BASIC DESIGN OF EXPERIMENTS USING CUSTOM DOE PLATFORM

Mastering JMP Webcast
April 24, 2015

Tom Donnelly, PhD, CAP
Sr. Systems Engineer
& Co-insurrectionist
JMP Federal Government Team

AGENDA

- JMP DOE Resources
- Multiple Response Optimization – Trade-Space Analysis
- Review of Classic DOE
- Real-World Experimental Issues – Custom DOE is about

  Making Designs Fit the Problem –

  NOT Making Problems Fit the Designs!

  - All continuous factors
  - Continuous, discrete numeric, categorical and blocking factors - with constraints

DOE RESOURCES

- Help in JMP Software
  - Help > Tutorials > DOE Tutorial
  - Help > Books > Design of Experiments Guide > Chapter 3: Building Custom Designs
- Three tabs across home page at www.jmp.com
  - Using JMP > Application Areas > Design of Experiments
  - News and Events > On-Demand Webcasts > Mastering JMP (see more in series)
  - Resources for Users >
    - JMP User Community
    - Learning Library – one-page guides & 1-3 minute videos
    - JMP Blog – search on Brad Jones, Ryan Lekivetz, and Joseph Morgan
    - File Exchange – Add-ins
- www.jmp.com/government-resources DOE Webcasts & Journal Articles
- DOE Books at http://www.jmp.com/software/books.shtml
  - *Design and Analysis of Experiments by Douglas Montgomery: A Supplement for Using JMP®* by Heath Rushing, Andrew Karl and James Wisnowski
  - *Optimal Design of Experiments: A Case Study Approach* by Brad Jones and Peter Goos
- Linked-In: JMP DOE subgroup (505 members) of JMP Professional Network
- YouTube: www.youtube.com/watch?v=R7Dh3RRleGo

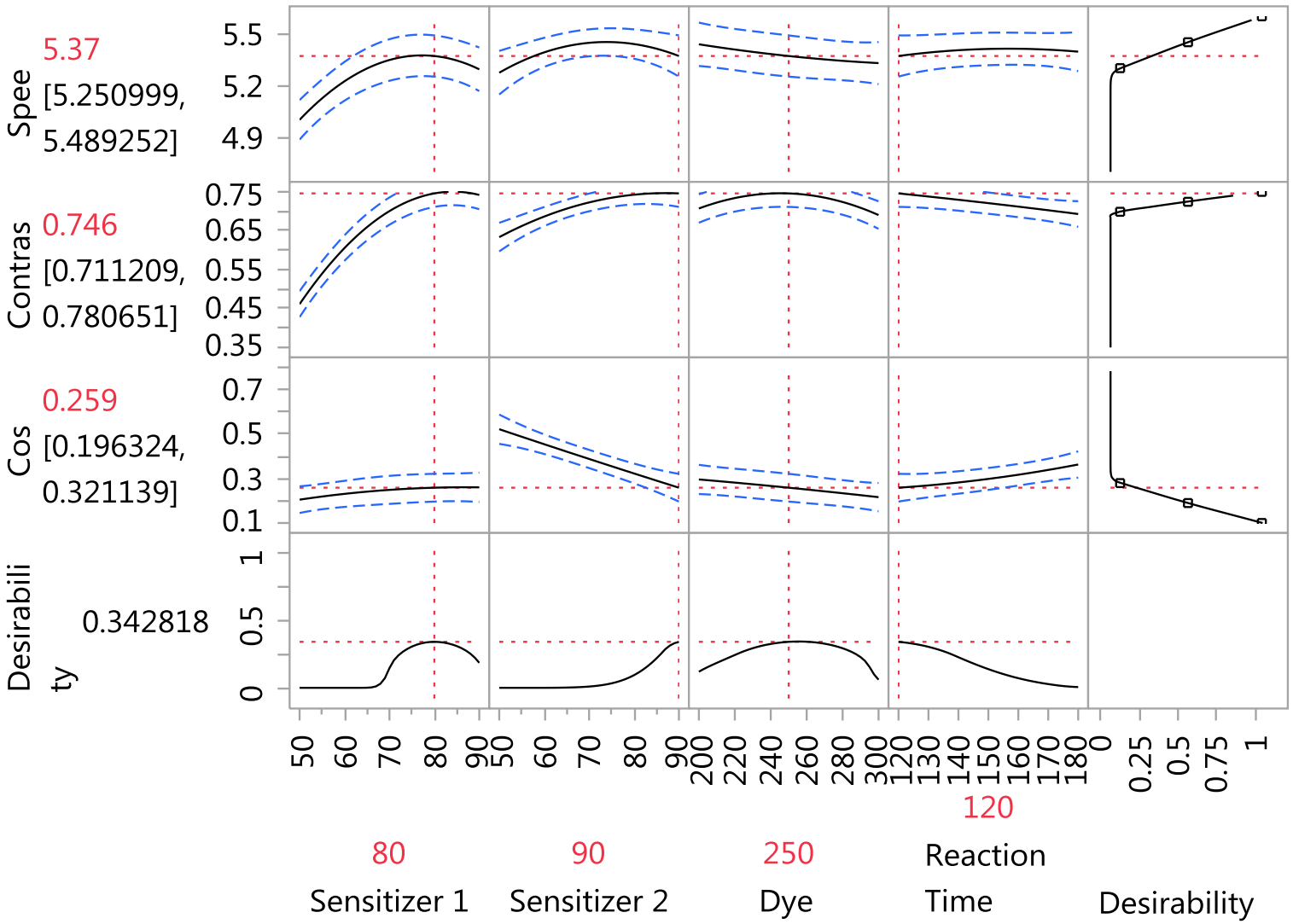
WHY USE DOE?

QUICKER ANSWERS,
LOWER COSTS,
SOLVE BIGGER PROBLEMS

- More rapidly answer “what if?” questions
- Do sensitivity and trade-space analysis
- Optimize across multiple responses
- By running efficient subsets of all possible combinations, one can – for the same resources and constraints – ***solve bigger problems***
- By running sequences of designs

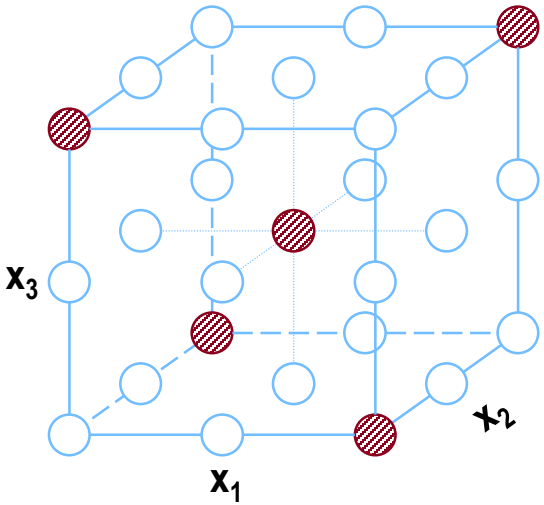
one can be as ***cost effective as possible*** and ***run no more trials than are needed*** to get a useful answer

USE JMP | TRADE-OFF AND OPTIMIZATION

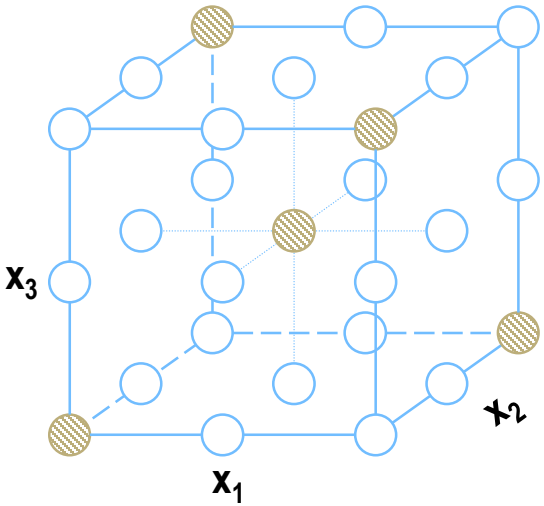


CLASSIC RESPONSE-SURFACE DOE IN A NUTSHELL

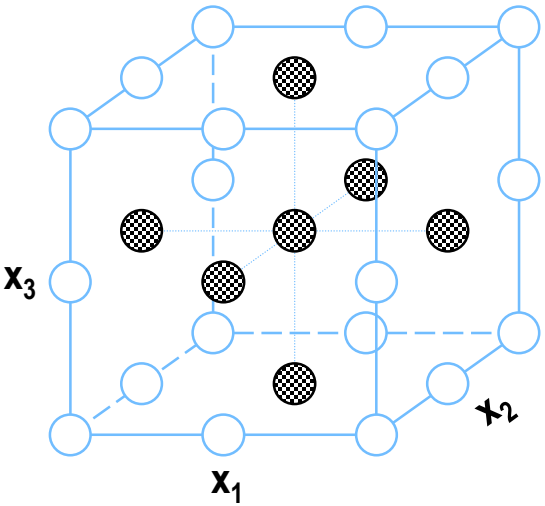
Block 1



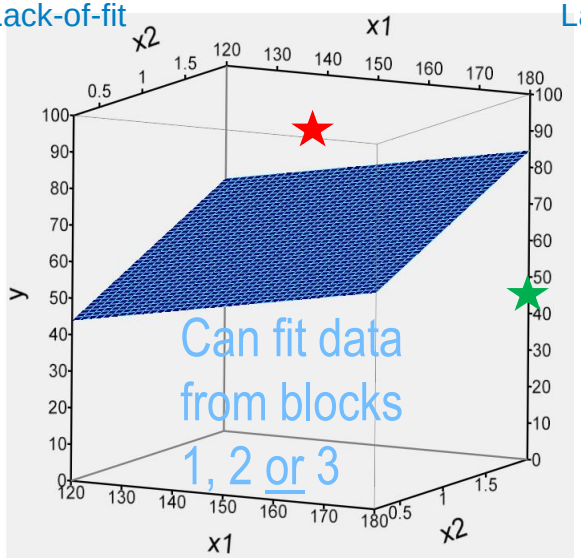
Block 2



Block 3



Lack-of-fit



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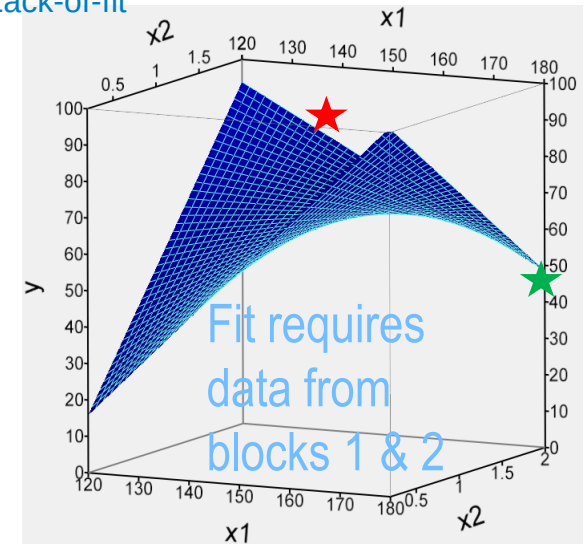
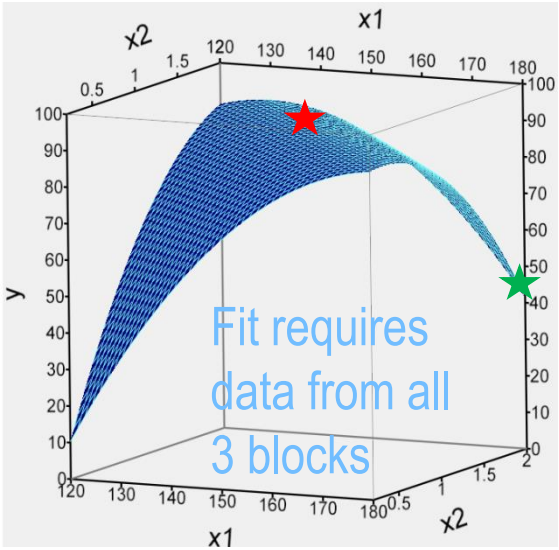
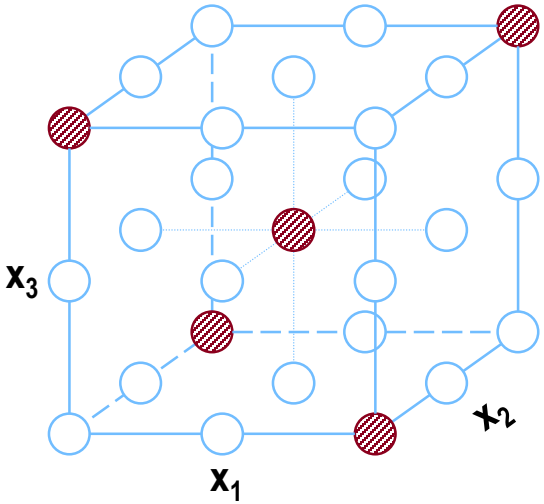




POLYNOMIAL MODELS USED TO CALCULATE SURFACES

Block 1

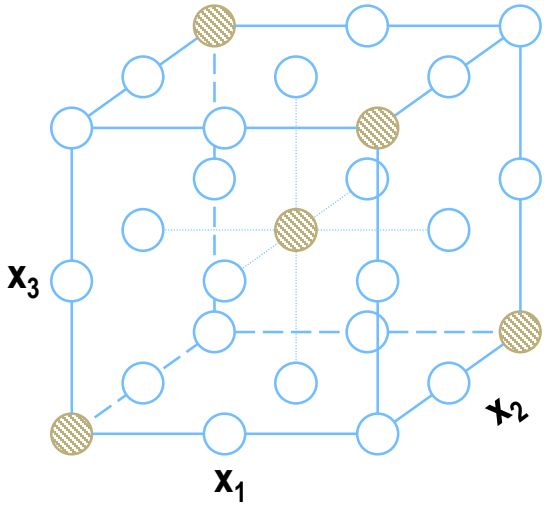


$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3$$

Run this block 1st to:

(i) estimate the main effects*
(ii) use center point to check for curvature.

Block 2

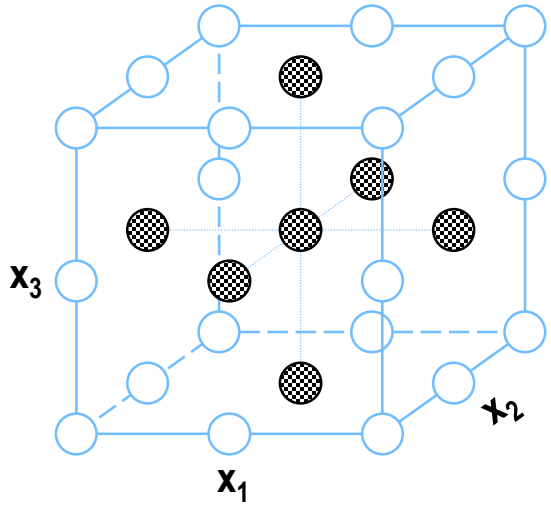


$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3$$
$$+ a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3$$

Run this block 2nd to:

(i) repeat main effects estimate,
(ii) check if process has shifted
(iii) add interaction effects to model <u>if needed.</u>

Block 3

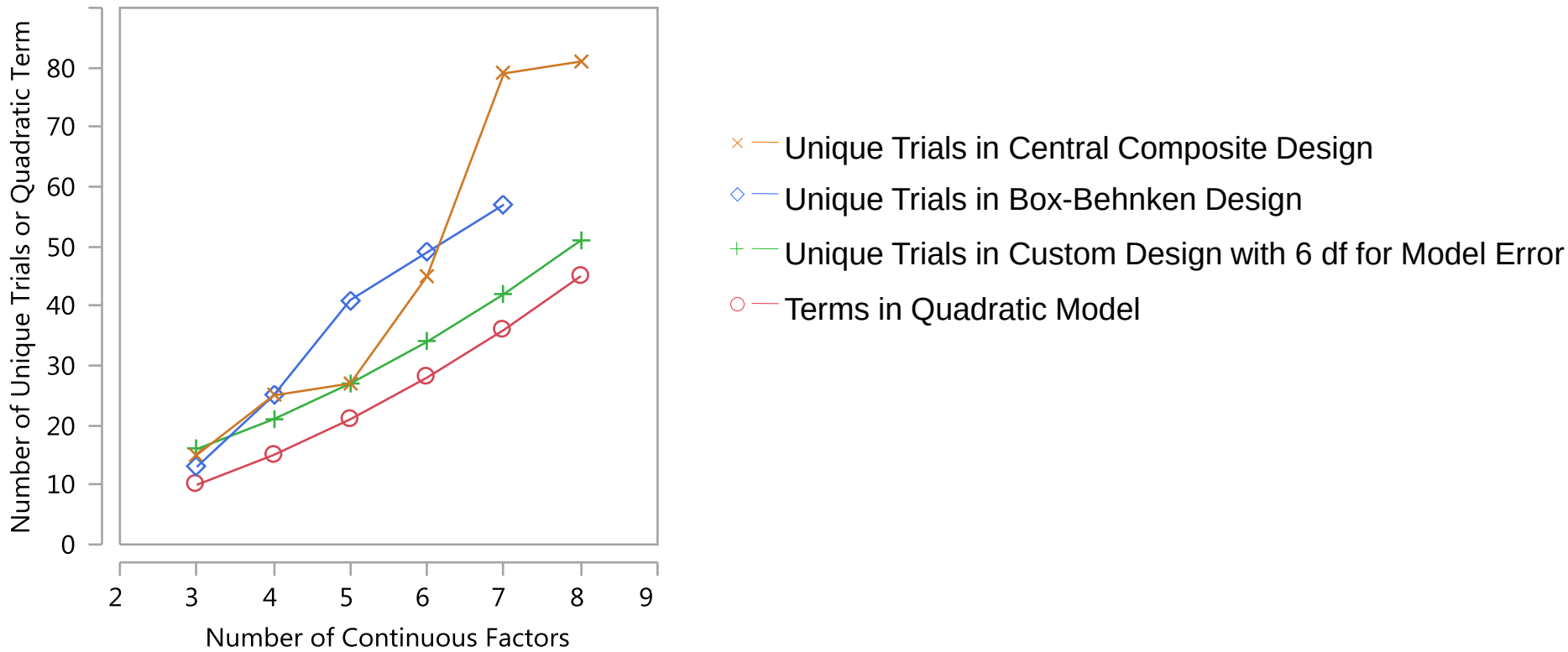


$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3$$
$$+ a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3$$
$$+ a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2$$

Run this block 3rd to:

(i) repeat main effects estimate,
(ii) check if process has shifted
(iii) add curvature effects to model <u>if needed.</u>

NUMBER OF UNIQUE TRIALS FOR 3 RESPONSE-SURFACE DESIGNS AND NUMBER OF QUADRATIC MODEL TERMS VS. NUMBER OF CONTINUOUS FACTORS



If generally running 3, 4 or 5-factor fractional-factorial designs…

1. How many interactions are you not investigating?
2. How many more trials needed to fit curvature?
3. Consider two stages: Definitive Screening + Augmentation

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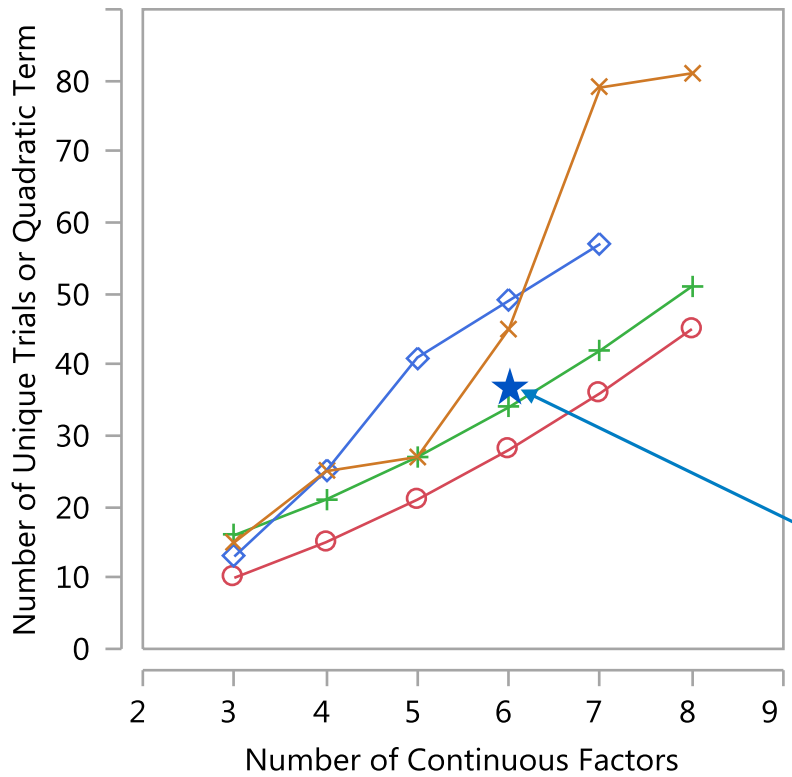


Unique Trials in Central Composite Design

Unique Trials in Box-Behnken Design

Unique Trials in Custom Design with 6 df for Model Error

Terms in Quadratic Model

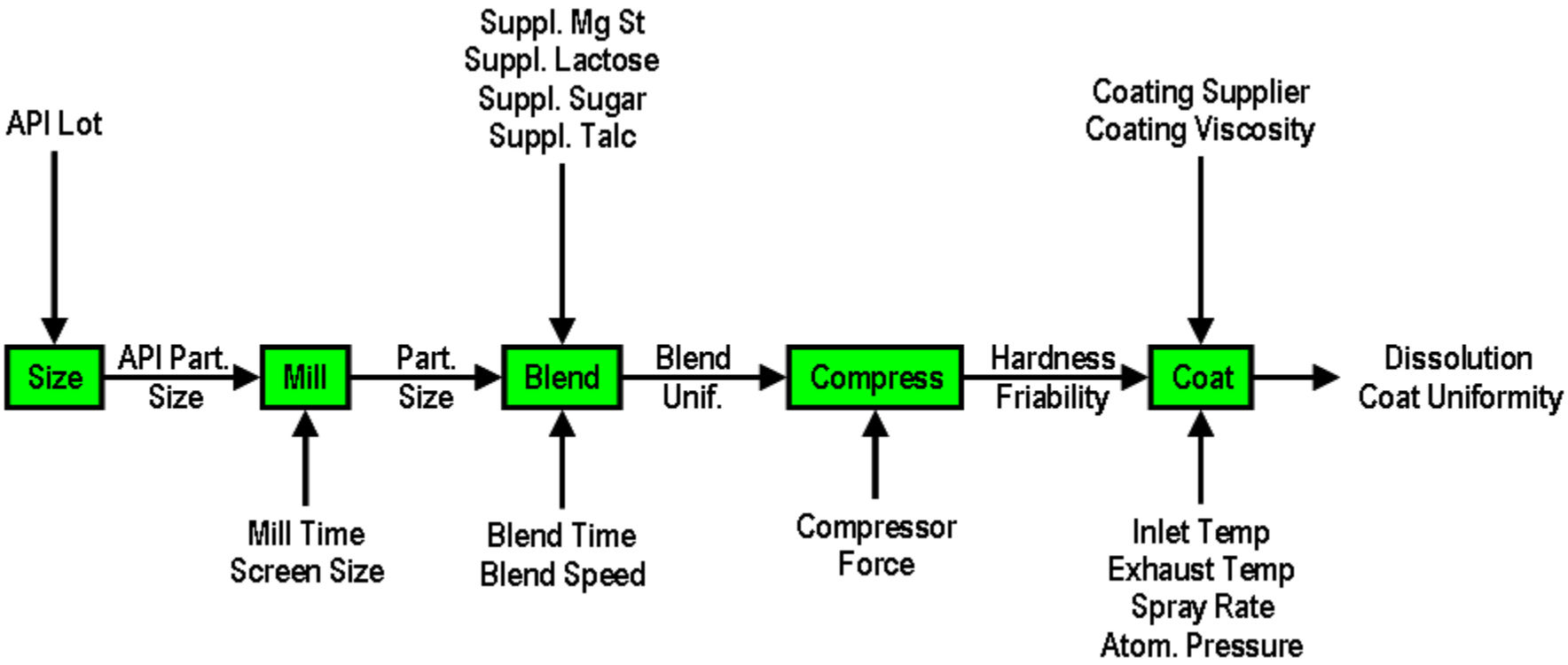
36 trial I-optimal response-surface design started as 10-factor DSD and was then augmented with 12 more trials in 6 most important factors

If generally running 3, 4 or 5-factor fractional-factorial designs…

1. How many interactions are you not investigating?
2. How many more trials needed to fit curvature?
3. Consider two stages: Definitive Screening + Augmentation

CLASSIC DEFINITION OF DOE

PURPOSEFUL CONTROL OF THE INPUTS (FACTORS) IN SUCH A WAY AS TO DEDUCE THEIR RELATIONSHIPS (IF ANY) WITH THE OUTPUT (RESPONSES).



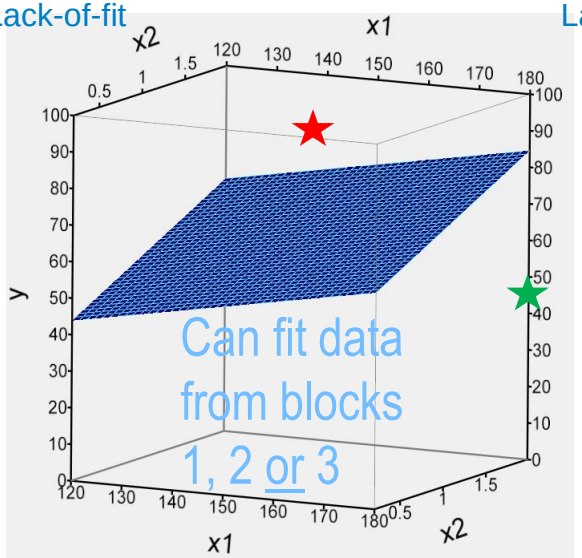
ALTERNATIVE DEFINITION OF DOE

A DOE IS THE SPECIFIC COLLECTION OF TRIALS RUN TO SUPPORT A PROPOSED MODEL.

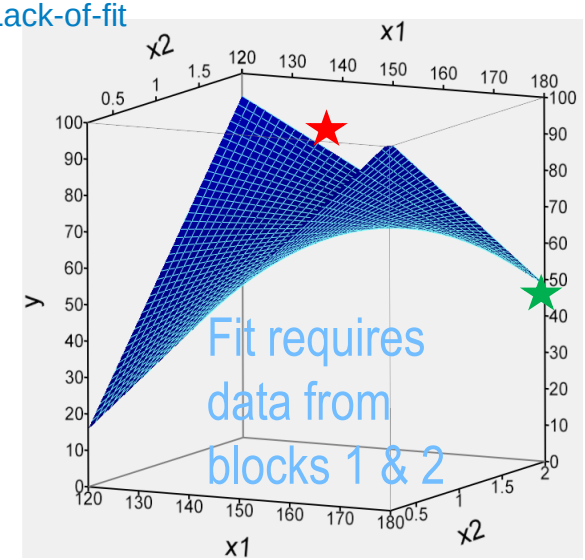
- If proposed model is ***simple***, e.g. just main effects or 1st order effects (x_1 , x_2 , x_3, etc.), the design is called a ***screening*** DOE
  - Goals include **rank factor importance** or find a "winner" quickly
  - Used with many (> 6?) factors at start of process characterization
- If the proposed model is ***more complex***, e.g. the model is 2nd order so that it includes two-way interaction terms (x_1x_2 , x_1x_3, x_2x_3, etc.) and in the case of continuous factors, squared terms (x_1^2, x_2^2, x_3^2 , etc.), the design is called a ***response-surface*** DOE
  - Goal is generally to develop a **predictive model** of the process
  - Used with a few (< 6?) factors after a screening DOE

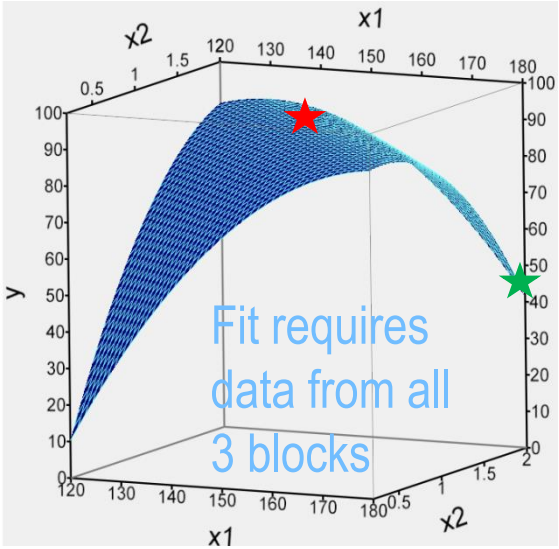
QUADRATIC MODEL NOT THAT MUCH BIGGER THAN INTERACTION MODEL

Lack-of-fit



Lack-of-fit





1st Order

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3$$

For k factors there are k main effects

For 3 factors Linear Model has 4 terms

For 6 factors Linear Model has 7 terms

For 10 factors Linear Model has 11 terms

2nd Order

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3$$

$$+ a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3$$

For k factors there are k(k-1)/2 interaction effects

For 3 factors Interaction Model has 7 terms

For 6 factors Interaction Model has 22 terms

For 10 factors Interaction Model has 56 terms

2nd Order

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3$$

$$+ a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3$$

$$+ a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2$$

For k factors there are k squared effects

For 3 factors Quadratic Model has 10 terms

For 6 factors Quadratic Model has 28 terms

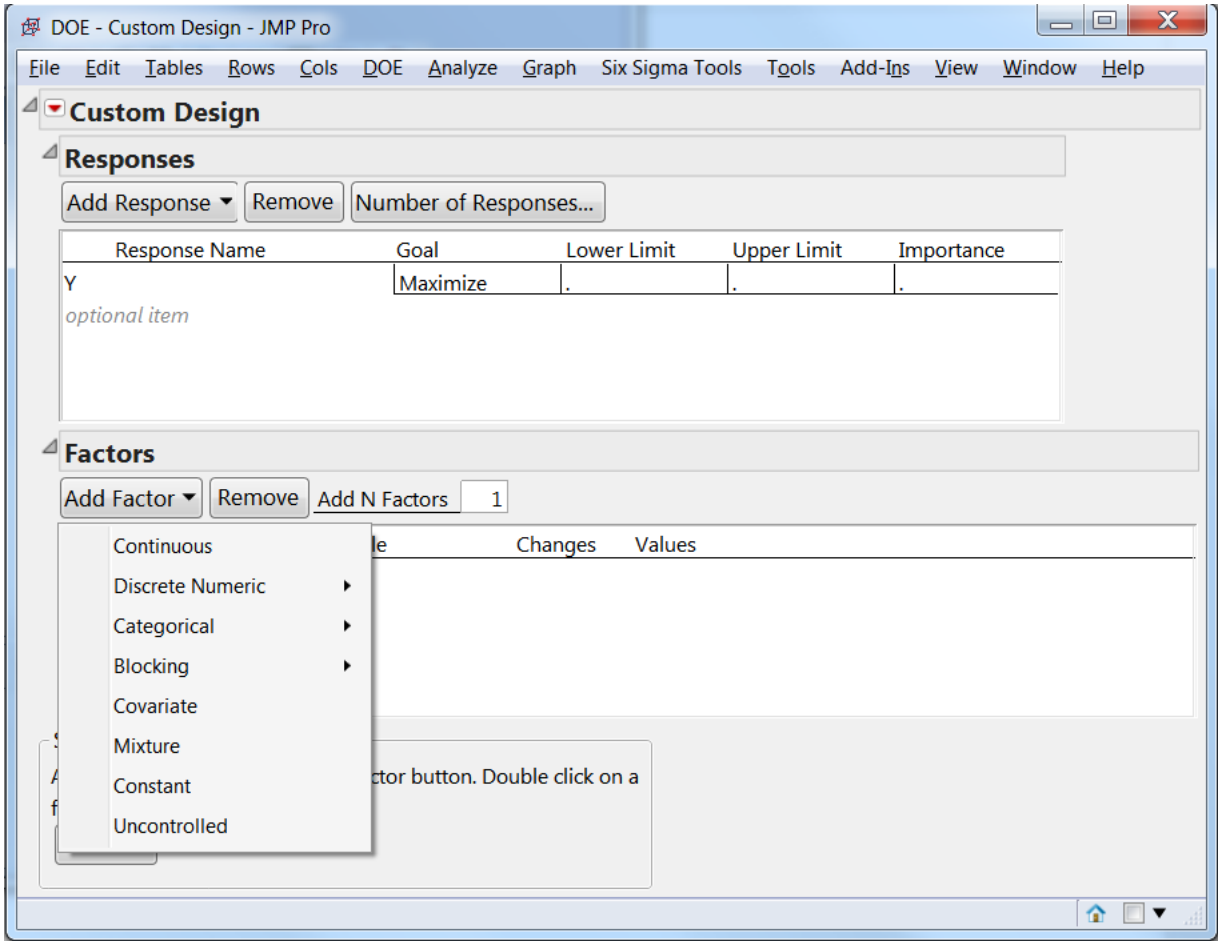
For 10 factors Quadratic Model has 66 terms

REAL-WORLD DESIGN ISSUES

How many experimenters have any of these issues?
Most of these are NOT well treated by classic DOE

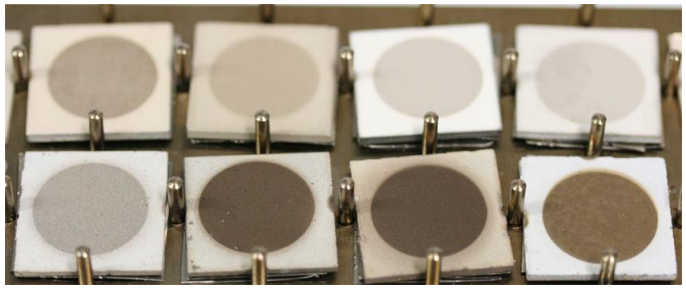
- Work with these different kinds of control variables/factors:
  - **Continuous/quantitative?** (Finely adjustable like *temperature*, *speed*, *force*)
  - **Categorical/qualitative?** (Comes in types, like material = *rubber*, *polycarbonate*, *steel* with mixed # of levels; 3 chemical agents, 4 decontaminants, 8 coupon materials…)
  - **Mixture/formulation?** (Blend different amounts of *ingredients* and the process performance is dependent on the *proportions* more than on the amounts)
  - **Blocking?** (e.g. "lots" of the same raw materials, multiple "same" machines, samples get processed in "groups" – like "eight in a tray," run tests over multiple days – i.e. variables for which there *shouldn't* be a causal effect
- Work with **combinations of these four kinds** of variables?
- Certain **combinations cannot be run**? (too costly, unsafe, breaks the process)
- Certain factors are **hard-to-change** (temperature takes a day to stabilize)
- Would like to **add onto existing trials**? (really expensive/time consuming to run)

REAL WORLD VARIABLES



CATEGORICAL FACTORS AND RESPONSES

- Agents
  - Agent 1
  - Agent 2
  - Agent 3
- Decontaminants
  - Decon 1
  - Decon 2
  - Decon 3
  - Decon 4
- Materials
  - Steel
  - Aluminum
  - Glass
  - Polycarbonate
  - CARC (Paint)
  - Viton
  - Kapton
  - Silicone



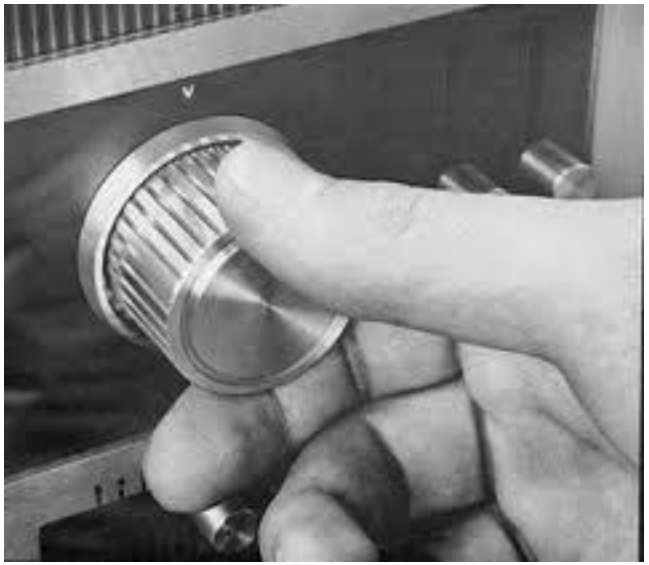
Responses

- Pass/Fail
- Yes/No
- Not Cracked/Cracked
- Safe/Caution/Unsafe
- Not Corroded/ Moderately Corroded/ Severely Corroded

CONTINUOUS FACTORS AND RESPONSES

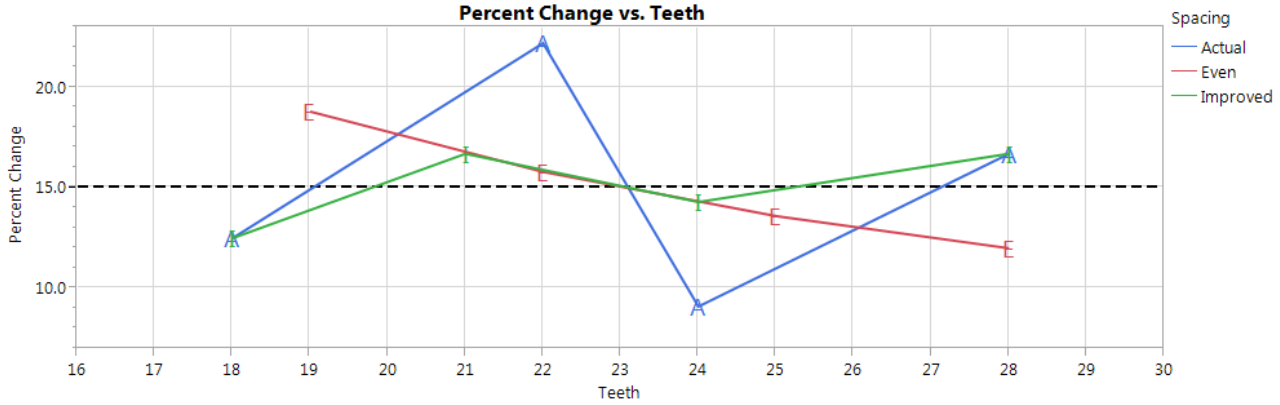
- Factors
  - Time
  - Temperature
  - Amount of Agent/Unit Area
  - Wind Speed
  - Humidity

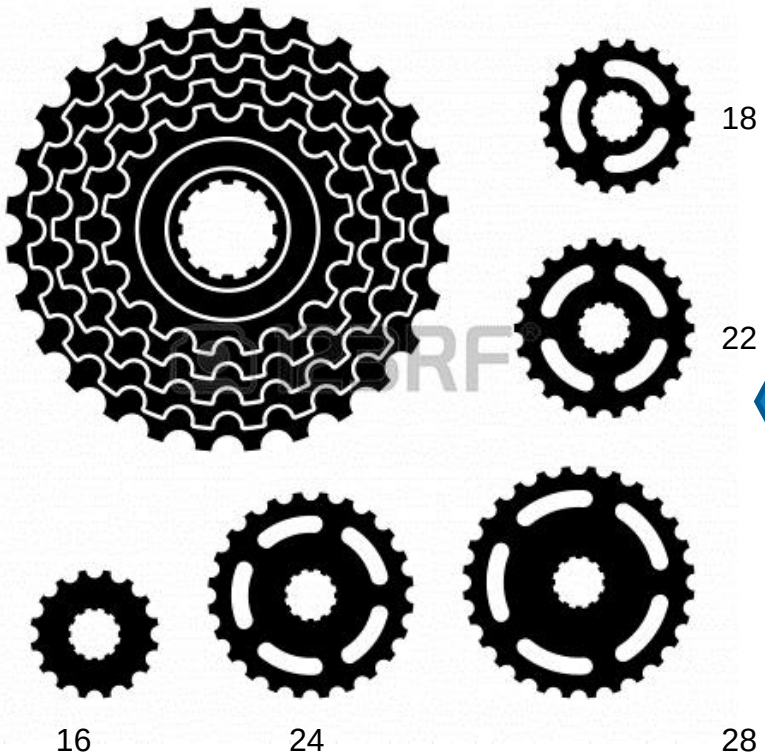
- Responses
  - Evaporation Rate
  - Absorption
  - Adsorption
  - Residual Concentration



DISCRETE NUMERIC VARIABLE

Example: Number of Teeth on Bicycle Sprockets





Evenly Spaced					
Teeth	16	19	22	25	28
Delta		3	3	3	3
% Change		18.8%	15.8%	13.6%	12.0%

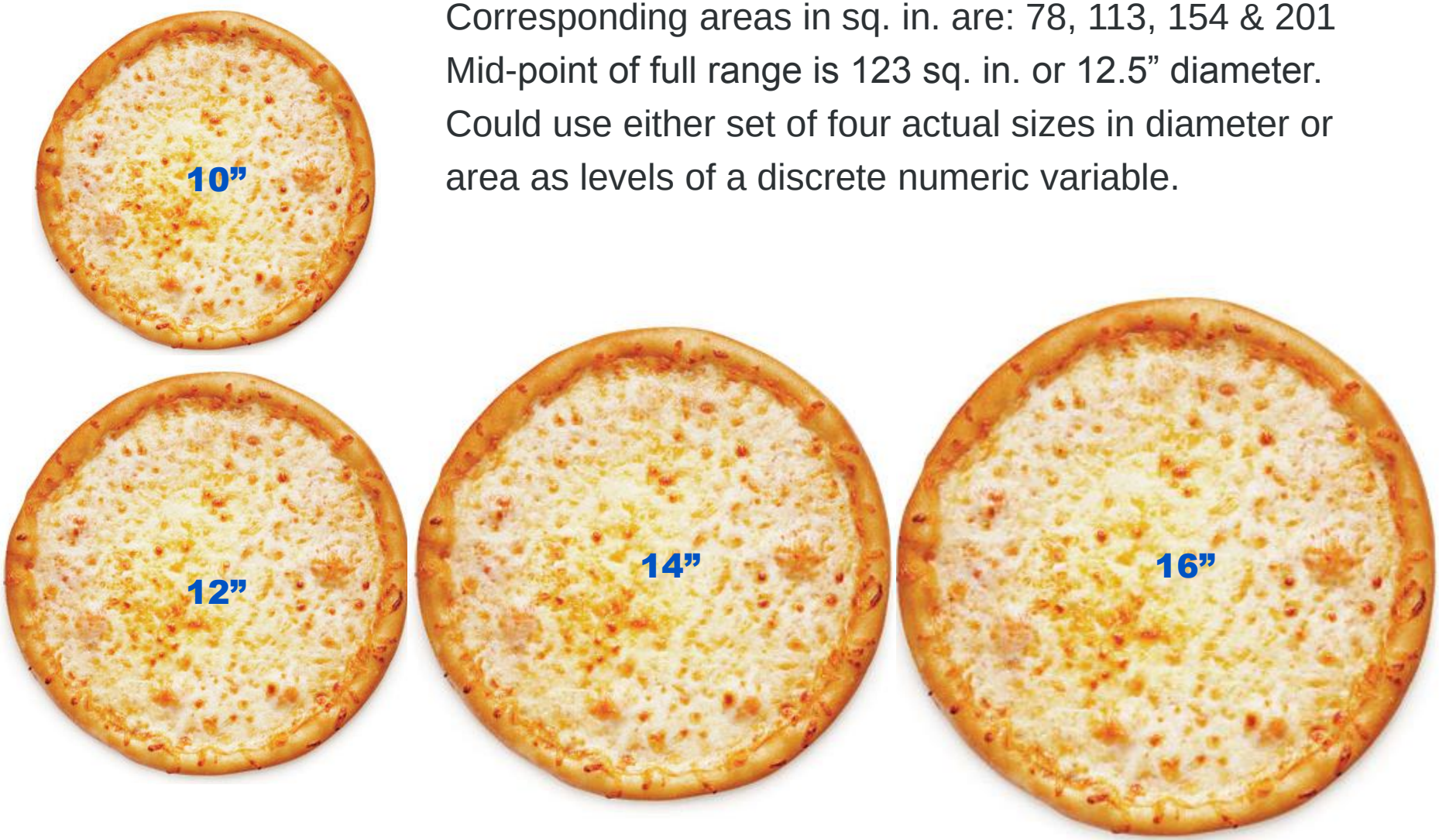
Actual Spacing					
Teeth	16	18	22	24	28
Delta		2	4	2	4
% Change		12.5%	22.2%	9.1%	16.7%

Improved Spacing					
Teeth	16	18	21	24	28
Delta		2	3	3	4
% Change		12.5%	16.7%	14.3%	16.7%

DISCRETE NUMERIC VARIABLE

Only make four sizes of pizza 10”, 12”, 14” & 16”
Mid-point of full range is 13” diameter.

Corresponding areas in sq. in. are: 78, 113, 154 & 201
Mid-point of full range is 123 sq. in. or 12.5” diameter.
Could use either set of four actual sizes in diameter or area as levels of a discrete numeric variable.



SIMPLE MIXTURE – MAKING SALAD DRESSING

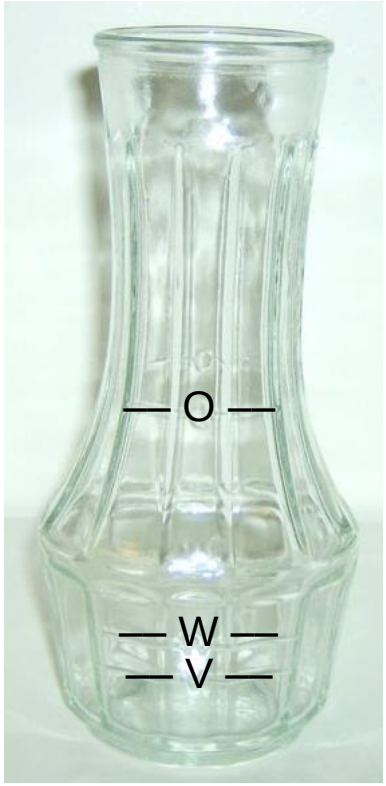
- Relative ***proportions*** of factors or components is more important than actual quantity
- Three liquid components - Oil, Water, and Vinegar
- 8 oz. in Cruet vs. 4 gal. in Jug

5 oz.	"O"	320 oz.	5/8
1 oz.	"W"	64 oz.	1/8
2 oz.	"V"	128 oz.	1/4

- To study these mixture components in a DOE use ranges that are proportions:

O:	0.500 to 0.750	(½ to ¾)
W:	0.000 to 0.250	(0 to ¼)
V:	0.125 to 0.375	(⅛ to ⅜)

- Sum of proportions ***constrained*** to equal 1.



100.0%

37.5%

25.0%

0%

1 = O + W + V so therefore…

W = 1 – (O + V), O = 1 – (V + W), & V = 1 – (O + W)

BLOCKING FACTOR LIKE "DAY" OR "BATCH"

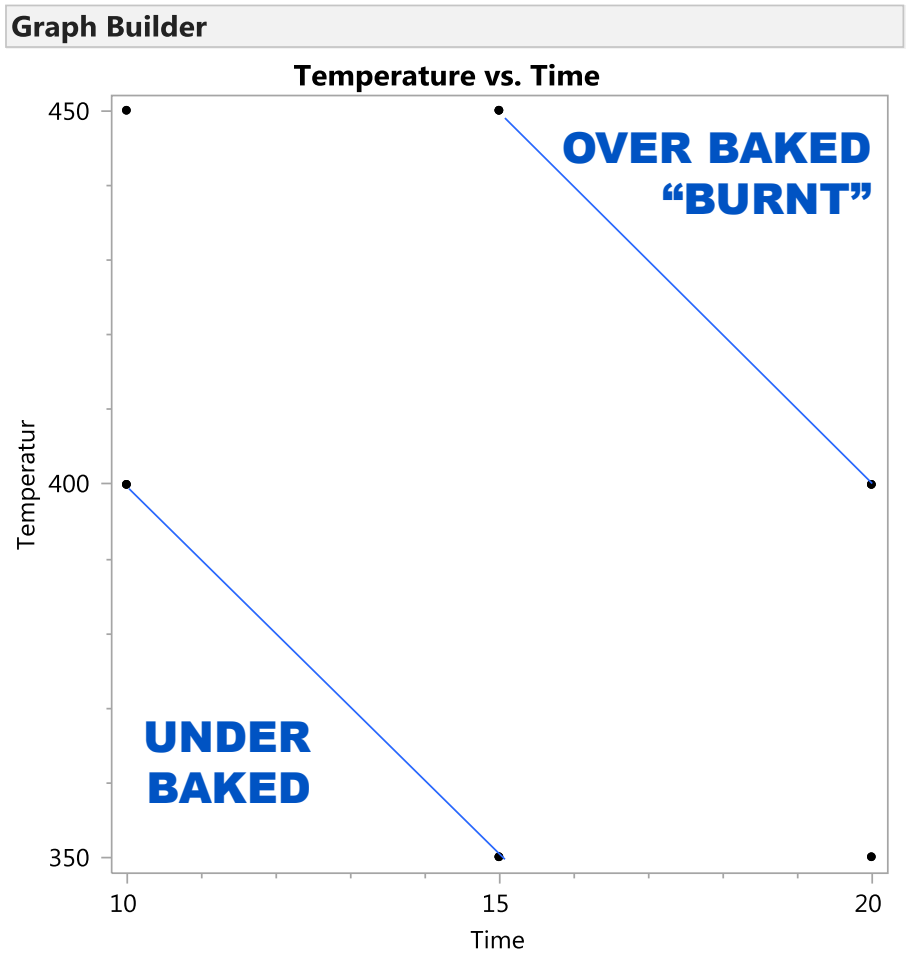
MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
				
Sunny Hi: 42 F Lo: 25 F	Sunny Hi: 42 F Lo: 33 F	Sunny Hi: 49 F Lo: 33 F	Showers and mild Hi: 52 F Lo: 30 F	Sunny and pleasant Hi: 57 F Lo: 39 F

- A design run over 5 days that is sensitive to humidity might SHIFT on Thursday
  - But what if because of the rain the tester from days 1, 2, 3 & 5 didn't make it to work?
  - What if that day the power went out briefly? Or, dept. meeting "paused" the work? Or…?
- The block variable doesn't tell you the cause of the effect - just that a shift has been detected among blocks.
- Hoping block variable has no effect. If it does then how can we reliably predict other blocks? If significant it probably means we are missing a factor.
- The only way to be sure that no "unknown" factor has crept into the experiment, is to test for it - and "blocking" your design is not expensive.
- Block variable is a categorical factor having only 1-way effects (no interactions)

TIME AND TEMPERATURE CONSTRAINTS UNCODED

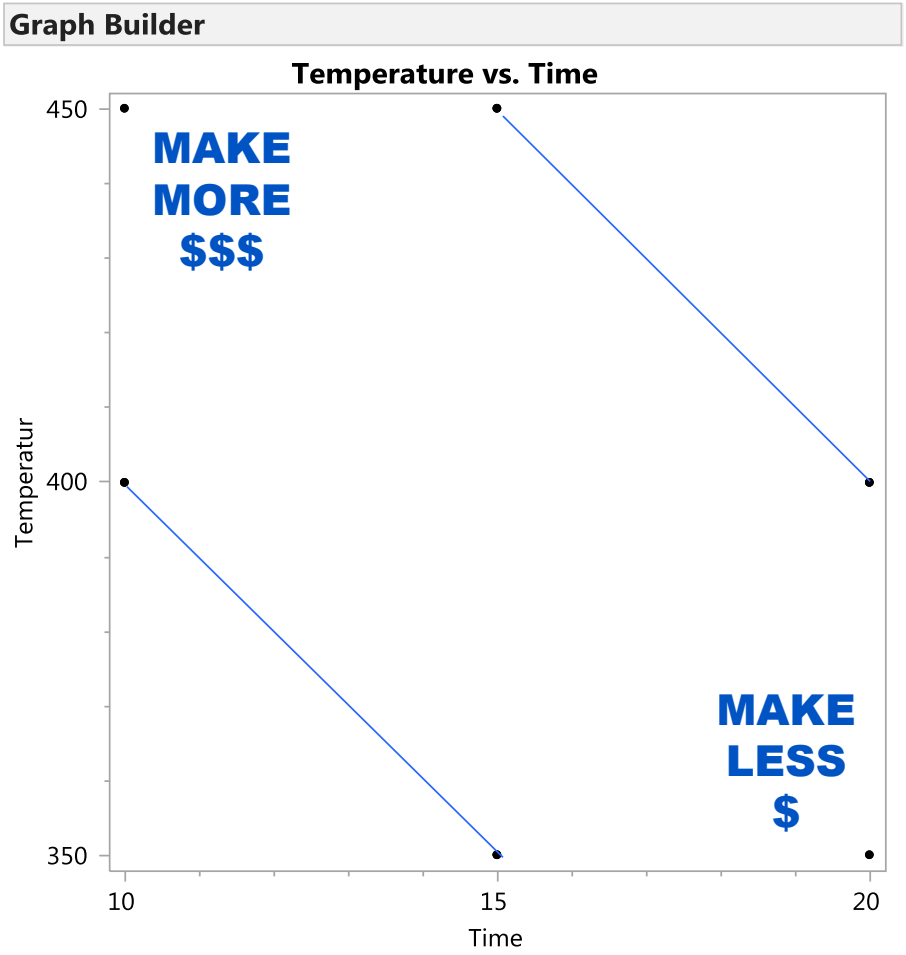
- Time: 10 20
- Temp: 350 450
- Pizza Size: 10, 12, 14 & 16
- Pizza Type
  - Cheese
  - Veggies
  - Meats
- Batch (Size = 5)

- Hi + Hi = “Burnt”
- Lo + Lo = “Not Done”

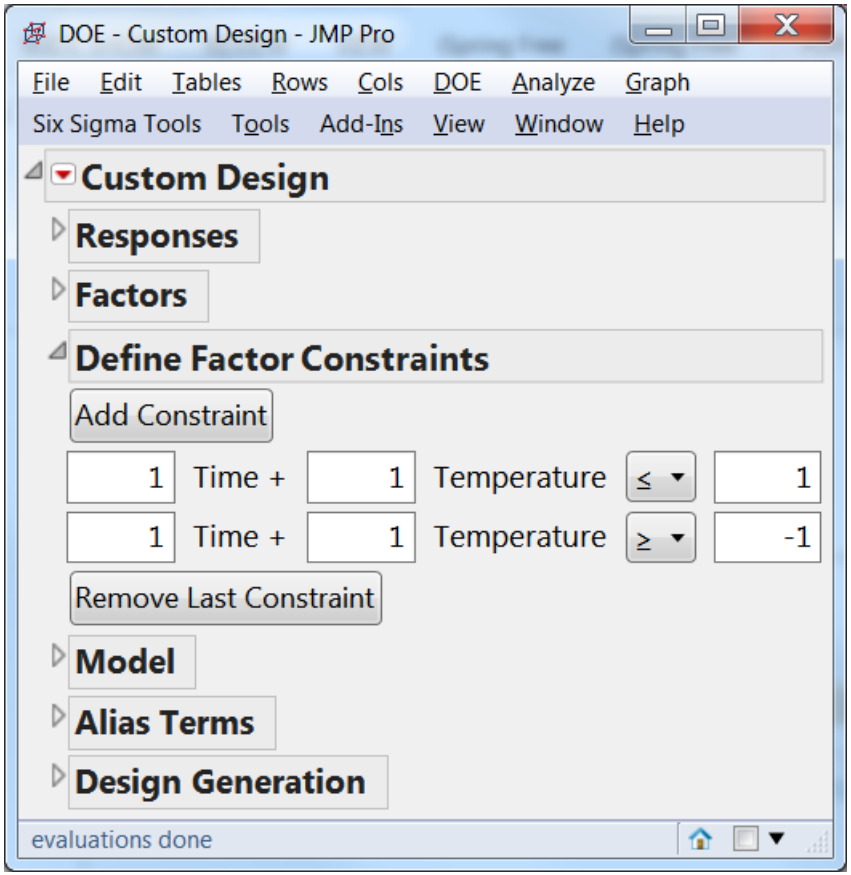


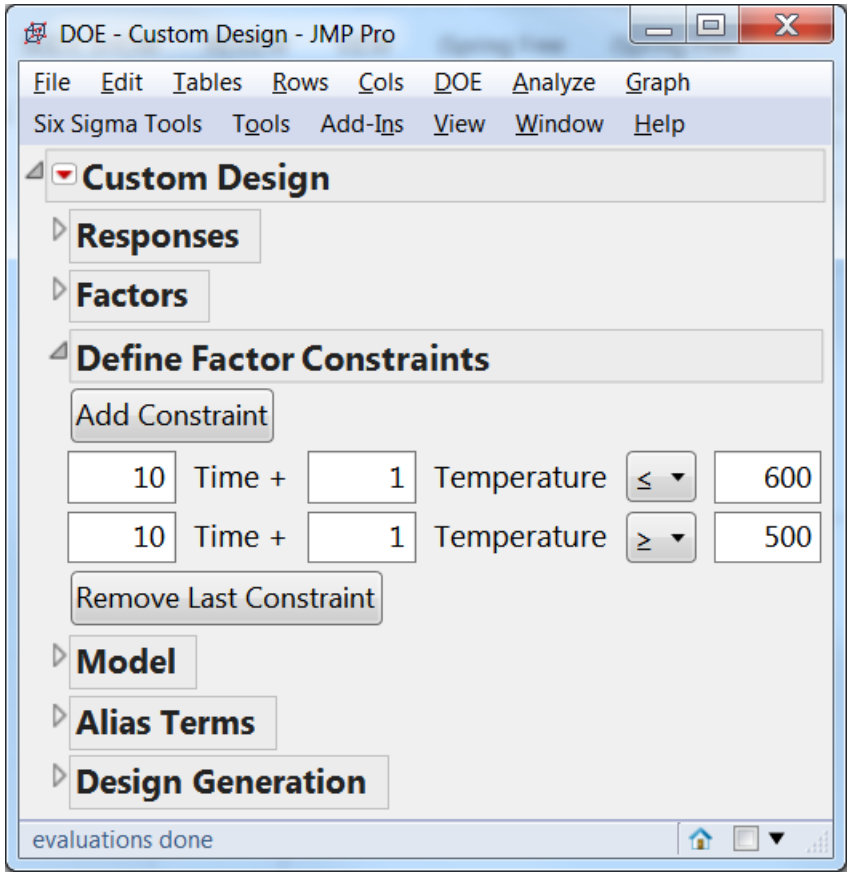
TIME AND TEMPERATURE CONSTRAINTS UNCODED

- Shorter times means more product produced per hour
- Make most of our money in a few hours each evening



TIME AND TEMPERATURE CONSTRAINTS CODED VS. UNCODED





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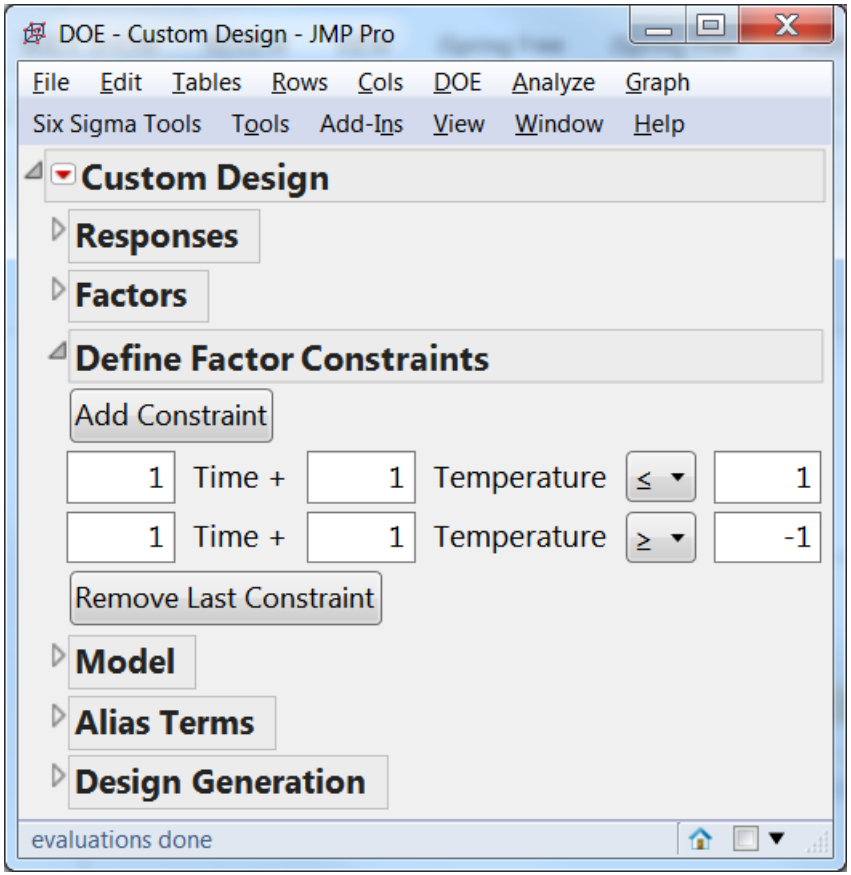
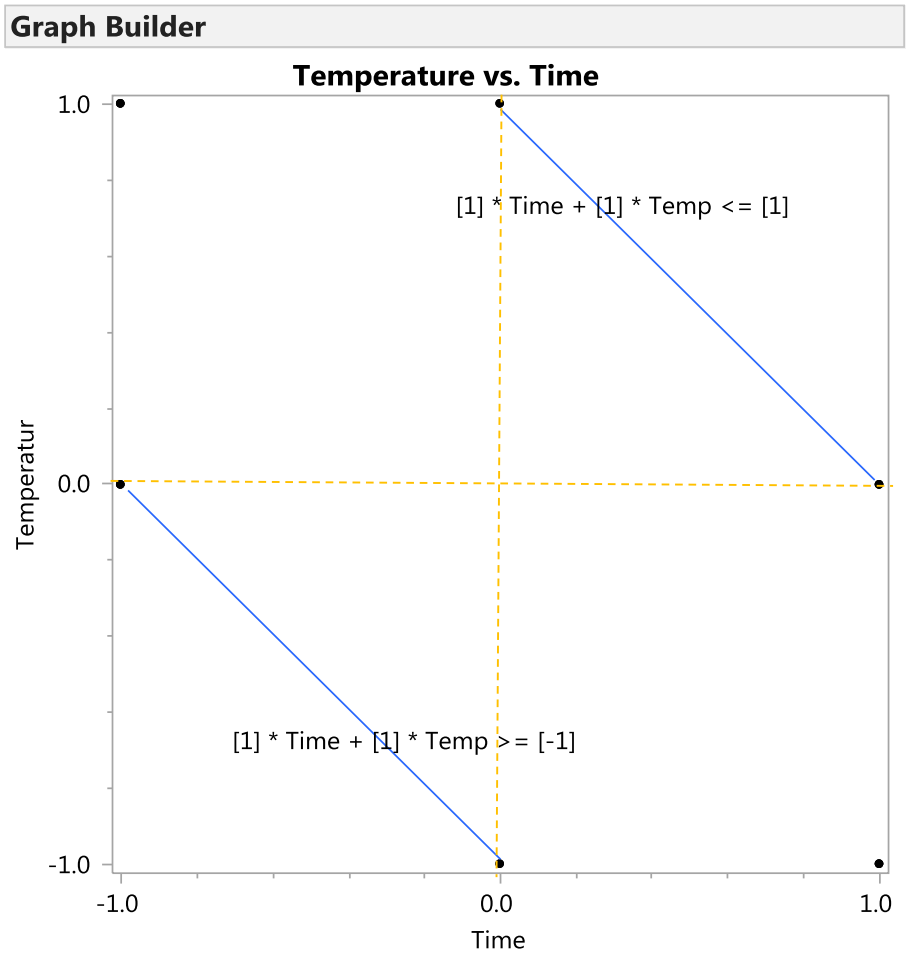




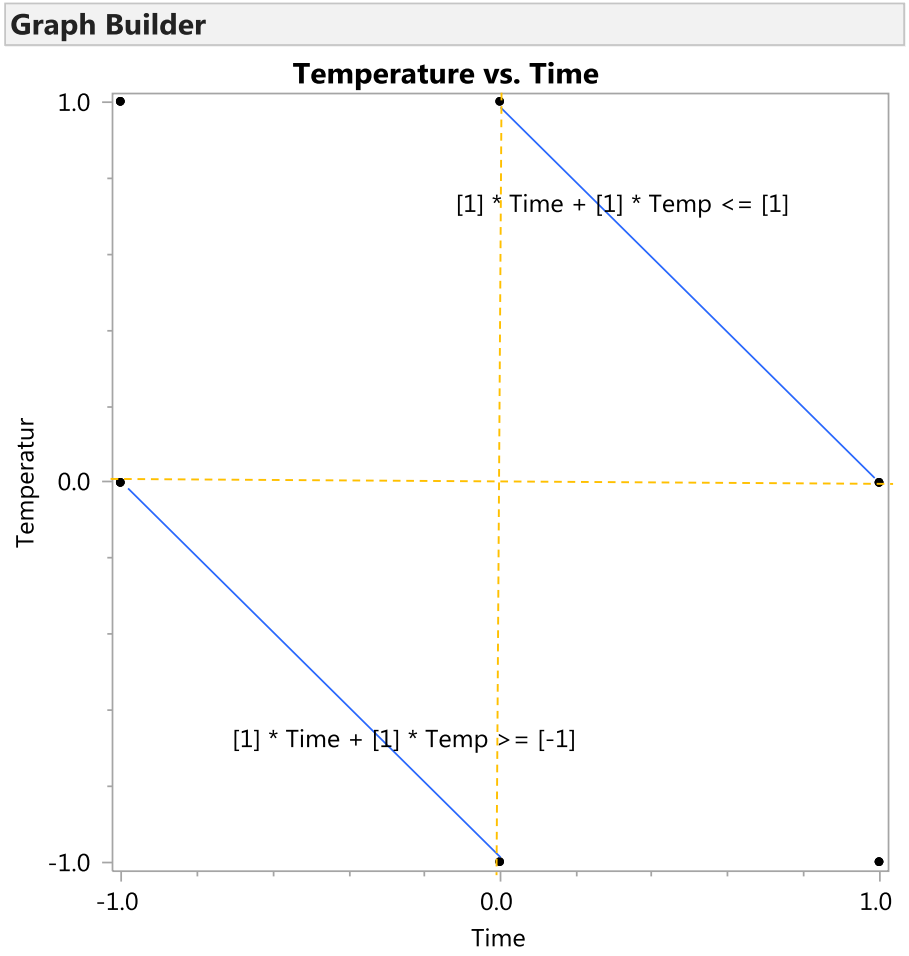
TIME AND TEMPERATURE CONSTRAINTS CODED

- y = mx + b
- Temp = m*Time + b
- [1]*Temp = [-1]*Time + [1]
- [1]*Time + [1]*Temp = [1]

- [1]*Time + [1]*Temp <= [1]

- y = mx + b
- Temp = m*Time + b
- [1]*Temp = [-1]*Time + [-1]
- [1]*Time + [1]*Temp = [-1]

- [1]*Time + [1]*Temp >= [-1]



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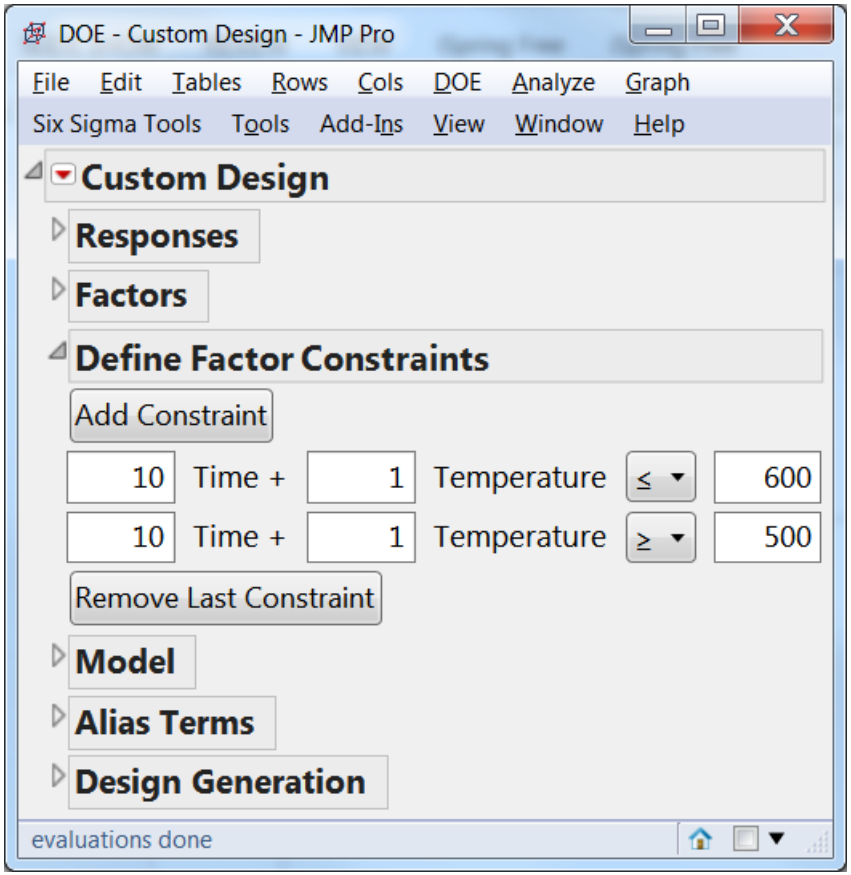
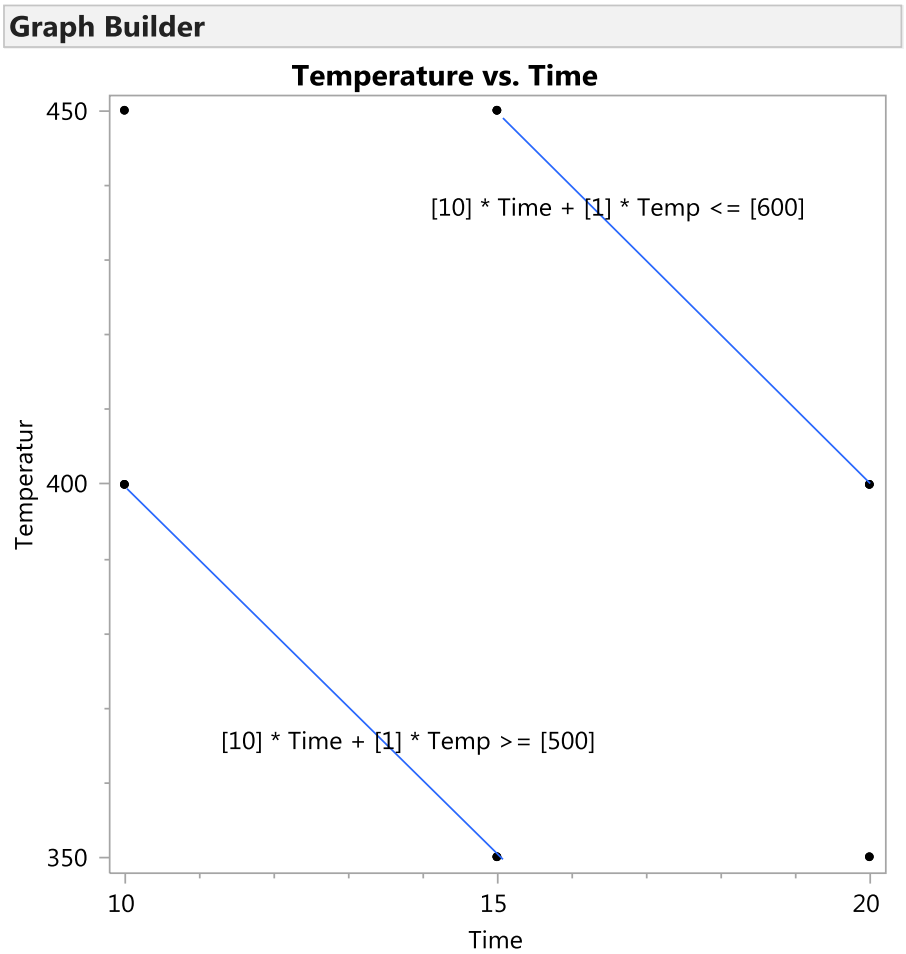
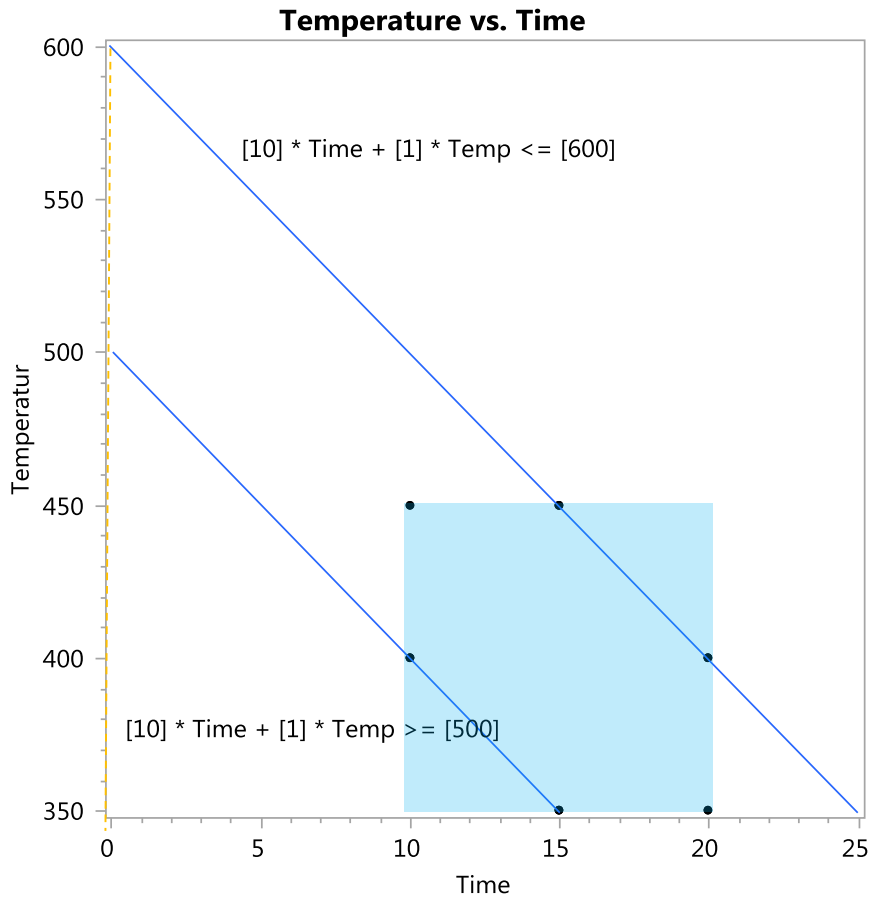


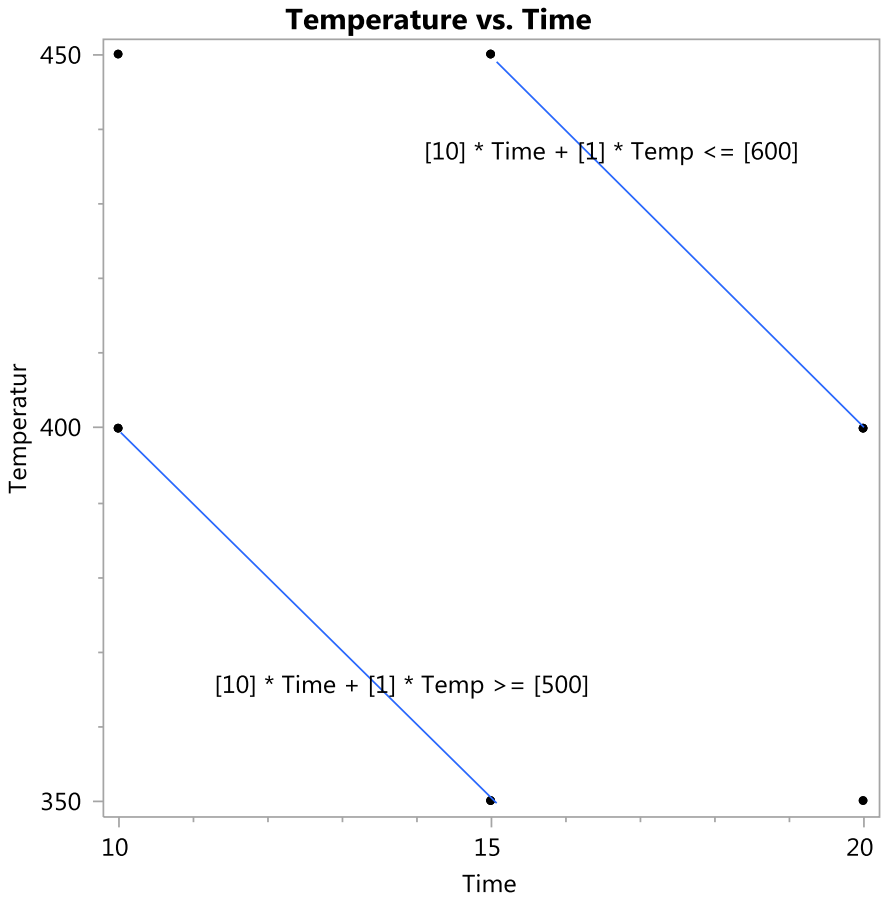


TIME AND TEMPERATURE CONSTRAINTS UNCODED

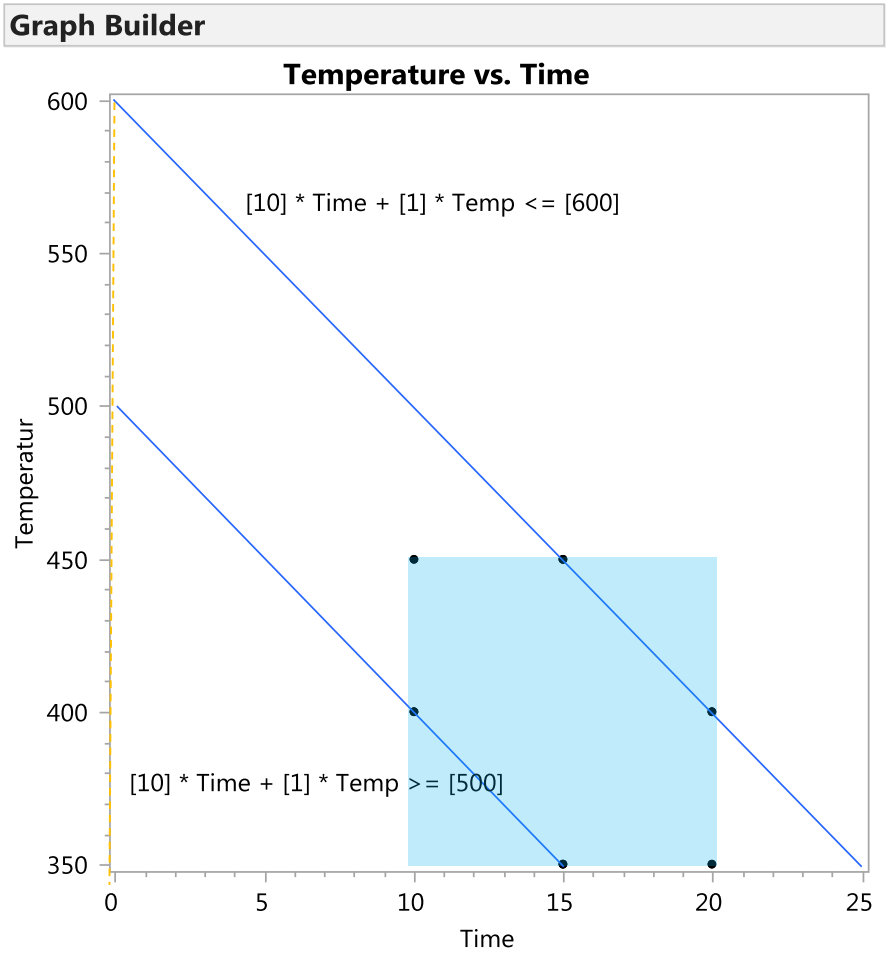
Graph Builder



Graph Builder



TIME AND TEMPERATURE CONSTRAINTS UNCODED



- y = mx + b
- Temp = m*Time + b
- [1]*Temp = [-10]*Time + [600]
- [10]*Time + [1]*Temp = [600]

- [10]*Time + [1]*Temp <= [600]

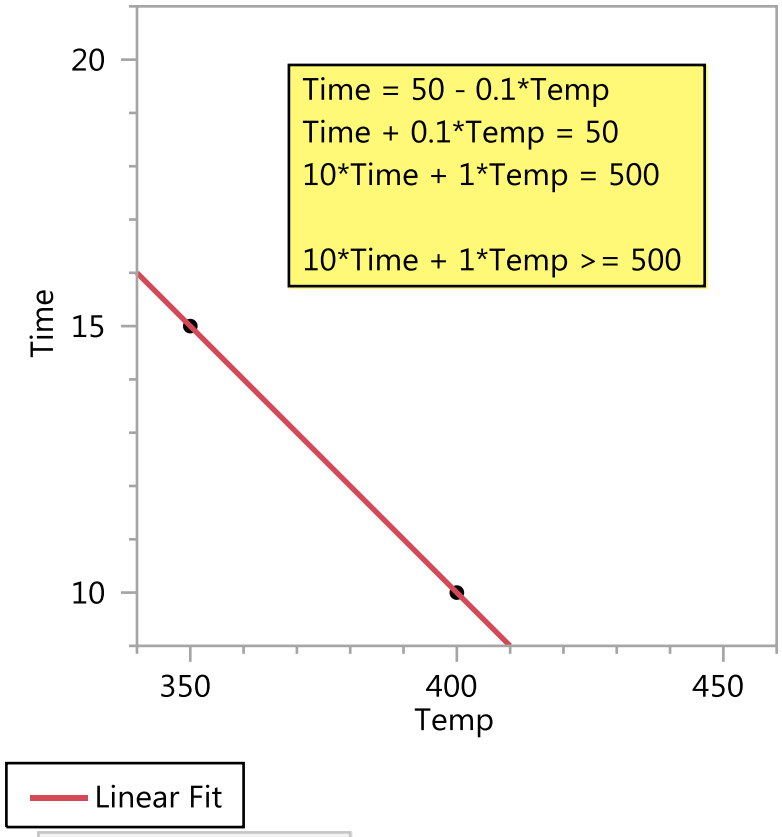
- y = mx + b
- Temp = m*Time + b
- [1]*Temp = [-10]*Time + [500]
- [10]*Time + [1]*Temp = [500]

- [10]*Time + [1]*Temp >= [500]

TIME AND TEMPERATURE CONSTRAINTS UNCODED

	Time	Temp	Constraint Location
1	15	450	Upper
2	20	400	Upper
3	15	350	Lower
4	10	400	Lower

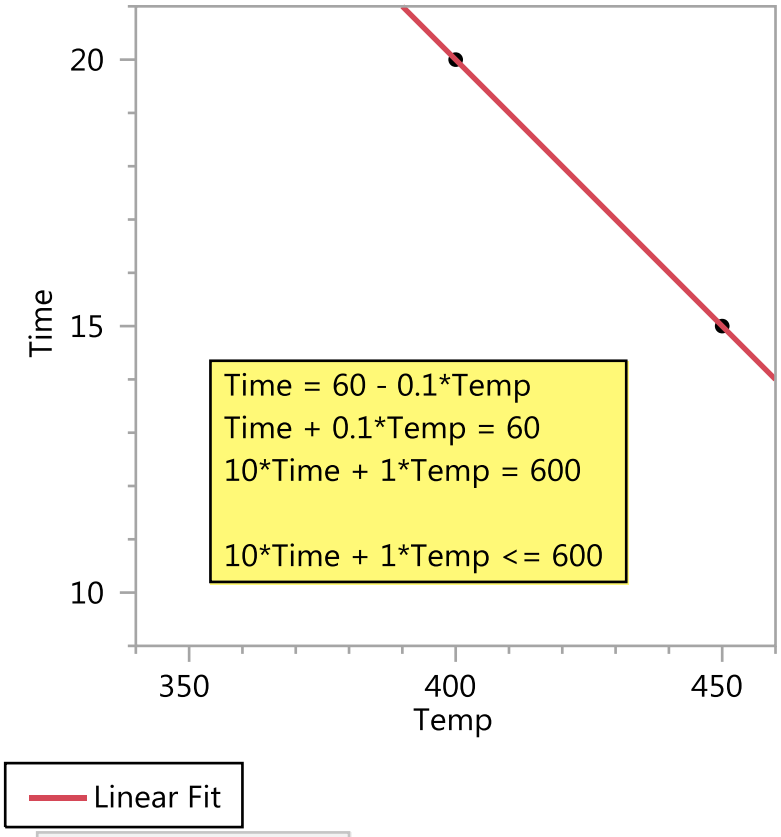
Bivariate Fit of Time By Temp Constraint Location=Lower



Linear Fit

Time = 50 - 0.1*Temp

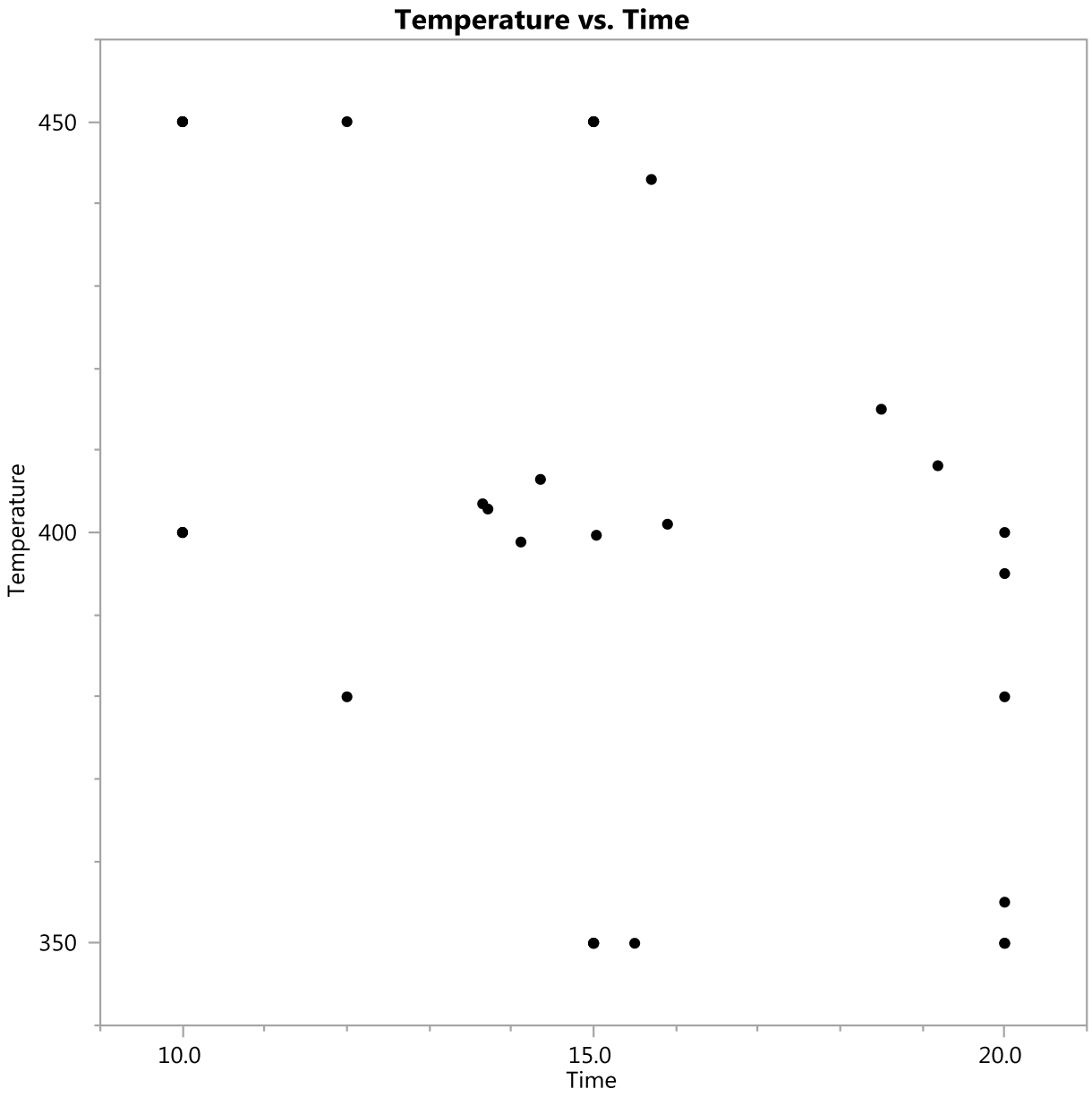
Bivariate Fit of Time By Temp Constraint Location=Upper



Linear Fit

Time = 60 - 0.1*Temp

FINAL DESIGN SHOWING CONSTRAINED REGIONS



FINAL DESIGN SHOWING CONSTRAINED REGIONS