Quality Corner

SPC: Measurement Systems Analysis, Process Capability, and Control Charts

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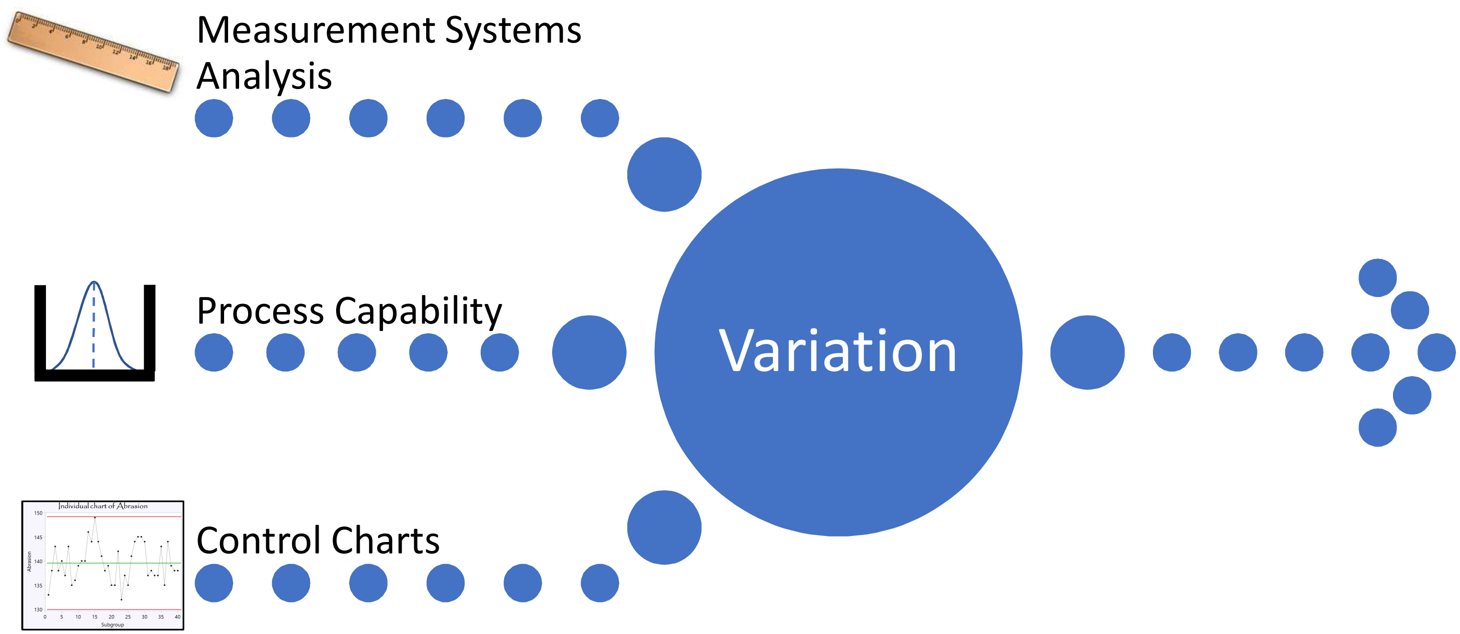
Statistical Process Control

- Methods and tools used to monitor and improve product quality by monitoring and reducing process variation
- First step to reduce variation is to understand the source:
  1. Common Cause Variation
     - Is stable over time?
     - Is a result of the system or process design?
     - Processes will have some natural level of variation over time
  2. Special Cause Variation
     - Is process unstable over time?
     - Is assignable to a specific event that is outside of the process?

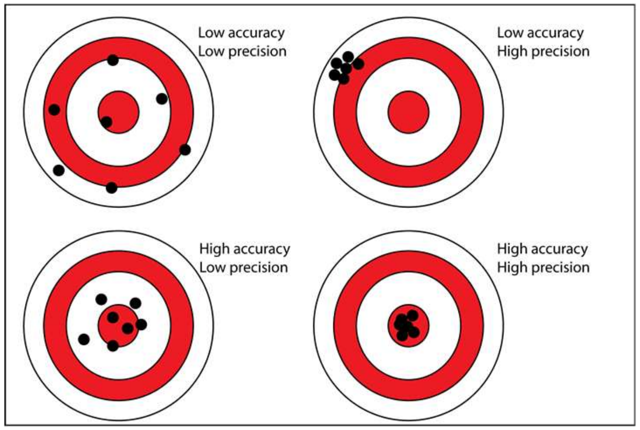
- Process is said to be operating in statistical process control when the only sources of variation are from common causes

SPC Common Theme



Measurement Systems Analysis

MSA



- Assess the measurement system
- What is the true value? (bias/accuracy)
- What is the precision? (variation)
- Range of techniques
  - Linearity
  - Resolution
  - Stability
  - Go/no go
- Helps determine how much variation is due to the measurement system rather than from product or process variation

Sources of Variation:

Part to Part
Operator
Device (equipment, gage)

$SS_{Total} = \sum(Y_i - \hat{Y})^2$

$$SS_{Total} = SS_{Part} + SS_{Measurement\ System}$$

SS: Measure the distance from the average, square it and sum them all together.

SS is a measure of variation.

Precision Errors

- **Repeatability** – the variation due to the measuring device. It is the variation observed when the same operator measures the same part repeatedly with the same device.
- **Reproducibility** – the variation due to the measurement system. It is the variation observed when different operators measure the same parts using the same device.

MSA – Gage R&R Studies

$$SS_{Total} = SS_{Part} + \underbrace{SS_{Operator}}_{\text{Reproducibility}} + \underbrace{SS_{Gage}}_{\text{Repeatability}}$$

$$SS_{Total} = SS_{Part} + SS_{R\&R}$$

For a good Measurement System

SS_{Part} / SS_{Total} >> $SS_{R\&R} / SS_{Total}$

$> 90\%$ $< 10\%$

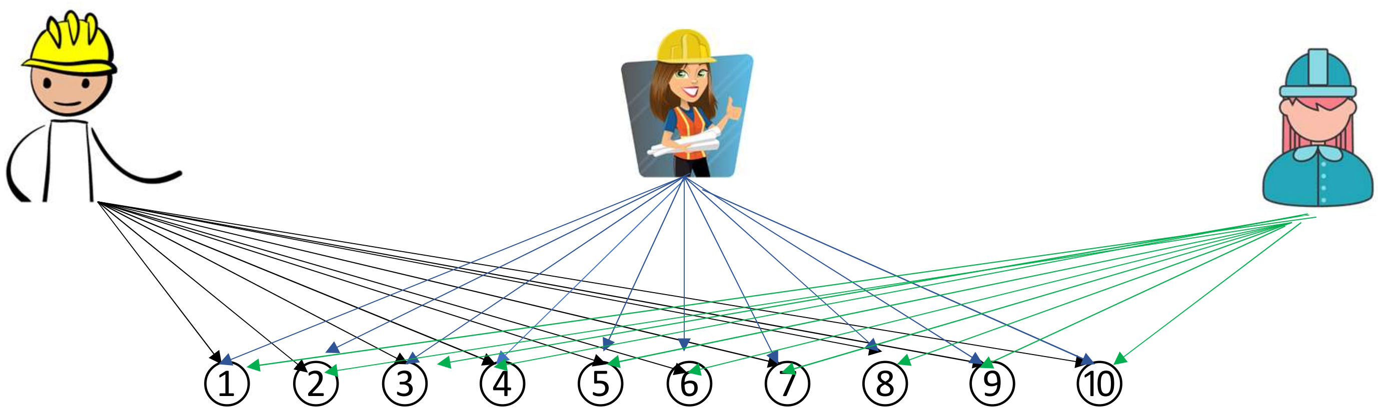
*** Variance SS Percent (%) of Total ***

Crossed Versus Nested Designs

- ***Crossed*** designs we assume that each operator can measure the same parts that other operators measure. We also plan on having each operator measure the same part more than once, (e.g. board that can be measured again)
- ***Nested*** designs we assume that the part is destroyed (e.g. breaking strength test, flammability test) to obtain a measurement, therefore, for repeatability the operator must have parts that are assumed to be the "same" to obtain repeatability variation.

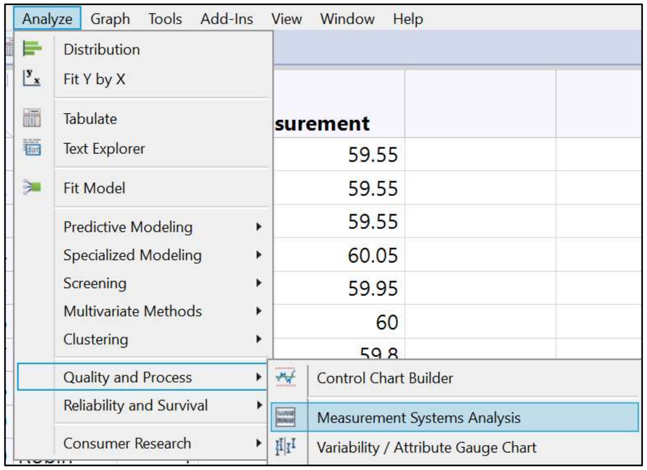
Example: Measuring Board Length

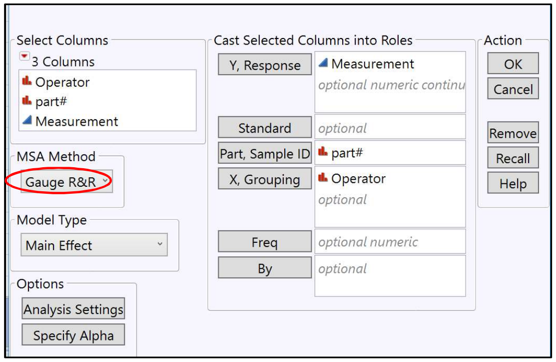
- Ten boards were selected that represent the expected range of the process variation (usually we want to assure at least 80% coverage).
- Three operators measured the ten parts, three times per part.



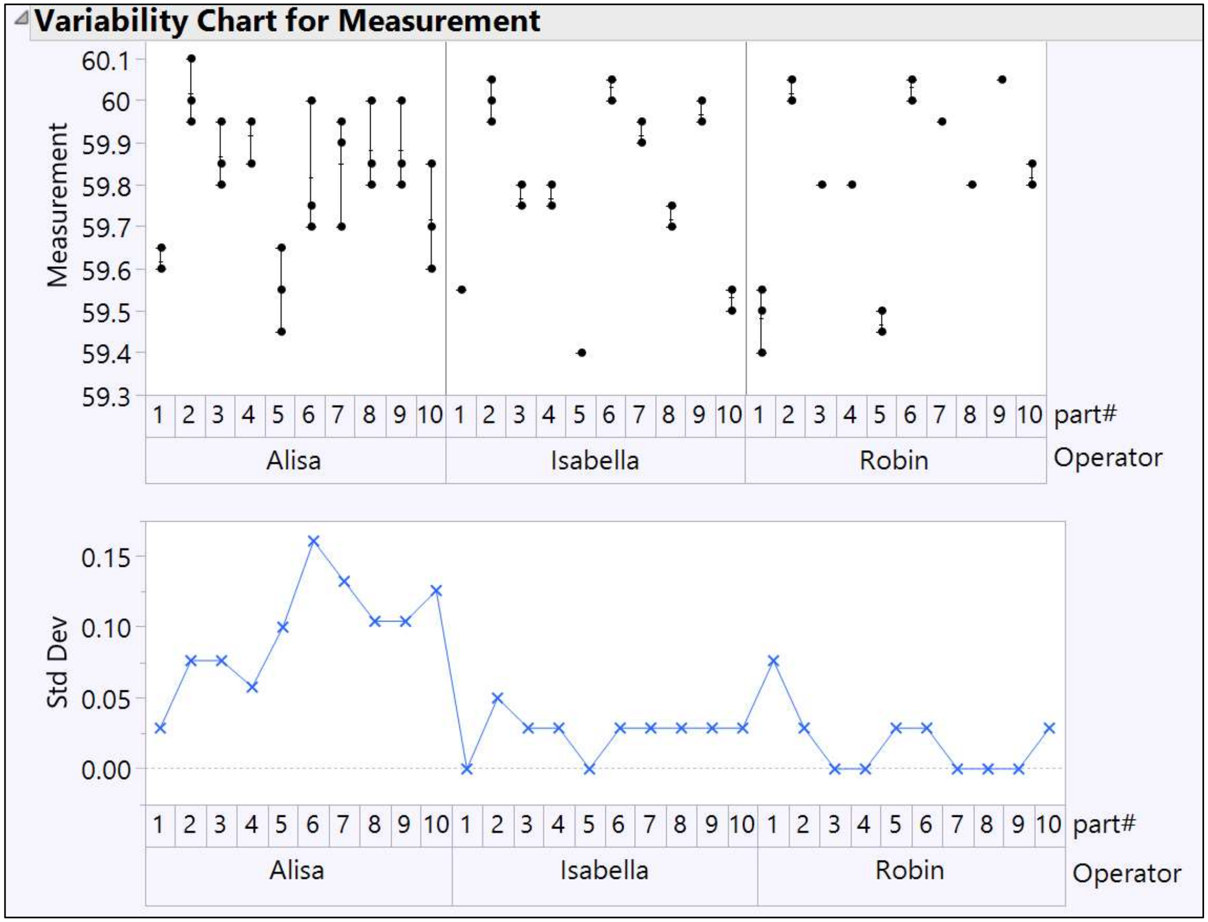
Instructions

1. Select **Analyze→ Quality and Process →Measurement Systems Analysis**
2. Select **Measurement** for **Y, Response**
3. Select **part#** for **Part, Sample ID**
4. Select **Operator** for **X, Grouping**
5. Make sure **MSA Method** is changed to **Gauge R&R**
6. Select **OK**.





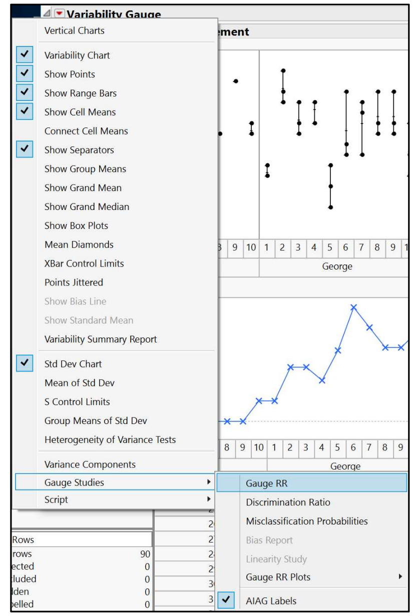
Output

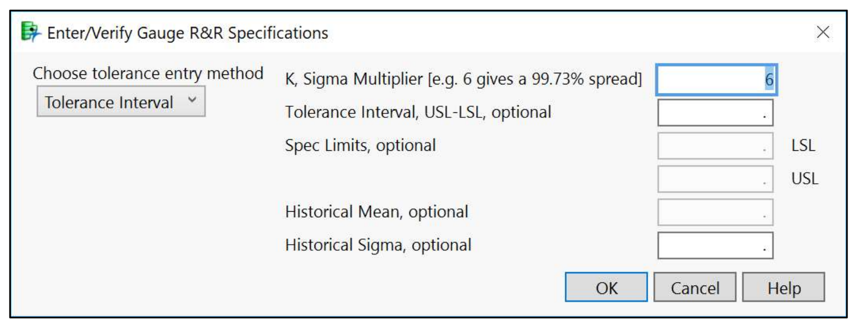


Does it look like there is variation within operator or between operators?

Instructions

1. Click on the red triangle next to **Variability Gauge**.
2. Select **Gauge Studies →Gauge RR**
3. Select **Crossed**
4. Select **OK**.





Output

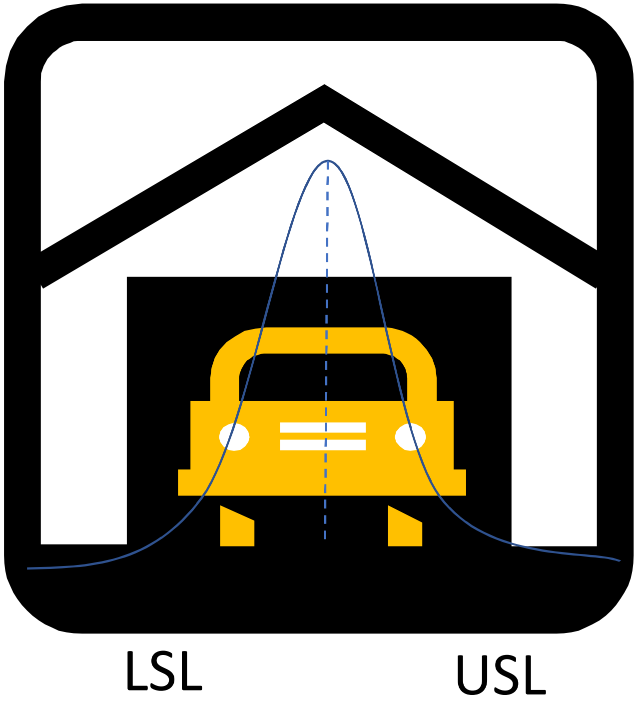
Variance Components for Gauge R&R

Component	Var Component	% of Total	20406080
Gauge R&R	0.00864815	21.47	
Repeatability	0.00800000	19.86	
Reproducibility	0.00064815	1.61	
Part-to-Part	0.03162449	78.53	

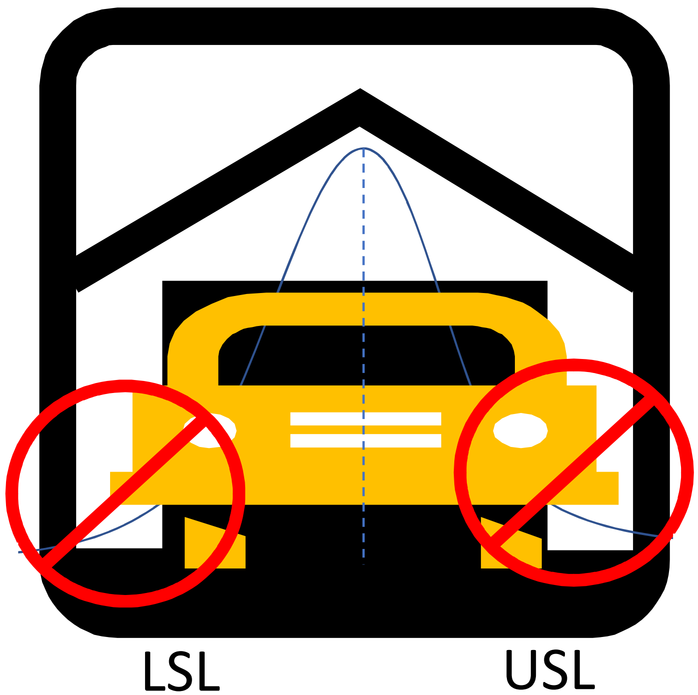
Look to see what % contribution of Gage R&R is.
Is this an acceptable Gage R&R?

Process Capability

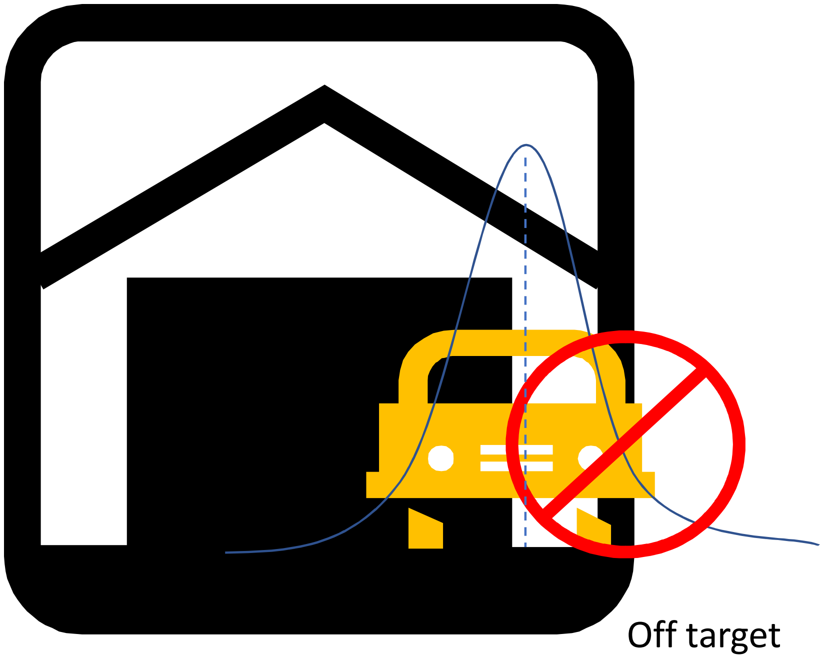
Driving into a Garage



Process Spread: Too Much Variation



Process is Not Centered: Creating Defects



Ideal: Centered and On Target

LSL=59.75”　　T=60”　　USL=60.25”

Capability Analysis

- Is my process capable of meeting specifications?
- Compare Voice of the Customer (Specifications) [USL-LSL] to Voice of the Process (Process Distribution) [6σ]
- Specifications:

  Internal, External; one-sided, two-sided

$$\hat{\sigma}_{\text{short-term}} = \frac{\overline{R}}{d_2} \qquad \hat{\sigma}_{\text{long-term}} = \sum_{i=0}^{N} \sqrt{\frac{(X_i - \overline{X})^2}{N-1}}$$

$$C_p = \frac{(USL - \text{LSL})}{6 * \hat{\sigma}_{\text{short−term}}} \qquad P_p = \frac{(USL - \text{LSL})}{6 * \hat{\sigma}\ \text{long−term}}$$

$$Cp_k = \frac{Min\ (USL - \overline{\text{x}}, \overline{\text{x}} - \text{LSL})}{3 * \hat{\sigma}_{\text{short−term}}} \qquad Pp_k = \frac{Min\ (USL - \overline{\text{x}}, \overline{\text{x}} - \text{LSL})}{3 * \hat{\sigma}\ \text{long−term}}$$

What are Good Process Capability Indices?

- It varies by industry, but generally anything higher than 1 is acceptable

Cp	Parts Per Million Defects
1.00	66,813
1.33	6,210
1.50	1,350
1.67	233
1.83	32
2.00	3.4

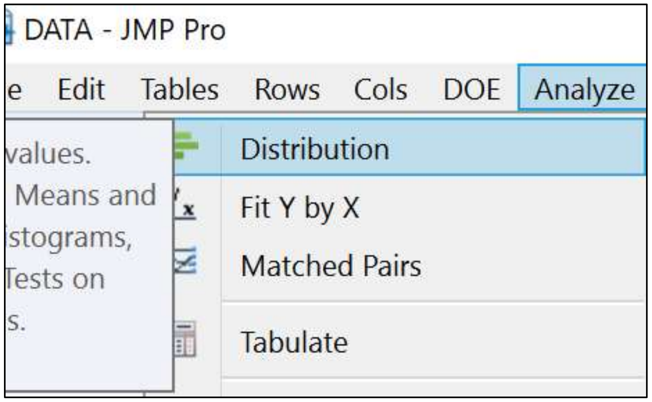
Assumptions

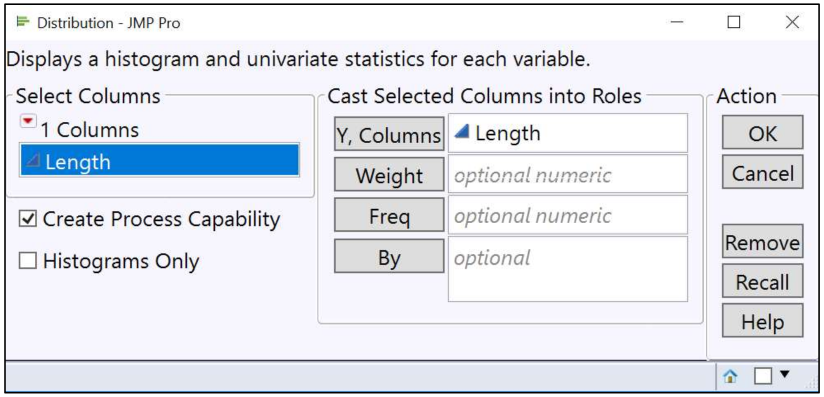
- Stable(No special cause variation)

- Process is normally distributed
  - Non-Normal distributions (Weibull, Gamma, Exponential, etc…) can be fit

Process Capability Example

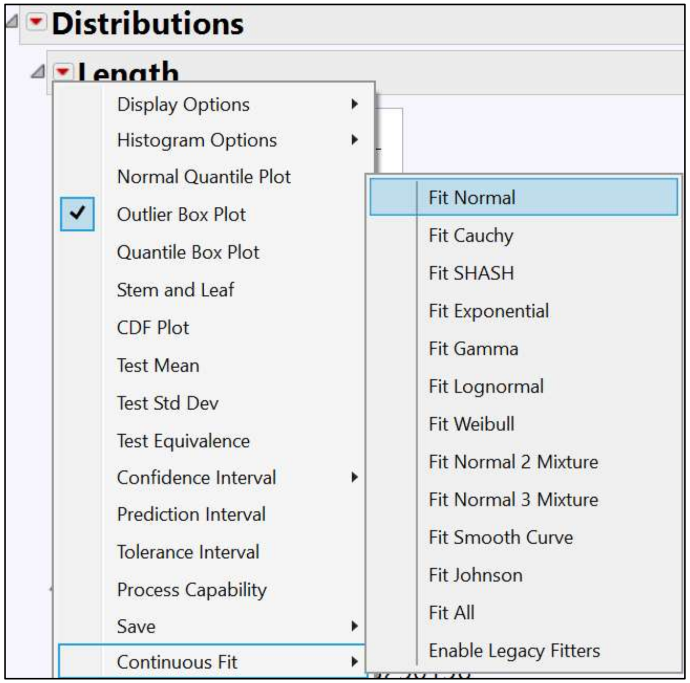
1. Open the data table JMP **Length.jmp**
2. Select **Analyze→ Distribution**
3. Select **Length** for **Y, Columns**
4. Select **OK.**

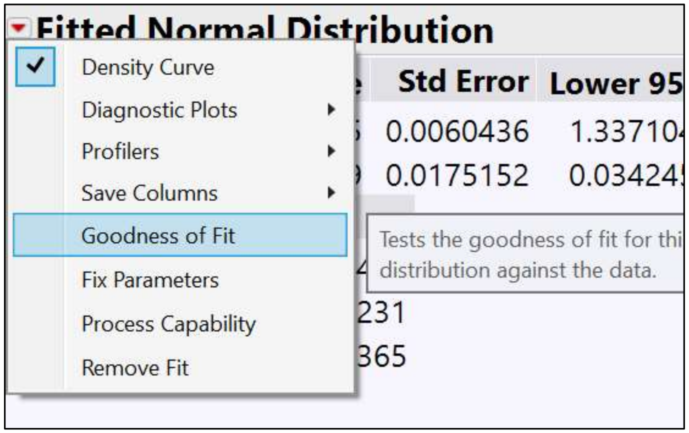




Process Capability Example

1. Click on the red triangle beside **Length**
2. Select **Continuous Fit →Fit Normal**
3. Click on the red triangle beside **Fitted Normal Distribution**→Select **Goodness of Fit**





Example: Length

Fitted Normal Distribution

Parameter		Estimate	Std Error	Lower 95%	Upper 95%
Location	μ	59.887387	0.0135685	59.860793	59.913981
Dispersion	σ	0.095944	0.0314502	0.0504657	0.0997013

Measures	
-2*LogLikelihood	-93.50524
AICc	-89.24992
BIC	-85.68119

Goodness-of-Fit Test

	W	Prob<W
Shapiro-Wilk	0.9818584	0.6325

	A2	Prob > A2
Anderson-Darling	0.3828168	0.3910

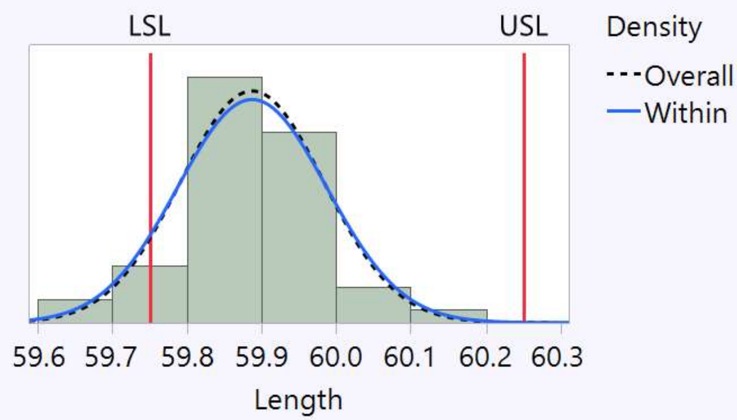
Is the data normal?
What’s next?

Process Capability Example

Process Capability

Length Capability

Histogram



Process Summary

LSL	59.75
USL	60.25
N	50
Sample Mean	59.88739
Within Sigma	0.099604
Overall Sigma	0.095944
Stability Index	0.963253

Within sigma estimated by average moving range.

Within Sigma Capability

Index	Estimate	Lower 95%	Upper 95%
Cpk	0.460	0.312	0.608
Cpl	0.460	0.311	0.606
Cpu	1.214	0.895	1.530
Cp	0.837	0.627	1.046

Overall Sigma Capability

Index	Estimate	Lower 95%	Upper 95%
Ppk	0.477	0.345	0.609
Ppl	0.477	0.344	0.608
Ppu	1.260	0.994	1.524
Pp	0.869	0.697	1.040

Is this process capable?

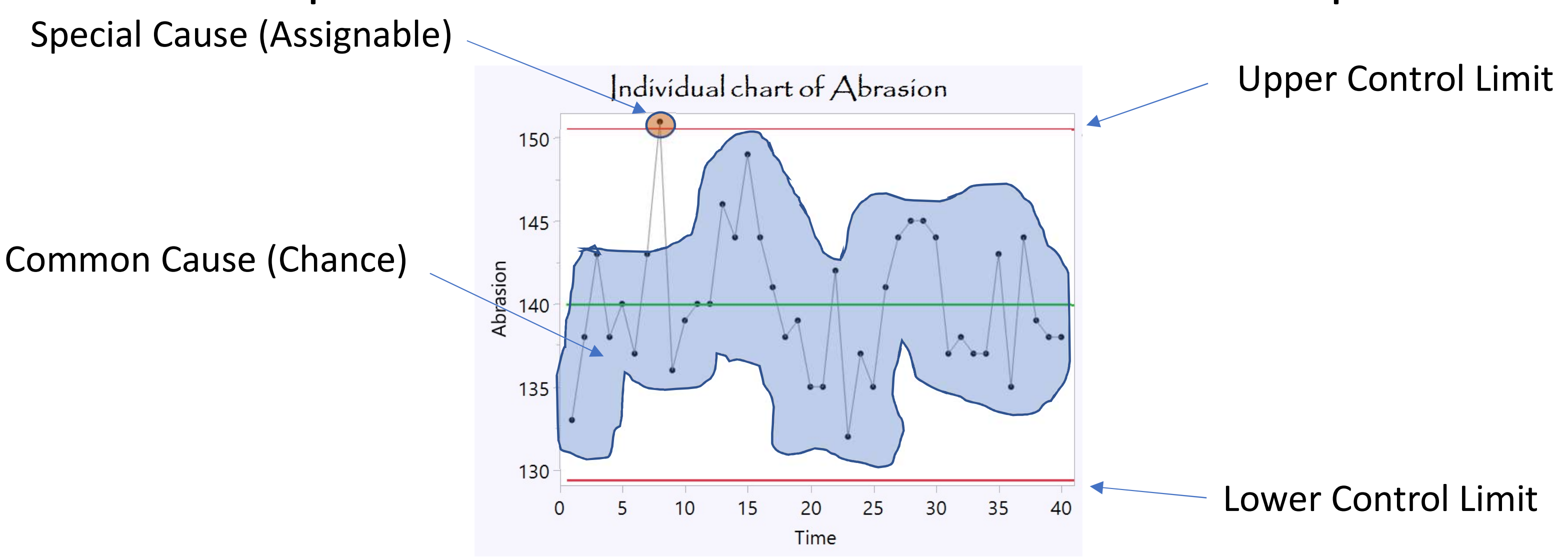
Control Charts

Control Charts

- Control Charts are used to monitor, control, and improve process performance over time
- Sophisticated time run chart
- What do they detect?
    - Changes in process average
    - Changes in process variation
    - Different sources of variation
- Benefits of Control Charts:
    1. Control Limits can distinguish ‘signals’ (special cause) from ‘noise’ (common cause)
    2. Allow you to evaluate the stability around the mean
    3. Allows for better decisions regarding process improvements – balance between acting when not necessary (overcontrol) vs. failing to act when necessary
    4. Control Limits are based on what the process is, not what it should be

Control Limits

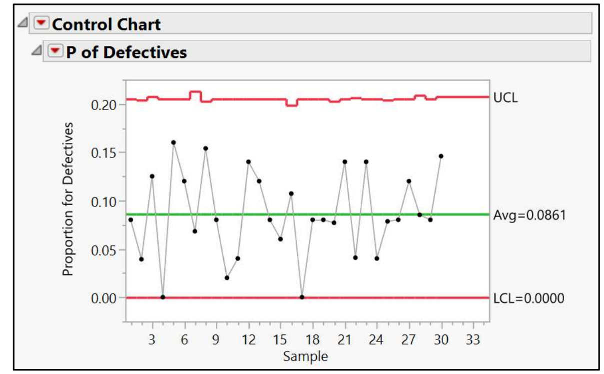
- Control Charts use limits to identify signs of trouble, or special cause variation that is not due to chance.
- These limits are expressed as lines plotted above (upper control limit) and below (lower control limit) the central tendency of the process.
- Plot points that fall outside of the limits indicate potential trouble



Western Electric Rules: How they detect changes

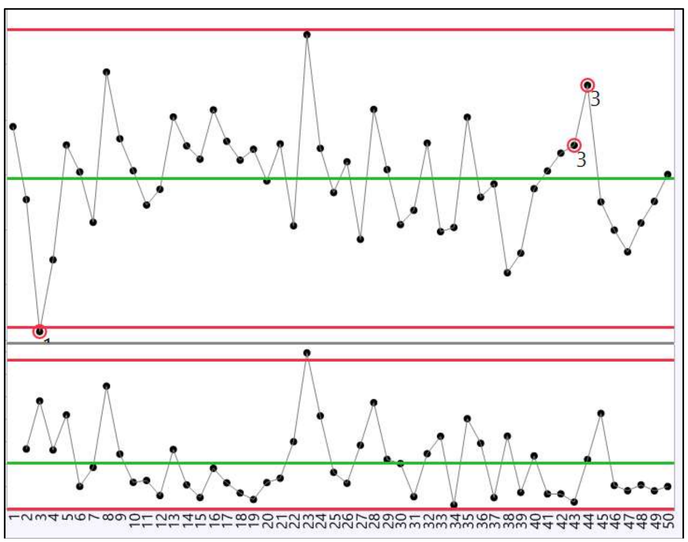
Test 1	One point beyond Zone A (upper or lower)	Detects a shift in the mean, an increase in the standard deviation, or a single aberration in the process. For interpreting Test 1, any dispersion chart (R, S, or MR) can be used to rule out increases in variation.
Test 2	Nine points in a row in a single (upper or lower) side of Zone C or beyond	Detects a shift in the process mean.
Test 3	Six points in a row steadily increasing or decreasing (anywhere on the chart)	Detects a trend or drift in the process mean.
Test 4	Fourteen points in a row alternating up and down (anywhere on the chart)	Detects systematic effects such as two alternately used machines, vendors, or operators.
Test 5	Two out of three points in a row in or beyond Zone A and the point itself is in or beyond Zone A; the two points must be on the same side (upper or lower)	Detects a shift in the process average or increase in the standard deviation. Any two out of three points provide a positive test.
Test 6	Four out of five points in a row in or beyond Zone B and the point itself is in or beyond Zone B; the four points must be on the same side (upper or lower)	Detects a shift in the process mean. Any four out of five points provide a positive test.
Test 7	Fifteen points in a row in Zone C, above and below the center line	Detects stratification of subgroups when the observations in a single subgroup come from various sources with different means. Also detects a reduction in variation.
Test 8	Eight points in a row on both sides of the center line with none in Zones C	Detects stratification of subgroups when the observations in one subgroup come from a single source, but subgroups come from different sources with different means.

What Type of Chart?



Continuous

- IR: n=1
- $\bar{X}$-R: 2<n<9
- $\bar{X}$-S: n≥10



Discrete

- np: sample size constant/counting defectives
- p: sample size not constant/counting defectives
- c: sample size constant/counting defects
- u: sample size not constant/counting defects

Observations in Sample, n	Chart for Averages: Factors for Control Limits			Chart for Standard Deviations: Factors for Center Line		Chart for Standard Deviations: Factors for Control Limits				Chart for Ranges: Factors for Center Line		Chart for Ranges: Factors for Control Limits				
	A	A_2	A_3	c_4	$1/c_4$	B_3	B_4	B_5	B_6	d_2	$1/d_2$	d_3	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	1.2533	0	3.267	0	2.606	1.128	0.8865	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	1.1284	0	2.568	0	2.276	1.693	0.5907	0.888	0	4.358	0	2.575
4	1.500	0.729	1.628	0.9213	1.0854	0	2.266	0	2.088	2.059	0.4857	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	1.0638	0	2.089	0	1.964	2.326	0.4299	0.864	0	4.918	0	2.115
6	1.225	0.483	1.287	0.9515	1.0510	0.030	1.970	0.029	1.874	2.534	0.3946	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	1.0423	0.118	1.882	0.113	1.806	2.704	0.3698	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	1.0363	0.185	1.815	0.179	1.751	2.847	0.3512	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	1.0317	0.239	1.761	0.232	1.707	2.970	0.3367	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	1.0281	0.284	1.716	0.276	1.669	3.078	0.3249	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	1.0252	0.321	1.679	0.313	1.637	3.173	0.3152	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	1.0229	0.354	1.646	0.346	1.610	3.258	0.3069	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	1.0210	0.382	1.618	0.374	1.585	3.336	0.2998	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	1.0194	0.406	1.594	0.399	1.563	3.407	0.2935	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	1.0180	0.428	1.572	0.421	1.544	3.472	0.2880	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	1.0168	0.448	1.552	0.440	1.526	3.532	0.2831	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	1.0157	0.466	1.534	0.458	1.511	3.588	0.2787	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	1.0148	0.482	1.518	0.475	1.496	3.640	0.2747	0.739	1.424	5.856	0.391	1.608
19	0.688	0.187	0.698	0.9862	1.0140	0.497	1.503	0.490	1.483	3.689	0.2711	0.734	1.487	5.891	0.403	1.597
20	0.671	0.180	0.680	0.9869	1.0133	0.510	1.490	0.504	1.470	3.735	0.2677	0.729	1.549	5.921	0.415	1.585
21	0.655	0.173	0.663	0.9876	1.0126	0.523	1.477	0.516	1.459	3.778	0.2647	0.724	1.605	5.951	0.425	1.575
22	0.640	0.167	0.647	0.9882	1.0119	0.534	1.466	0.528	1.448	3.819	0.2618	0.720	1.659	5.979	0.434	1.566
23	0.626	0.162	0.633	0.9887	1.0114	0.545	1.455	0.539	1.438	3.858	0.2592	0.716	1.710	6.006	0.443	1.557
24	0.612	0.157	0.619	0.9892	1.0109	0.555	1.445	0.549	1.429	3.895	0.2567	0.712	1.759	6.031	0.451	1.548
25	0.600	0.153	0.606	0.9896	1.0105	0.565	1.435	0.559	1.420	3.931	0.2544	0.708	1.806	6.056	0.459	1.541

For $n > 25$.

Introduction to Statistical Quality Control, 4th Ed. Montgomery, pg. 761

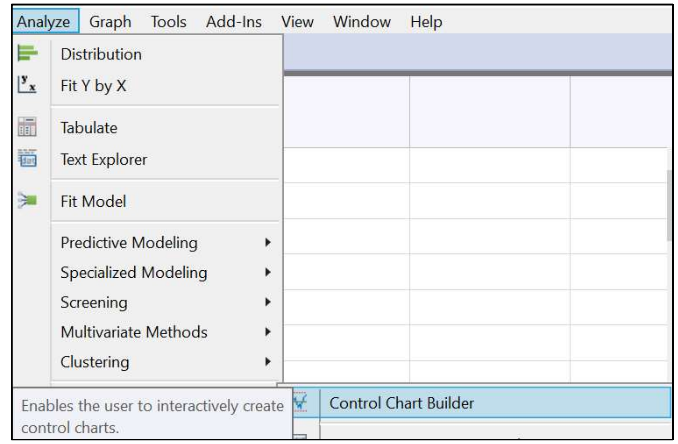
$$A = \frac{3}{\sqrt{n}} \qquad A_3 = \frac{3}{c_4\sqrt{n}} \qquad c_4 \simeq \frac{4(n-1)}{4n-3}$$

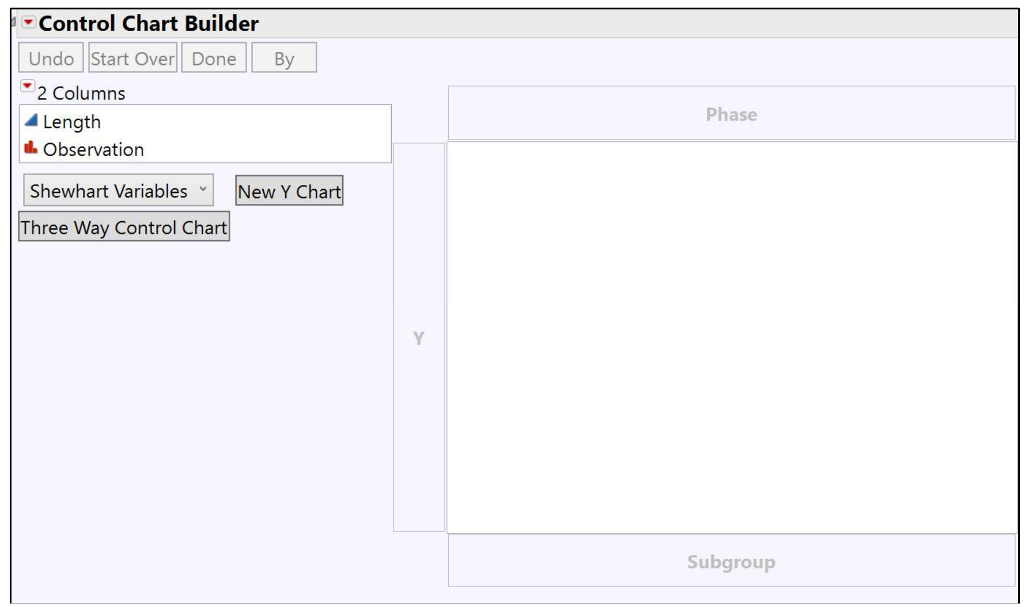
$$B_3 = 1 - \frac{3}{c_4\sqrt{2(n-1)}} \qquad B_4 = 1 + \frac{3}{c_4\sqrt{2(n-1)}}$$

$$B_5 = c_4 - \frac{3}{\sqrt{2(n-1)}} \qquad B_6 = c_4 + \frac{3}{\sqrt{2(n-1)}}$$

Example-Cont'd

1. Open **Length.jmp.**
2. Select **Analyze→ Quality and Process →Control Chart Builder.**
3. Drag **Length** for **Y** and **Observation** to **Subgroup.**
4. Select **OK.**

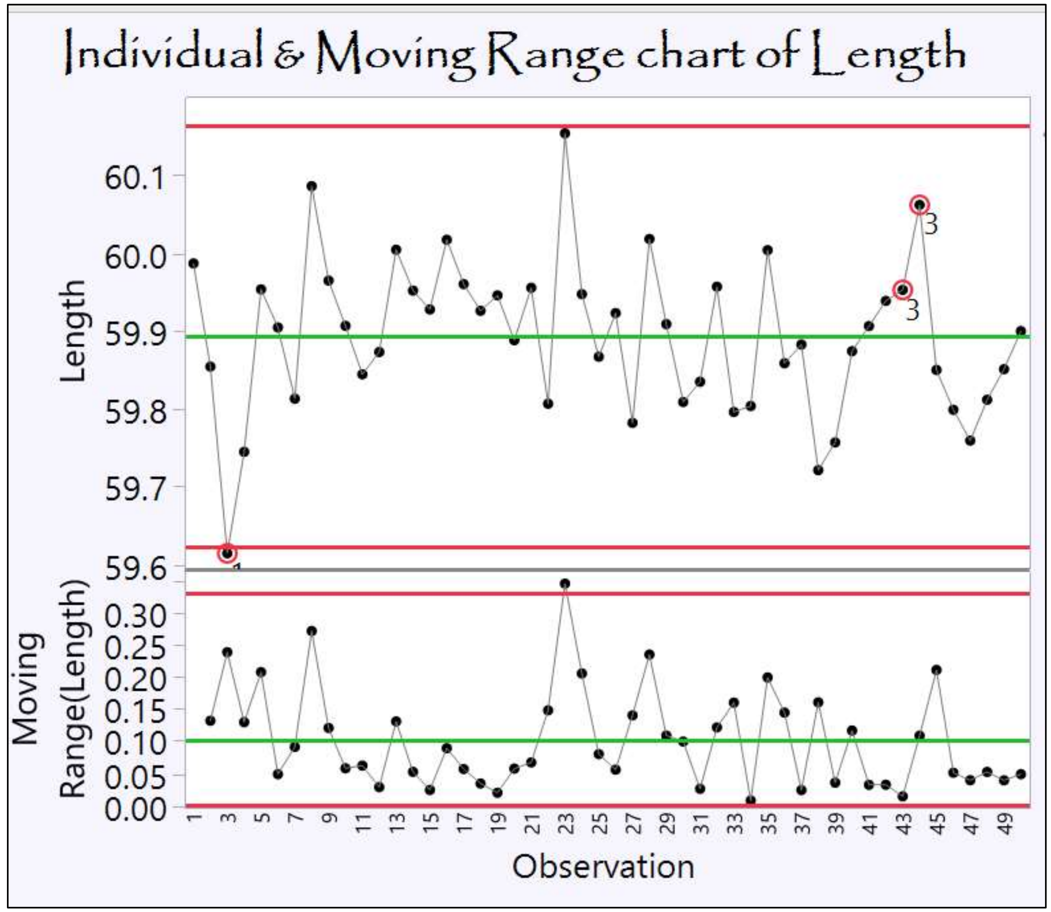




Example

1. Right click on top chart and select **Warnings→Tests →All Tests.**
2. Click **Done.**

Is the process in control?



Example

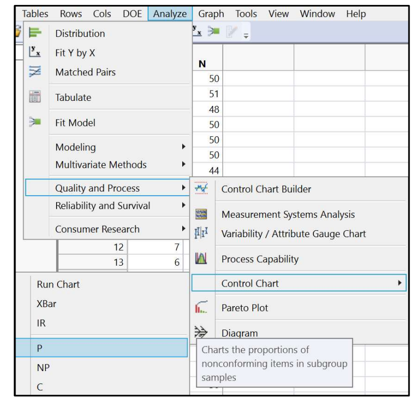
You decide to count the number of product returns over the last month from varying production lot sizes.

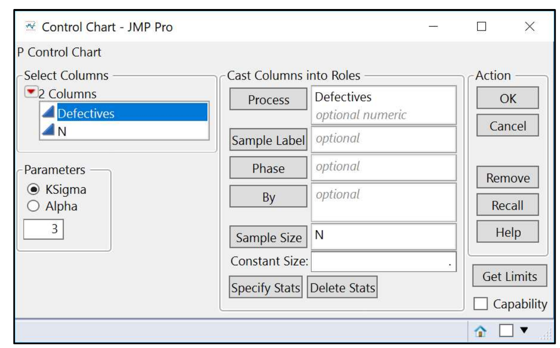
Which control chart would you use? Is it in control?

Open **Returns.jmp**

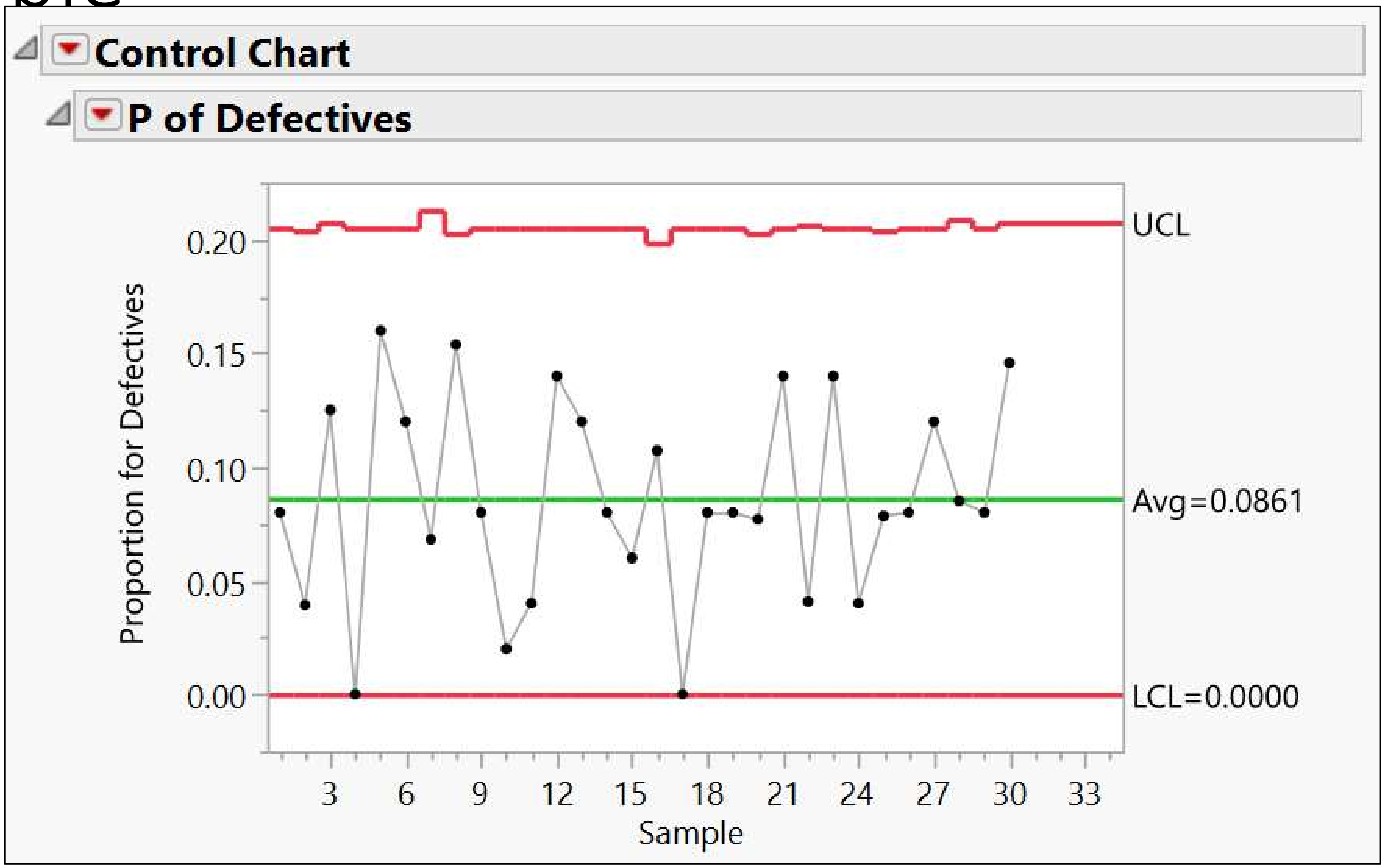
Example-Cont’d

1. Open **Returns.xlsx**
2. Select **Analyze→Quality and Process→ Control Chart →P.**
3. Select **Defectives** for **Process.**
4. Select **N** for **Sample Size.**
5. Select **OK.**





Example



Control Chart
P of Defectives
UCL
Avg=0.0861
LCL=0.0000
Proportion for Defectives
0.20
0.15
0.10
0.05
0.00
3
6
9
12
15
18
21
24
27
30
33
Sample

Summary

- SPC looks at variation
- MSA assesses the measurement system
- Process capability assesses whether meeting customer specifications
- Control charts identify whether process is stable and in control

Just in case you were wondering….

For more information, please check out the free SAS Training course, Statistical Thinking for Industrial Problem Solving