Structural Equation Modeling: Path Analysis and Structural Regression

JMP Academic Webinar

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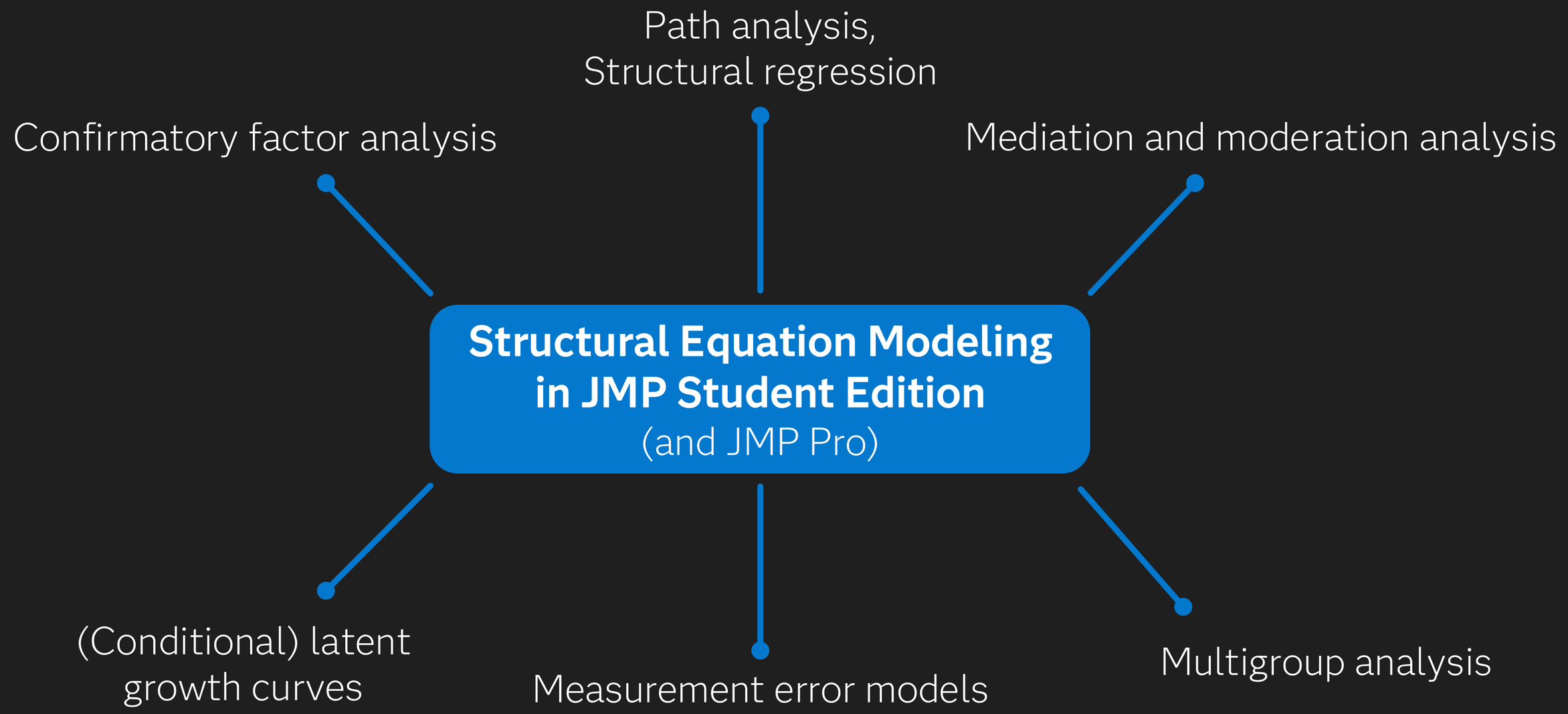
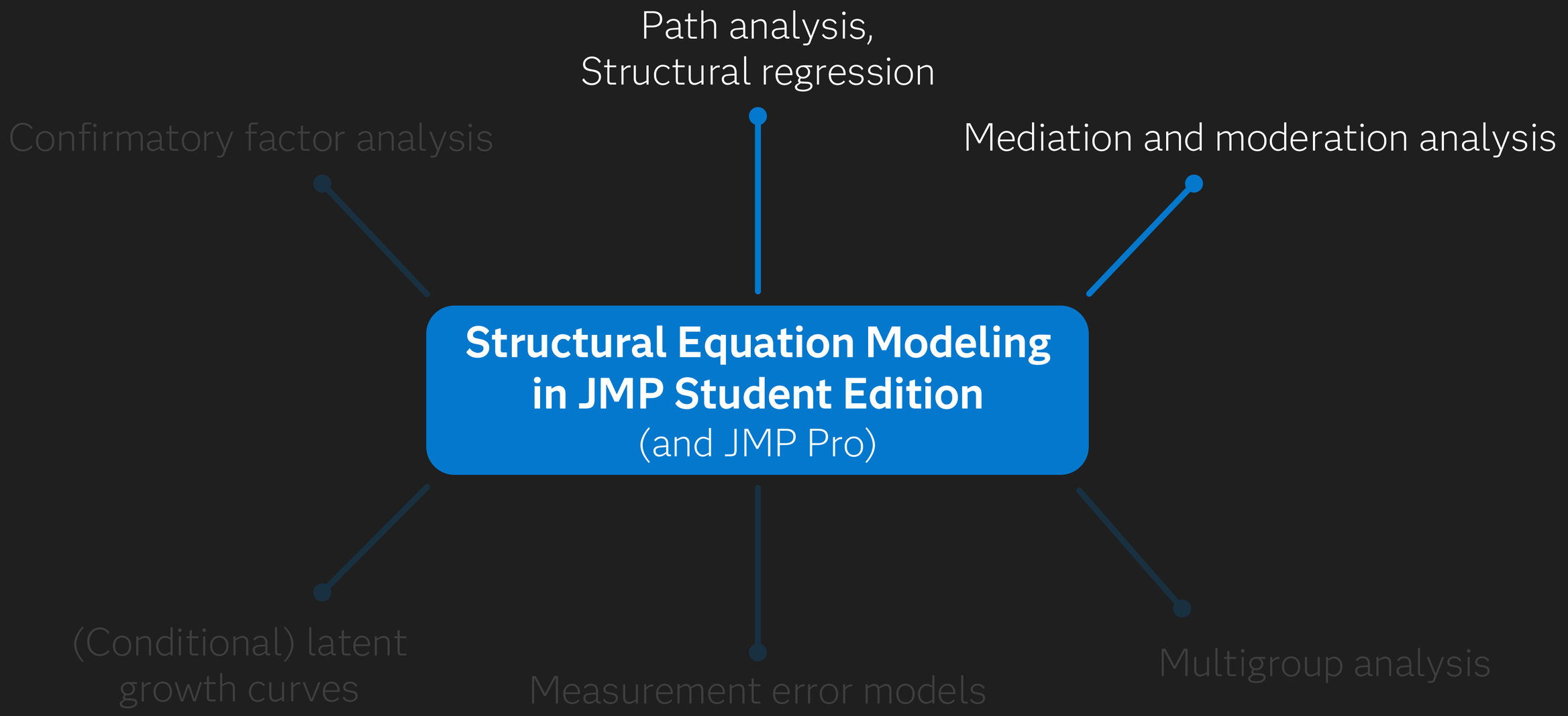
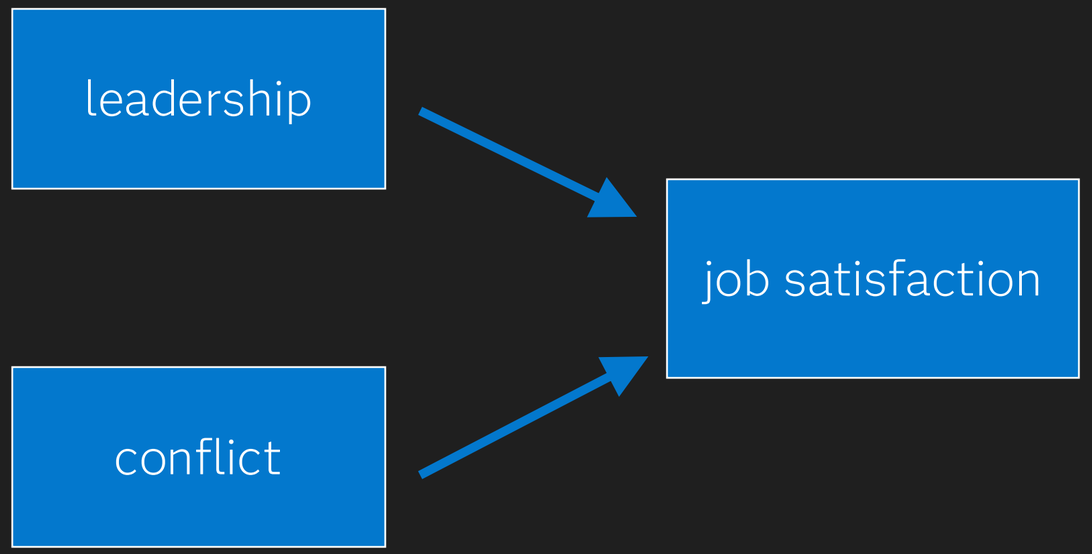
JMP STATISTICAL DISCOVERY





How do leadership and conflict influence job satisfaction?

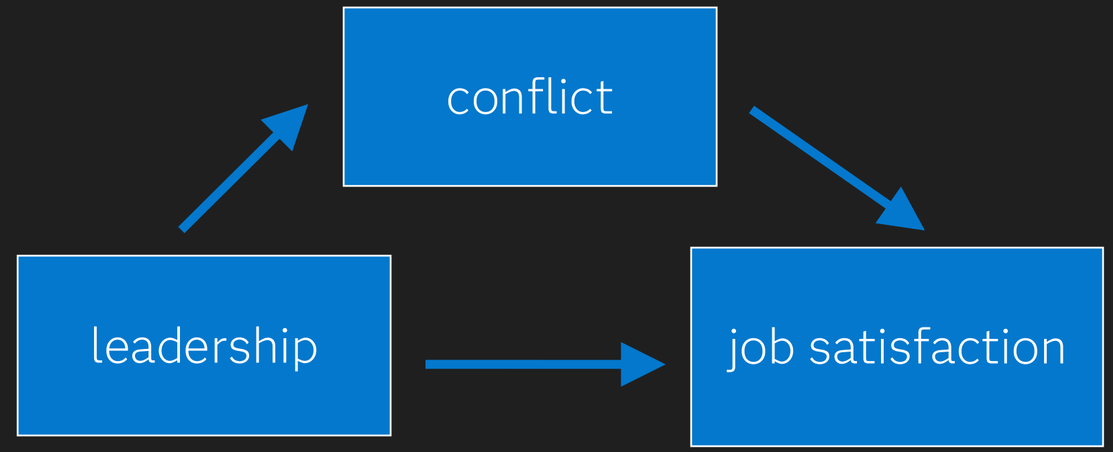
Theory #1



Answer: Multiple Regression

$$Satisfaction = \alpha + \beta_1 * Leadership + \beta_2 * Conflict$$

Theory #2



~~Answer: Multiple Regression~~

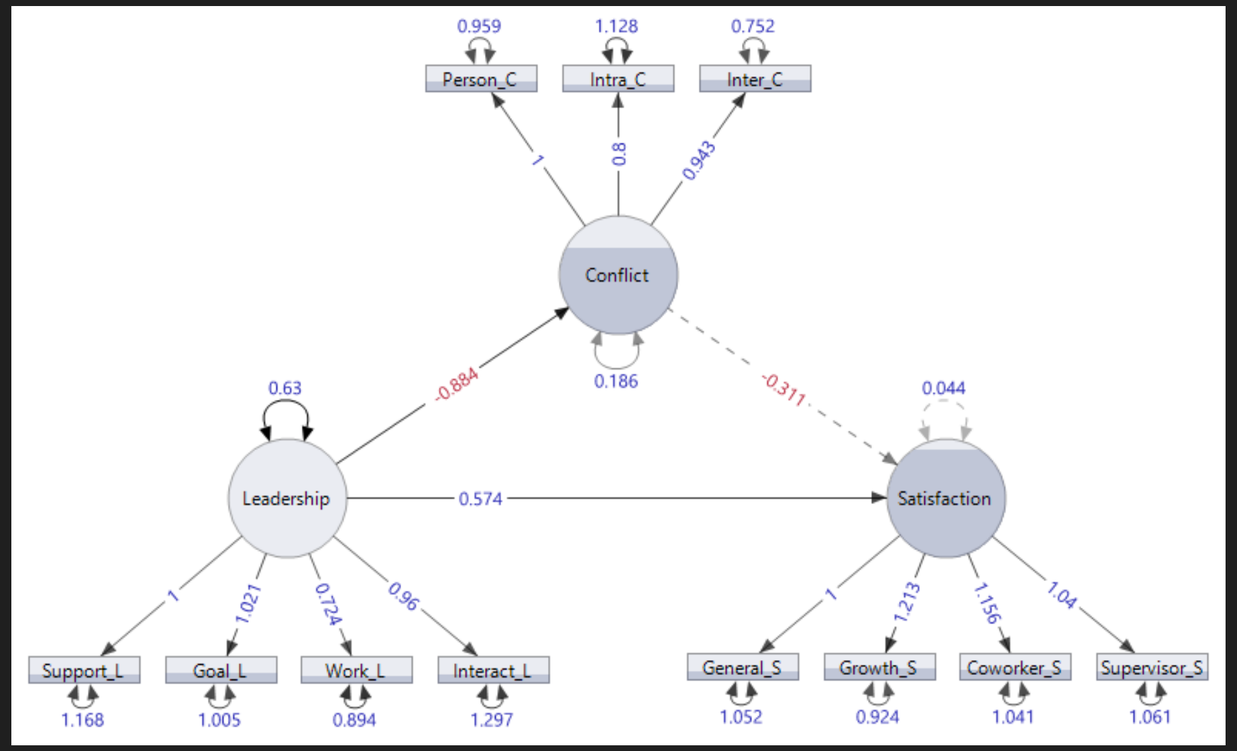
Answer: Structural Equation Modeling

$$Conflict = \alpha_c + \beta_{1c} * Leadership$$

$$Satisfaction = \alpha_s + \beta_{1s} * Leadership + \beta_{2s} * Conflict$$

Path analysis and structural regression

Testing explanatory theories about complex relationships between variables



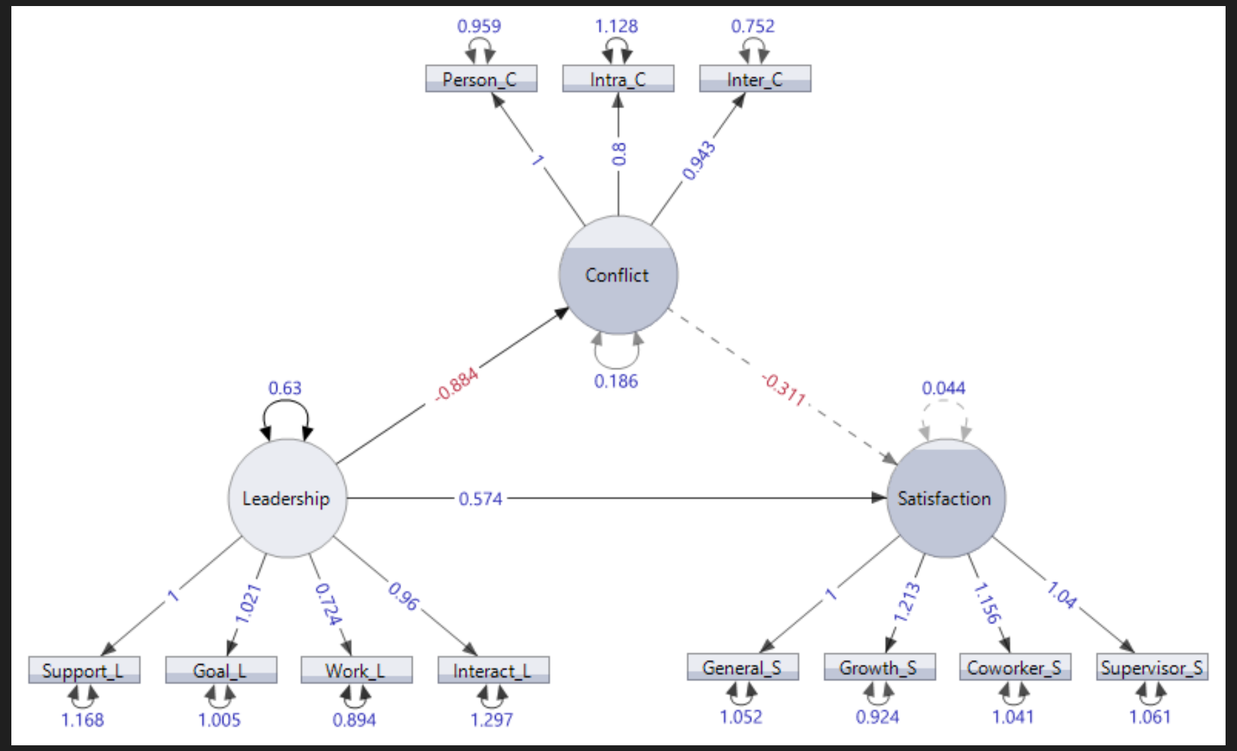
Model both manifest (observed) and latent (unobserved) variables

A single variable can both predict, and be predicted by, other variables

Fit and compare multiple competing theoretical models

Path analysis and structural regression

Testing explanatory theories about relationships between variables



Path diagram elements

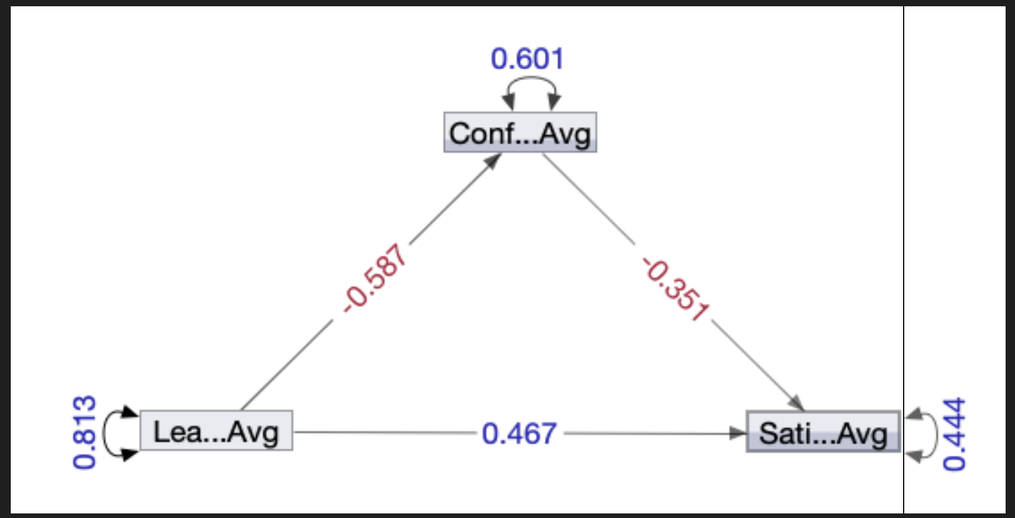
observed variable

latent variable

variance or covariance

regression or loading

Path model for mediation analysis



"The effect of leadership on satisfaction is mediated by conflict"

$$Conflict_i = \tau_1 + \beta_{1c} * Leadership_i + \varepsilon_{ci}$$

$$Satisfaction_i = \tau_2 + \beta_{1s} * Leadership_i + \beta_{2s} * Conflict_i + \varepsilon_{si}$$

$$Leadership_i = \tau_3 + \varepsilon_{li}$$

Parameter Estimates

Means/Intercepts	Estimate
Constant → Leadership_Avg	5.9425
Constant → Conflict_Avg	5.6235364
Constant → Satisfaction_Avg	3.7076041

Regressions	Estimate
Leadership_Avg → Satisfaction_Avg	0.4668926
Conflict_Avg → Satisfaction_Avg	-0.351107
Leadership_Avg → Conflict_Avg	-0.587049

Variances	Estimate
Leadership_Avg ↔ Leadership_Avg	0.8129438
Conflict_Avg ↔ Conflict_Avg	0.6010578
Satisfaction_Avg ↔ Satisfaction_Avg	0.4436317

Indirect Effects

Predictor	Outcome	Estimate
Leadership_Avg	Satisfaction_Avg	0.2061168

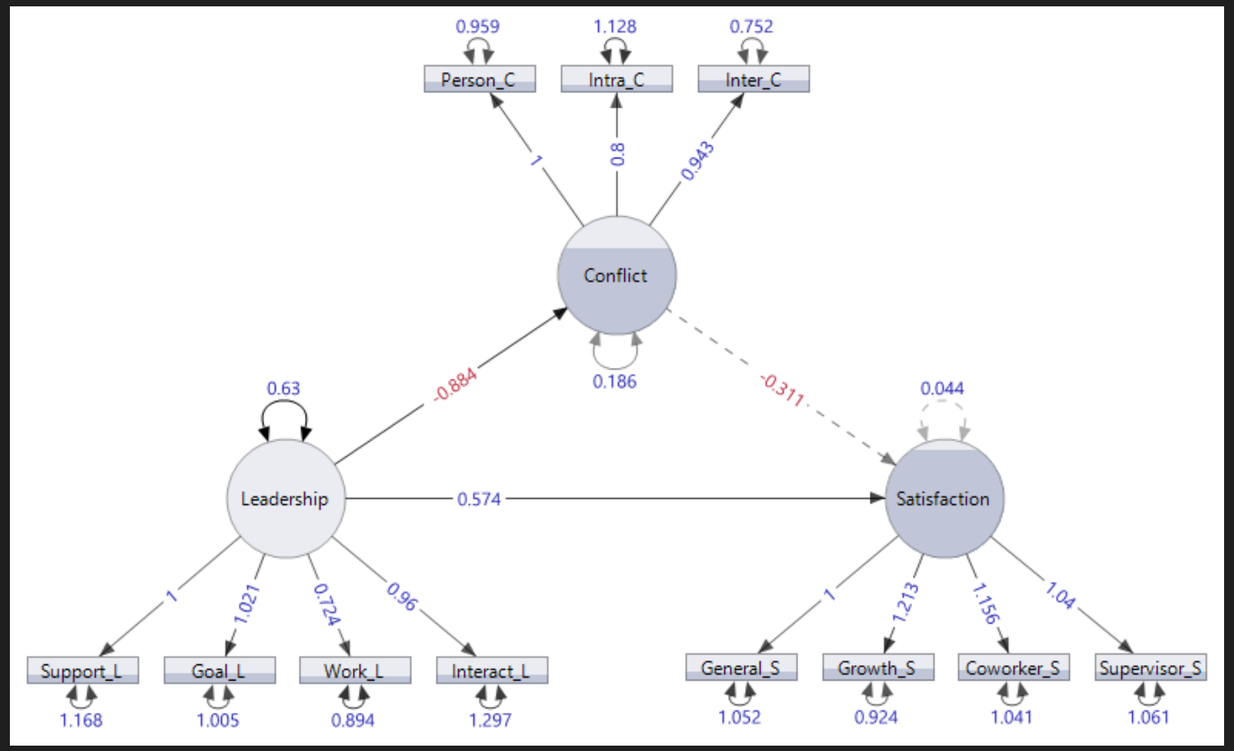
Total Effects

Predictor	Outcome	Estimate
Leadership_Avg	Conflict_Avg	-0.587049
Leadership_Avg	Satisfaction_Avg	0.6730094
Conflict_Avg	Satisfaction_Avg	-0.351107

$$\beta_{1indirect} = \beta_{1c} * \beta_{1s}$$

$$\beta_{1total} = \beta_{1indirect} + \beta_{1s}$$

Structural regression



Like path analysis, but incorporating latent factors.

Observed variables often called “indicators”. Latent factors often called “constructs”.

Precede with confirmatory factor analysis to validate measurement model.

“The effect of leadership* on satisfaction* is mediated through conflict* ”

* latent factor measured via multiple survey items

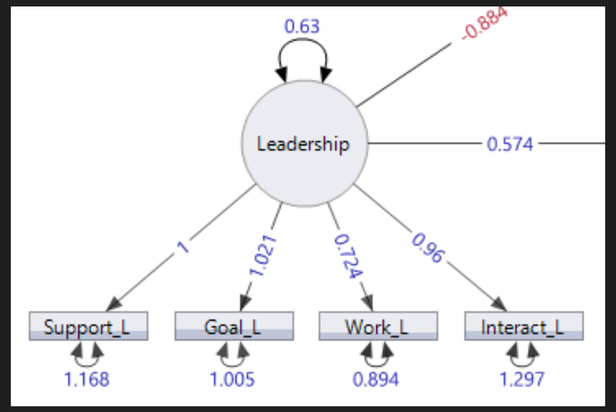
Structural regression

Notes on factor loadings

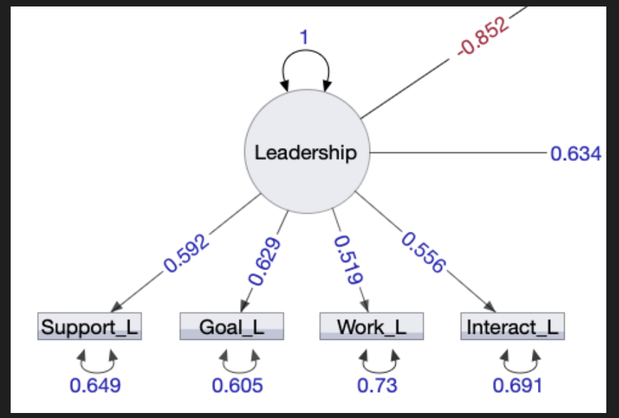
Unstandardized loadings (default) interpreted as a regression slope

Standardized loadings can be interpreted as correlation between factor and observed variable (as in exploratory factor analysis)

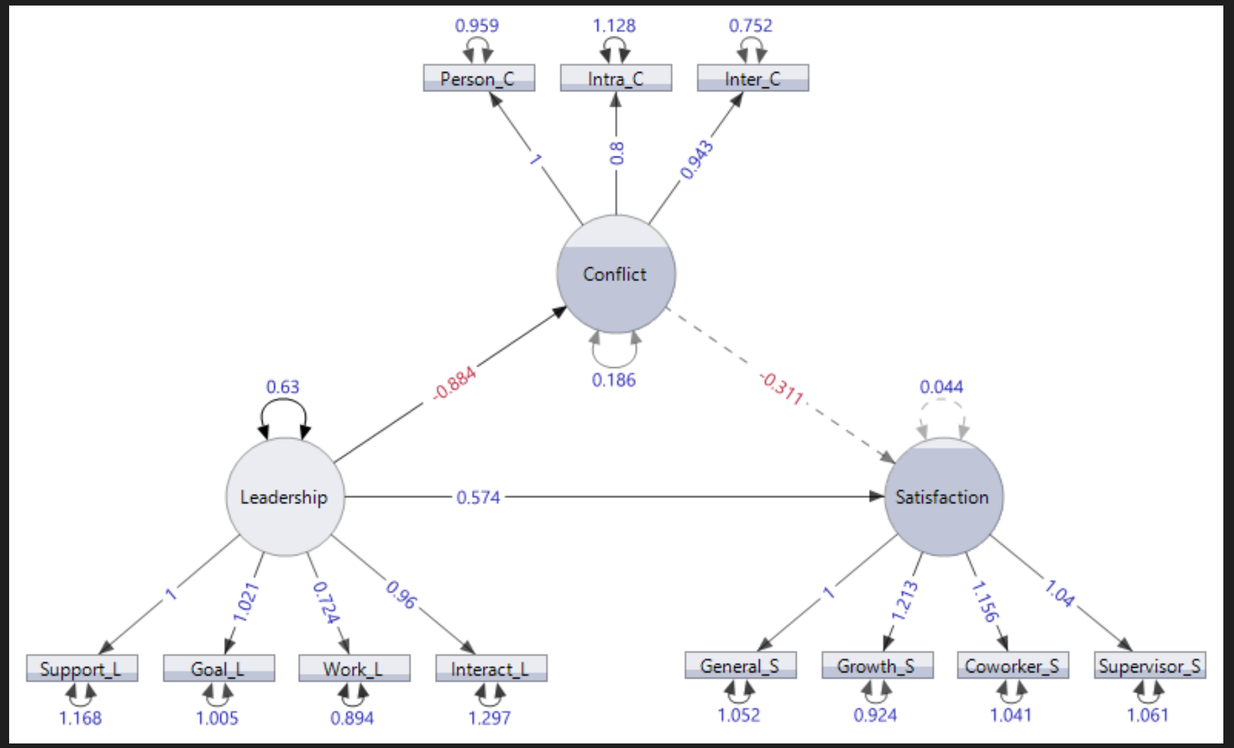
unstandardized



standardized



Structural regression



Loadings	Estimate
Leadership → Support_L	1
Leadership → Goal_L	1.0211907
Leadership → Work_L	0.7236143
Leadership → Interact_L	0.9598656
Conflict → Person_C	1
Conflict → Intra_C	0.8002214
Conflict → Inter_C	0.9433541
Satisfaction → General_S	1
Satisfaction → Growth_S	1.2129164
Satisfaction → Coworker_S	1.156314
Satisfaction → Supervisor_S	1.0403482
Regressions	**Estimate**
Leadership → Conflict	-0.884428
Leadership → Satisfaction	0.5740423
Conflict → Satisfaction	-0.311419

Indirect Effects

Predictor	Outcome	Estimate
Leadership	Person_C	-0.884428
Leadership	Intra_C	-0.707739
Leadership	Inter_C	-0.834329
Leadership	General_S	0.8494701
Leadership	Growth_S	1.0303362
Leadership	Coworker_S	0.9822542
Leadership	Supervisor_S	0.8837447
Leadership	Satisfaction	0.2754278
Conflict	General_S	-0.311419
Conflict	Growth_S	-0.377725
Conflict	Coworker_S	-0.360098
Conflict	Supervisor_S	-0.323984

Total Effects

Predictor	Outcome	Estimate
Leadership	Support_L	1
Leadership	Goal_L	1.0211907
Leadership	Work_L	0.7236143
Leadership	Interact_L	0.9598656
Leadership	Person_C	-0.884428
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Conflict	Coworker_S	-0.360098
Conflict	Supervisor_S	-0.323984
Conflict	Satisfaction	-0.311419
Satisfaction	General_S	1
Satisfaction	Growth_S	1.2129164
Satisfaction	Coworker_S	1.156314
Satisfaction	Supervisor_S	1.0403482

Assessing model fit

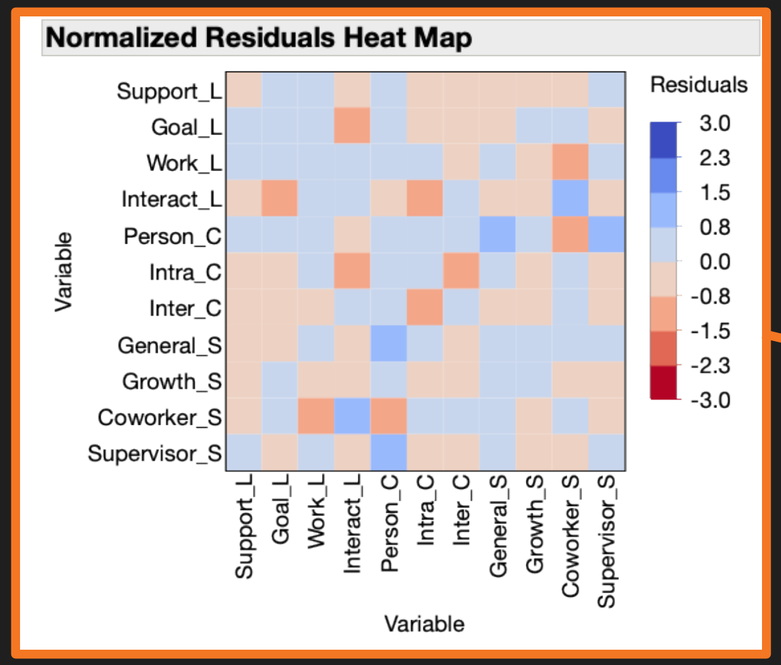
Path diagram depicts a model

implies

Expected covariance structure

Estimation tries to match

Observed covariances



Compare model implied covariances to observed covariances

Assess model fit

Assessing model fit

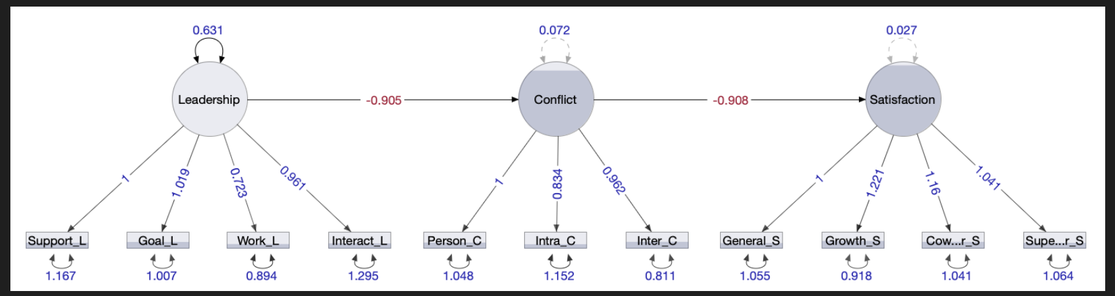
Many measures of fit, including…

Fit indices documentation

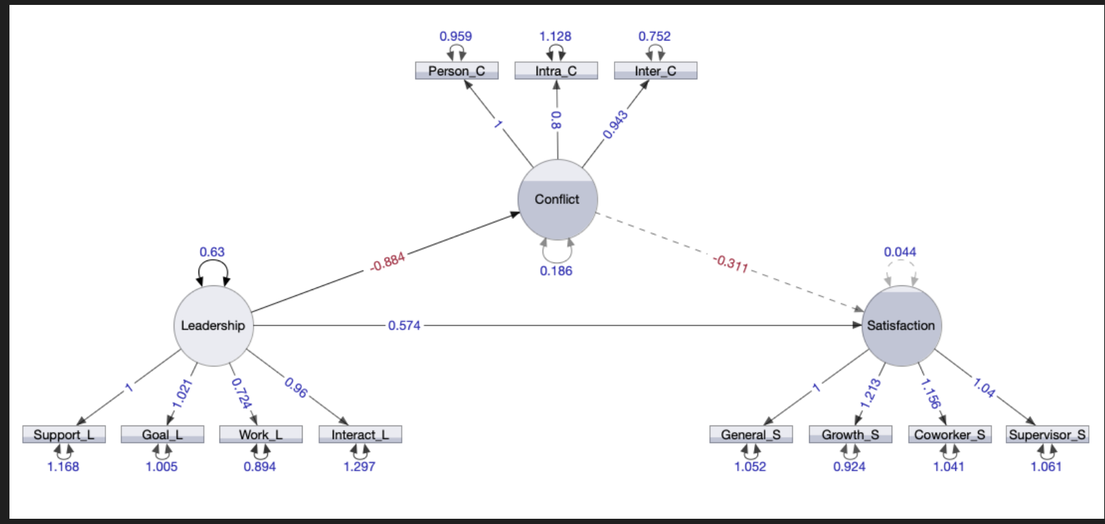
Fit index	Rough cutoff for "good" model	Notes
Chi-square test	p > 0.05 (*don't* reject null)	• Tests null hypothesis that the model fits the data perfectly. • Can be overly conservative with large N.
Comparative Fit Index (CFI)	CFI > 0.9 or 0.95	• Compare model to "independence" baseline model. • Range from 0 to 1. Higher is better. • Model complexity is penalized. TLI penalty is stronger.
Tucker Lewis Index (TLI)	TLI > 0.9 or 0.95	
Root Mean Square Error of Approximation (RMSEA)	RMSEA < 0.08	• Absolute measure of fit (i.e., no comparison to a baseline model). • Lower values are better. • Penalizes more complex models. • Can be overly sensitive with small N or df.

Nested models can be compared directly

Model A is “nested” in Model B when it consists of only a subset of the parameters of Model B



is nested in



Model Comparison

	Model Name	-2 Log Likelihood	Number of Parameters	AICc	AICc Weight	.2.4.6.8	BICu	ChiSquare	DF	Prob>ChiSq	CFI	RMSEA	Lower 90%	Upper 90%	SRMR
1	Unrestricted (Saturated)	6708.0203	77	6960.4793	0.0000		0.0000	0.0000	0	.	1.0000	0.0000	0.0000	0.0000	0.0000
2	Independence	7278.1696	22	7327.8871	0.0000		278.7418	570.1493	55	<.0001*	0.0000	0.2164	0.2005	0.2327	0.2859
3	Mediation	6742.2869	36	6830.6305	0.5548		-182.9644	34.2666	41	0.7624	1.0000	0.0000	0.0000	0.0350	0.0308
4	Indirect Only	6745.7047	35	6831.0706	0.4452		-184.8449	37.6845	42	0.6608	1.0000	0.0000	0.0000	0.0402	0.0323

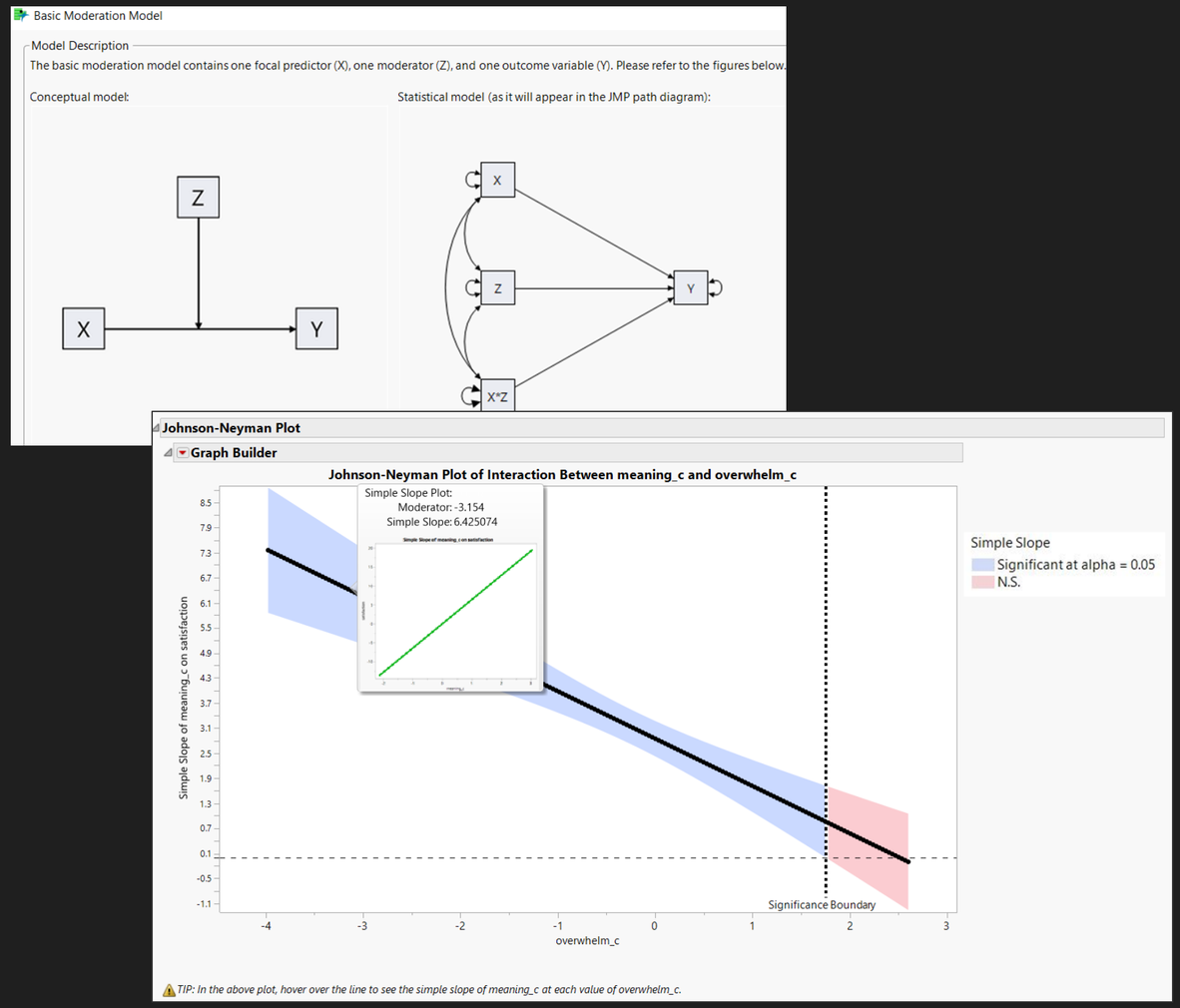
Chi-Square Difference Test

Model nested...	...in model	ΔChiSquare	ΔDF	Prob>ChiSq	ΔCFI	ΔRMSEA
Indirect Only	Mediation	3.4178	1	0.0645	0.0000	0.0000

Difference tests are meaningful only for nested models

JMP’s Moderation and Mediation add-in

https://marketplace.jmp.com/appdetails/Moderation+and+Mediation+for+JMP%C2%AE+Pro



Specify moderation, first-stage moderated mediation, second-stage moderated mediation

Test simple slopes, conditional indirect effects

Johnson-Neyman boundaries, plots