 |
| AICc | 432.94702 |
| BIC | 436.00045 |

## Covariance Matrix

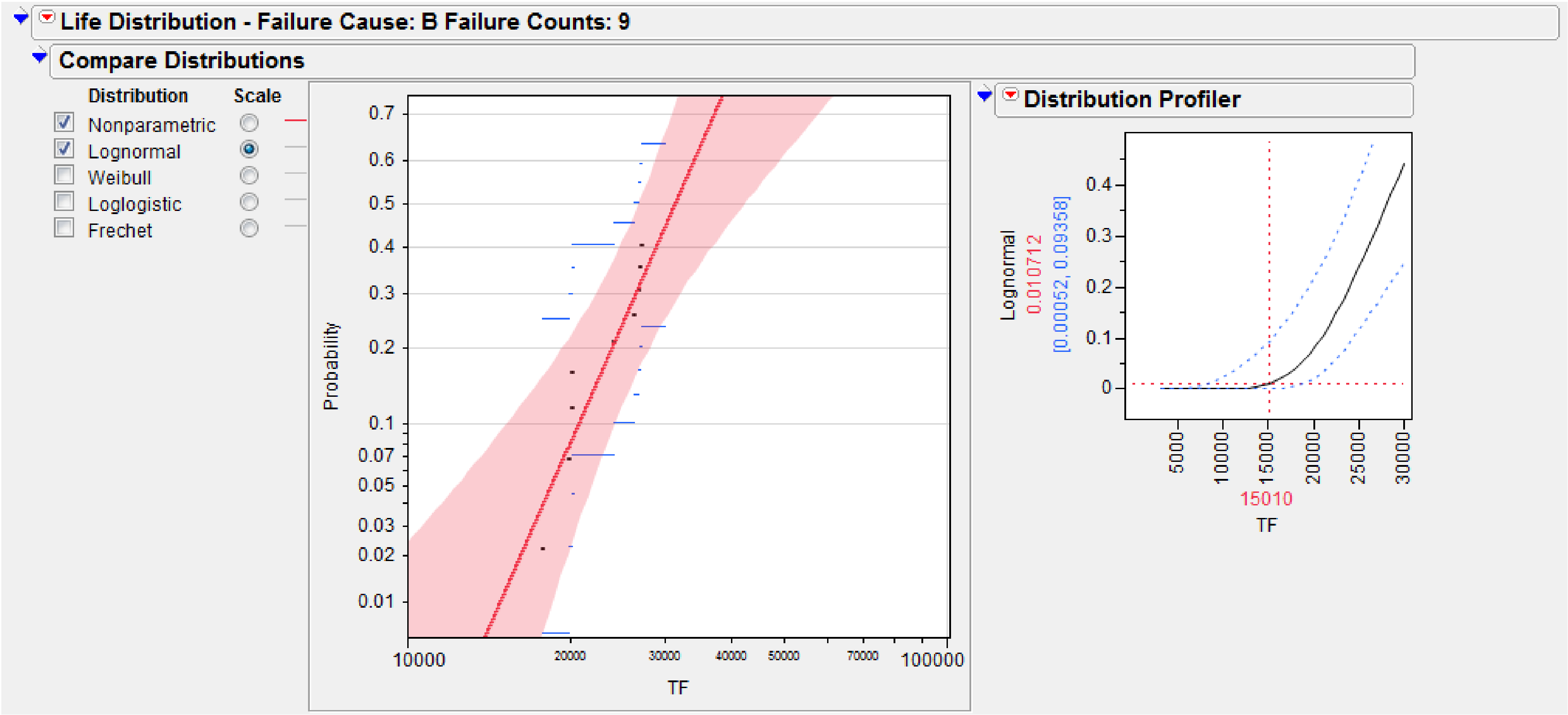
Distribution Profiler | Quantile Profiler | Hazard Profiler | Density Profiler



# Nonparametric Plot Mode B



# Lognormal Probability Plot Mode B



# Statistics and Profiler Mode B

**Statistics**

**Model Comparisons**

| Distribution | AICc | -2Loglikelihood | BIC |
|---|---|---|---|
| Lognormal | 206.03633 | 201.71201 | 209.08977 |

**Summary of Data**

**Nonparametric Estimate**

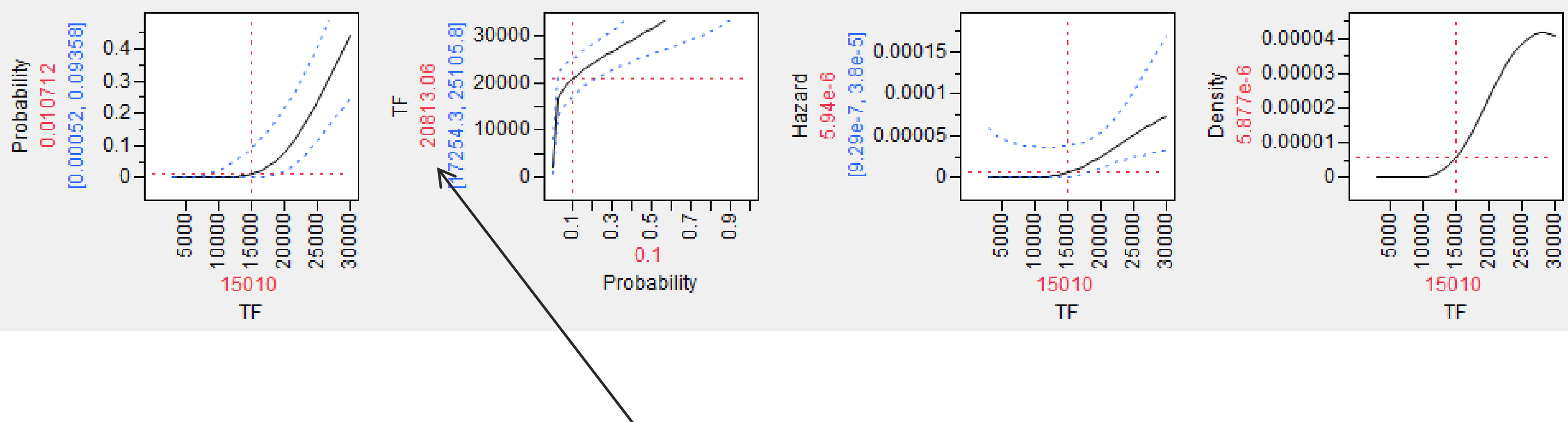
**Parametric Estimate - Lognormal**

| Parameter | Estimate | Std Error | Lower 95% | Upper 95% |
|---|---|---|---|---|
| location | 10.354463 | 0.09481169 | 10.168636 | 10.540291 |
| scale | 0.320804 | 0.08395323 | 0.156259 | 0.485350 |

| Criterion | |
|---|---|
| -2 LogLikelihood | 201.71201 |
| AICc | 206.03633 |
| BIC | 209.08977 |

**Covariance Matrix**

**Distribution Profiler** | **Quantile Profiler** | **Hazard Profiler** | **Density Profiler**




Time to Reach 10% Fraction Failing for Mode B

# Likelihood Contours

**Parametric Estimate - Lognormal**

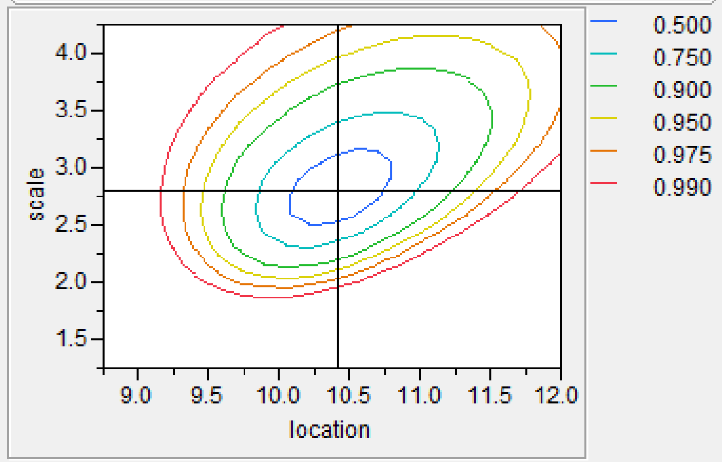
| Parameter | Estimate | Std Error | Lower 95% | Upper 95% | Criterion | |
|---|---|---|---|---|---|---|
| location | 10.415732 | 0.56091492 | 9.3163587 | 11.515105 | -2 LogLikelihood | 428.62269 |
| scale | 2.797599 | 0.50600249 | 1.8058525 | 3.789346 | AICc | 432.94702 |
| | | | | | BIC | 436.00045 |

Mode A

T50 = 33,381 cycles

$\sigma$ = 2.80

**Loglikelihood Contour**




**Parametric Estimate - Lognormal**

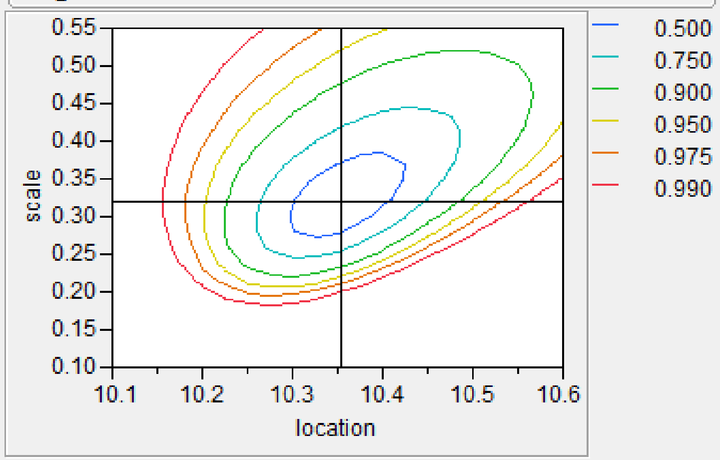
| Parameter | Estimate | Std Error | Lower 95% | Upper 95% | Criterion | |
|---|---|---|---|---|---|---|
| location | 10.354463 | 0.09481169 | 10.168636 | 10.540291 | -2 LogLikelihood | 201.71201 |
| scale | 0.320804 | 0.08395323 | 0.156259 | 0.485350 | AICc | 206.03633 |
| | | | | | BIC | 209.08977 |

Mode B

T50 = 31,397 cycle

$\sigma$ = 0.321

**Loglikelihood Contour**



# Lessons Learned

- Although the early failure mode showed a decreasing hazard rate, the wearout mode began appearing late in product life and had to be remediated.
- Analysis of multiple failure modes, called ***competing risk analysis***, can provide valuable information for improving product reliability.
- Engineers can see the effect of eliminating different failure modes on product reliability.

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