Fundamental Strategies to Handle Noise In Experimental Situations, Part II

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Introduction

Experimentation is the manipulation of controllable factors (independent variables) at different levels to see their effect on responses (dependent variables) in the face of noise (noise is referred to as unit structure in this paper). In the context of this paper, experimentation is done with the intent of understanding causality represented by the symbolic model Y=*f*(x). Some experimental designs, such as randomized complete block designs, are more effective in understanding both design factor effects **and** noise effects. The efficiency and effectiveness of an experimental design is a function of how noise is handled during the experiment. Noise is the set of factors one is unwilling to manage or control. This may be for reasons of convenience, difficulty or cost.

Robustness is the consistent performance of products or processes in the face of changing noise. In order to understand robustness, noise must be included *and* must vary in the experimental studies (i.e., not held constant). The identification and understanding of noise is an opportunity to increase the robustness of products and processes.

There are a number of ways to estimate or *manage* noise during the execution of an experiment. Tests of significance (e.g., F-test) are comparisons between the factors explicitly manipulated in an experiment (herein referred to as design factors) and the factors that change during the experiment but are NOT explicitly manipulated (herein referred to as unit structure). The recommended strategy is to partition the unit structure in some sensible way to allow for better precision in detecting design factor effects and also to allow for estimation of the noise effects. This needs to be done while not compromising the ability to extrapolate the results (i.e., negatively affecting the inference space of the study). A host of techniques such as blocking, efficiency split-plots, cross-product arrays and nesting are effective at accomplishing this. Selection of which technique to use is dependent on the situation.

The focus of this paper will be on three fundamental strategies to handle noise in a designed experiment: repeats, replicates and split-plots. These approaches will be illustrated with a hypothetical situation where two variables (X1 and X2) are the factors manipulated in a designed experiment.

The intent is to:

- clarify the differences in strategies,
- describe the mechanics of using each strategy,
- explain how each strategy estimates the effect of noise,
- provide guidance as to when each strategy is applicable, and
- demonstrate the differences with a data set.

The target audience for this paper is individuals applying experimentation to the fields of science and engineering.

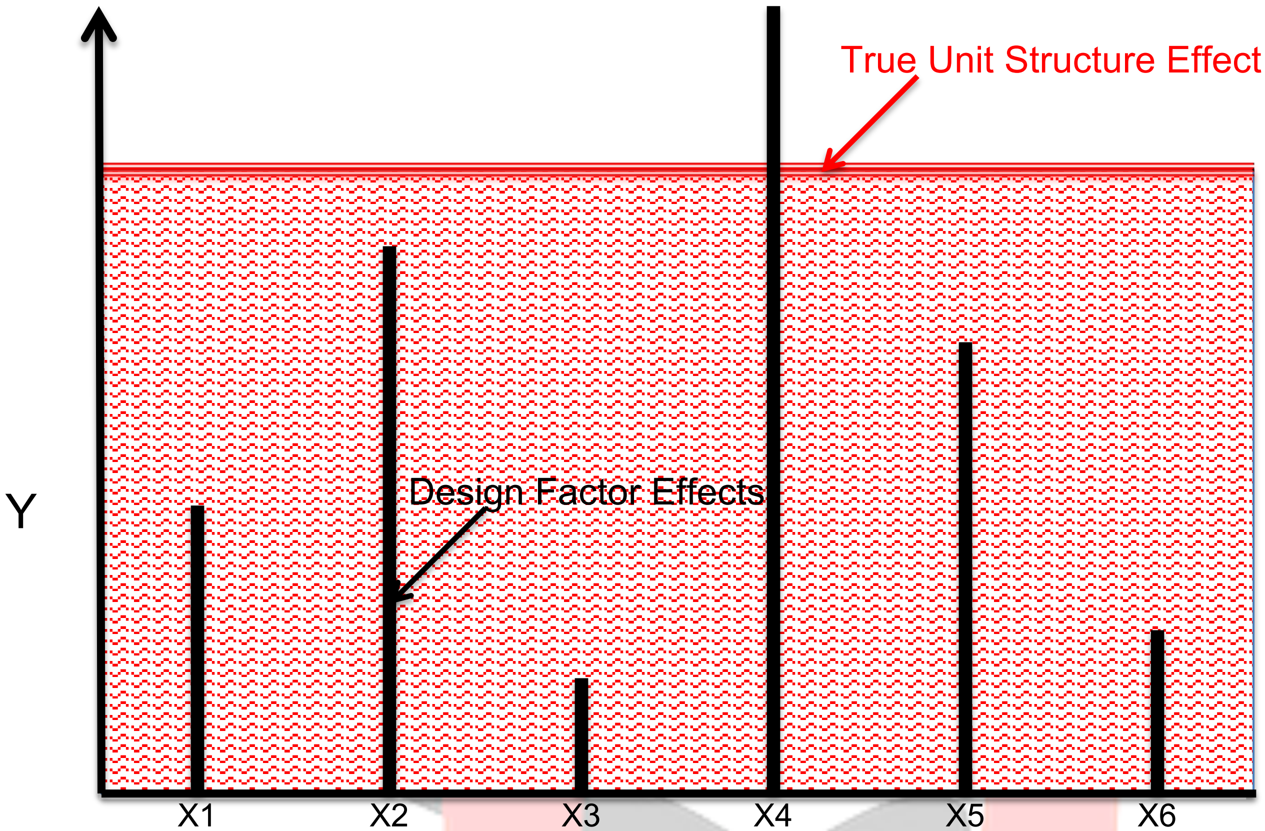
Conceptual Overview

Designed experiments (e.g., factorials) are powerful tools to understand factor effects. In designed experiments, factors (x's) are manipulated at multiple levels to quantify their effect on response variables (Y's) in an attempt to understand the causal relationship between the factors and the responses. This relationship is often expressed as a polynomial, $Y = f(x) + N$[1]. Noise can have a significant influence on the effectiveness of experiments.

To illustrate the impact of unit structure on detecting factor effects, imagine the noise is analogous to the water level of a lake (figure 1, shown in red). The factor effects are the change in Y represented by black lines emanating from the bottom of the lake. To determine whether the design factors are significant, the effects of the design factors are compared to the estimated effect of the unit structure represented by the red line. As the water level rises as a function of the unit structure, the ability to detect the factor effects (referred to as precision) is compromised.

[1] For experimental design, noise is referred to as unit structure

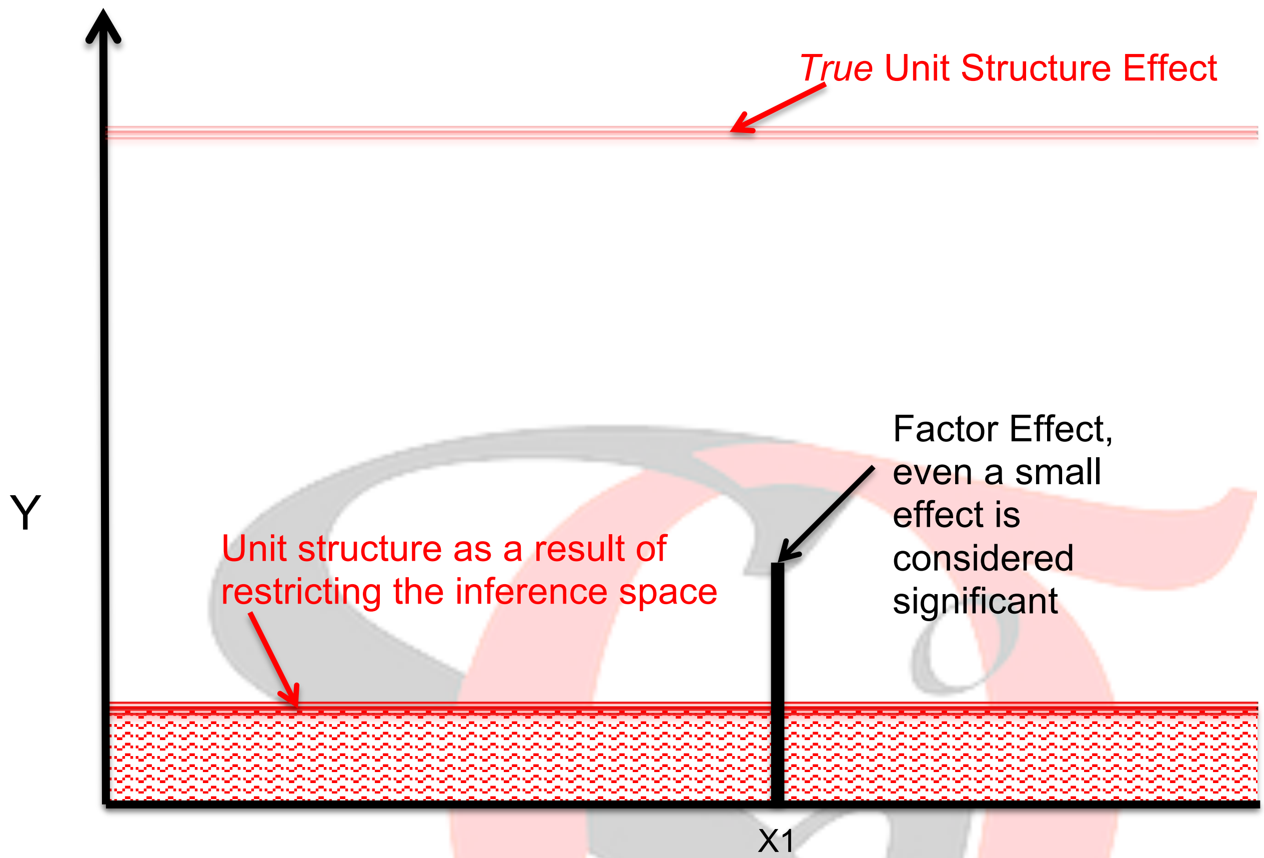
Figure 1: Graphic Representation of Unit Structure and Design Factor Effects



In figure 1, only factor X4 would be considered to have a significant effect on Y. The other factor effects are concealed by the unit structure. Since significance is a result of the comparison between unit structure and design structure, there are two ways to show significance: reduce the unit structure effects or increase the design factor effects typically accomplished by manipulating the design factors at “bold” level settings. The different approaches to reduce the unit structure effects will be discussed.

A commonly acceptable thought is to vary one factor while holding all other factors constant. While this approach is significantly flawed, it has the effect of reducing the noise in the experiment and thus increases the precision for estimating the factor effect (figure 2). This is referred to as one-factor-at-a-time experimentation or OFAT for short.

Figure 2: Graphic Representation of the Unit Structure and the Design Factor Effect in an OFAT



The OFAT methodology has some extremely negative issues and consequences:

- ✓ Unable to estimate the effects of changing one factor while other factors are changing. Since results are obtained when all factors are fixed, the inference space is extremely narrow and unrealistic. This has a negative effect on the ability to extrapolate the results into the future.
- ✓ Unrealistic and unreasonable to hold **all** factors constant while changing one.
- ✓ Inefficient and costly. Requires more resources.
- ✓ Potential sub-optimization. Factors are "locked" in based on the levels of other factors that have yet to be optimized.
- ✓ Impossible to identify & quantify interaction effects.

The critical question: how can the precision of detecting factor effects be increased while not negatively impacting the inference space?

Planning Tools & Strategies

The Factor Relationship Diagram[2] (FRD) is a graphical schematic tool for displaying the relationship between manipulated (design factors) and un-manipulated factors (unit structure). Note both types of factors are potential sources of variation. The FRD keeps track of partitioning of the unit structure and the degrees of freedom. It is used to:

- plan the experiment by visualizing the potential comparisons between the different sources of variability in the study, and
- guide the analysis revealing the appropriate comparisons to make.

An FRD will have design factors, unit structure (US) and at least one line of restriction (LOR)[3]. It will also include the model assigning the degrees of freedom to each partition of the unit structure.

The design factors are shown in black and their levels are coded equidistant, centered on zero (e.g., for two level designs: -1 & +1 or - & +).

The unit structure is the noise related to the experiment. The unit structure is red and coded using whole numbers to show whether the noise is constant or varying[4]. The unit structure shows:

1. Noise held constant during the experiment. Conditions under which the experiment is run (i.e., inference space) which impacts the ability to extrapolate the results, and
2. Noise that varies during the execution of the experiment. This is represented by the word treatment[5] in the FRD. This is what the design factors will be compared against to determine the significance of their effects (the water level in figure 1).

Lines of restriction are shown in green and are used to show:

1. Partitioning of the unit structure. Subdividing the noise into smaller "chunks".
2. Partitioning of the degrees of freedom for analysis. Necessary to determine the appropriate model and comparisons to make.

Solid lines of restriction partition both unit structure & degrees of freedom; dashed lines of restriction partition only the unit structure.

[2] Sanders, Doug and Jim Coleman (1999), "Considerations Associated with Restrictions on Randomization in Industrial Experimentation", *Quality Engineering*, Volume 12, No. 1
[3] The phrase comes from restrictions on randomization.
[4] The exception to this is when the noise is manipulated (e.g., Blocks or split-plots). Then the coding is -1 & +1 or - & +
[5] Treatment represents the entirety of the set of unit structure potentially changing for each set of factor combinations

Strategy 1: Repeats

Multiple data points are collected for the same treatment where the treatment combinations are ***not changed*** between those data points. The data points could be multiple measures of the same location on the same sample (indicative of measurement error), multiple measures at different locations on the same sample (indicative of within sample variation), multiple samples made consecutively (indicative of sample-to-sample variation), etc. The data points are nested within treatments. Data points from repeats are not considered independent events and therefore do not increase the degrees of freedom in the study.

The benefit of such a strategy is the ability to:

- assess within treatment stability, and
- estimate and analyze response variables for the mean and variation.

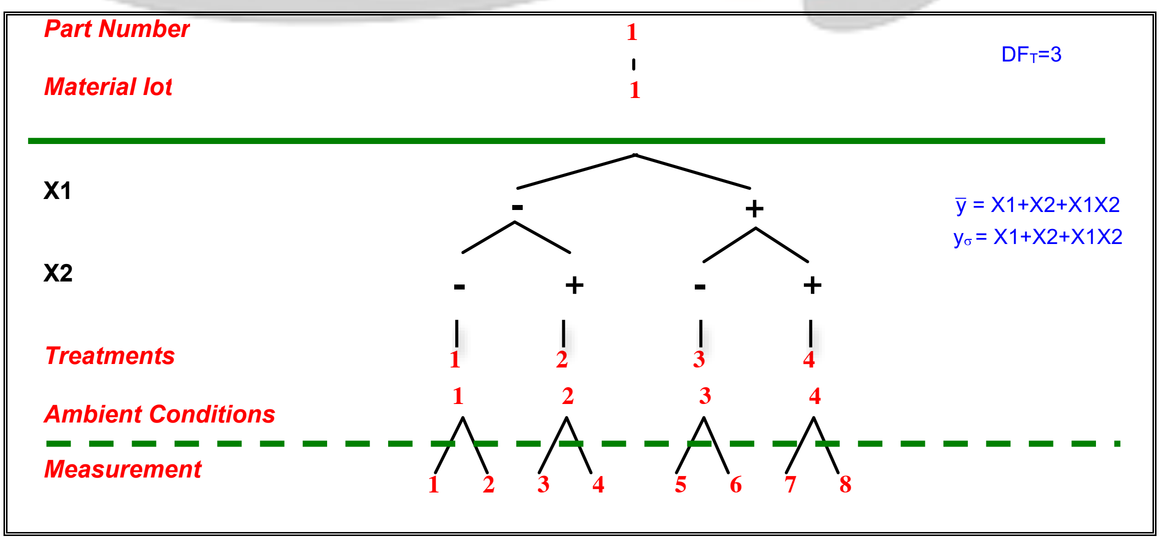
Averaging the measures of repeated units reduces the variation of the within treatment component. This will decrease the level of unit structure the factors will be compared to and increase the precision. The variation of the repeated units may be quantified (e.g., range, standard deviation, variance, etc.) and will quantify the effect of the unit structure within treatment. This response variable is extremely useful in determining factors that affect variability. Both the mean and the variation can be analyzed as responses of the experiment.

Models:

$$\bar{y} = X1+X2+X1X2$$

$$y_{\sigma} = X1+X2+X1X2$$

Figure 3: FRD for Repeats (measurements nested in treatments)



In this situation, the x's associated with the measurement system are partitioned from the treatments thereby reducing the variation due to unit structure the treatments will be compared to. This will increase the precision of the design factors (e.g., reduce the level of the unit structure). It is worth noting multiple "layers" of repeats can be made for assessing and separating within treatment components of variation.

Strategy 2: Replicates

Multiple, independent, Experimental Units (EU's) are created for each treatment combination. Treatment combinations ***change*** between each experimental unit (each treatment requires breakdown and set-up). This will increase the amount of unit structure in the experiment (e.g., ambient conditions, lots of raw material, multiple machines, other factors one is not willing to manage) and the degrees of freedom available. Three scenarios:

1. **Completely Randomized Replicates** (CRR). The run order is randomly selected. Since the treatments are randomly selected, it is not possible to assign the unit structure. Therefore an unassigned *error* term is added to the model. Model:

$$y = X1+X2+X1X2+ \varepsilon$$

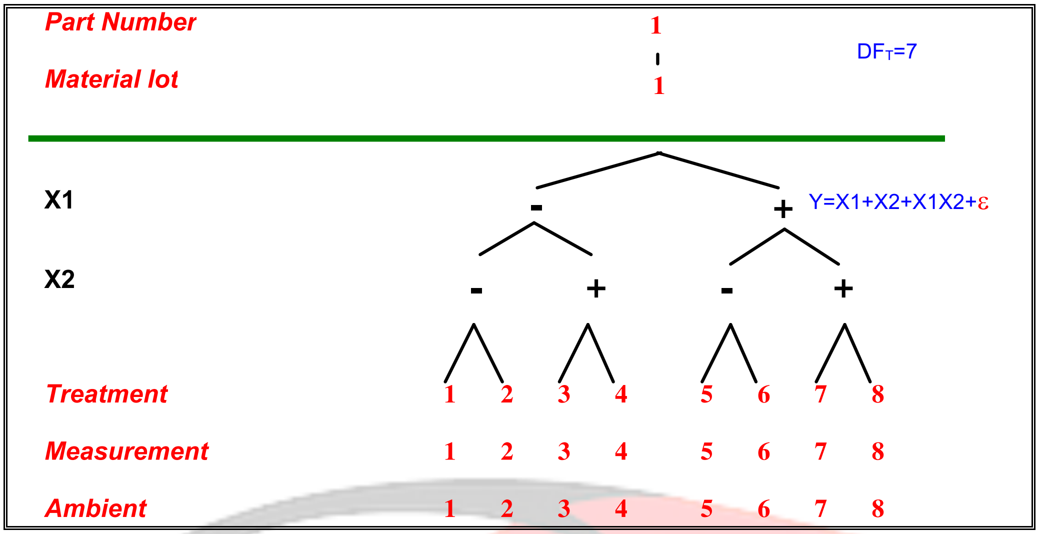
The benefit of such a strategy is:

- *Theoretically*, an unbiased estimate of the effect of unit structure may be gotten during the experiment (*pure* error). This may be used as a basis for comparison in analysis (e.g., mean square error in ANOVA).
- Having unassigned degrees of freedom allows some flexibility in adding random variables to the model (e.g., measurable noise factors called covariates).

The disadvantages are:

- It is unknown what the factor effects are being compared to (they are un-assignable due to the nature of how they were obtained), and
- Specific identification and assignment of the noise factors cannot be accomplished. Therefore when the error is large it is difficult to know which factors to work on for further improvement opportunities.
- There is a potential for the unit structure to have a large effect and therefore reduces the precision of detecting design factor effects.

Figure 4: FRD for CRR



The inference space is much broader than OFAT having a positive impact on the ability to extrapolate the results. The unit structure effect is greater between treatments decreasing the ability to detect factor effects (the unit structure level is higher). This strategy is most useful when the noise has not been identified and therefore cannot be partitioned or assigned.

2. **Randomized Complete Block Design** (RCBD). First introduced by Sir Ronald Fisher, blocking combines the object of reducing the effect of unit structure to increase the precision (within block) while NOT negatively impacting the inference space (the changing unit structure is still contained in the experiment). Figure 5 depicts the effect blocking has on unit structure in an experiment.

   Blocks are created by first identifying the unit structure using tools such as the Process or Product Map[6] and the Thought Map[7]. Once identified, the unit structure may be controlled or sampled over during the experiment. It is held or remains constant **within** the block, and is allowed to vary or is explicitly changed **between** the blocks. Blocks are aliased with unit structure: lots of raw material, machines, shifts, ambient conditions, etc.

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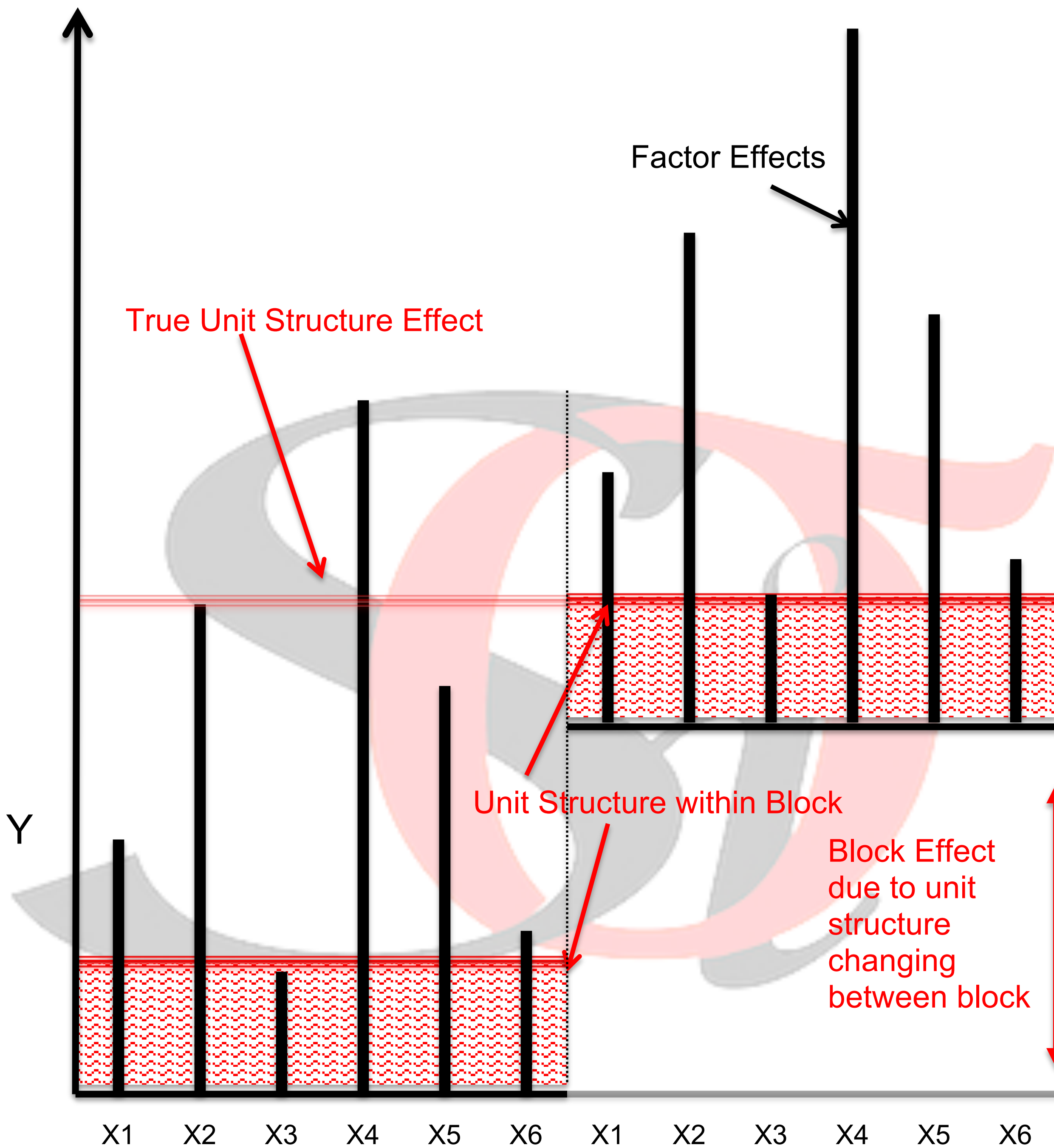
[6] Sanders, Doug, W. Ross, and J. Coleman (2000), "The Process Map", *Quality Engineering*, Vol. 11, No. 4,.
[7] Hild, Cheryl, D. Sanders (2000) "The Thought Map", *Quality Engineering*, Vol. 12, No. 1.

Figure 5: Graphic Representation of Unit Structure and Factor Effects with Blocks



The benefit of such a strategy is:

- better precision (lowers the unit structure level within block),
- the ability to estimate all block and block-by-factor interaction effects, and
- increased inference space.

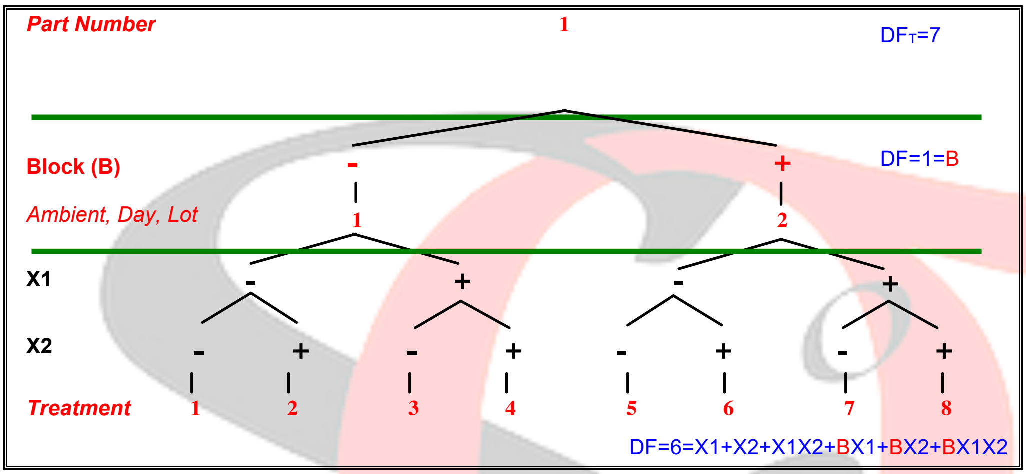
This doubles the size of an un-replicated experiment, but is a superb strategy for the estimation of noise-by-factor interactions as it does this with increased precision.

The block degree of freedom is partitioned from the other degrees of freedom reducing the unit structure used as a basis to compare the design factor effects to and increases the precision of estimating noise-by-factor interaction effects improving the ability to create robust products and processes.

Model:

y = X1+X2+X1X2+B+BX1+BX2+BX1X2

Figure 6: FRD for RCBD



In this situation, material lot is included in the experiment thereby increasing the inference space (there is increased confidence the results will hold true as lots change). In addition, the ambient conditions are confounded with the block not with the treatments. This decreases the unit structure between treatments and thus increases the precision. The added benefit of the RCBD is the block-by-factor interactions occur with the design structure so they are estimated with increased precision. The ability to estimate noise-by-factor interactions is required for robust design.

3. **Randomized Incomplete Block Designs**[8] (RIBD). RIBD are fractional blocks. They achieve much of what a RCBD does in terms of inference space and ability to assign the block effect, but instead of running the entire experiment twice, two different orthogonal fractions are run. An incomplete block is created by aliasing the block with one of the degrees of freedom in the experiment, most likely a higher

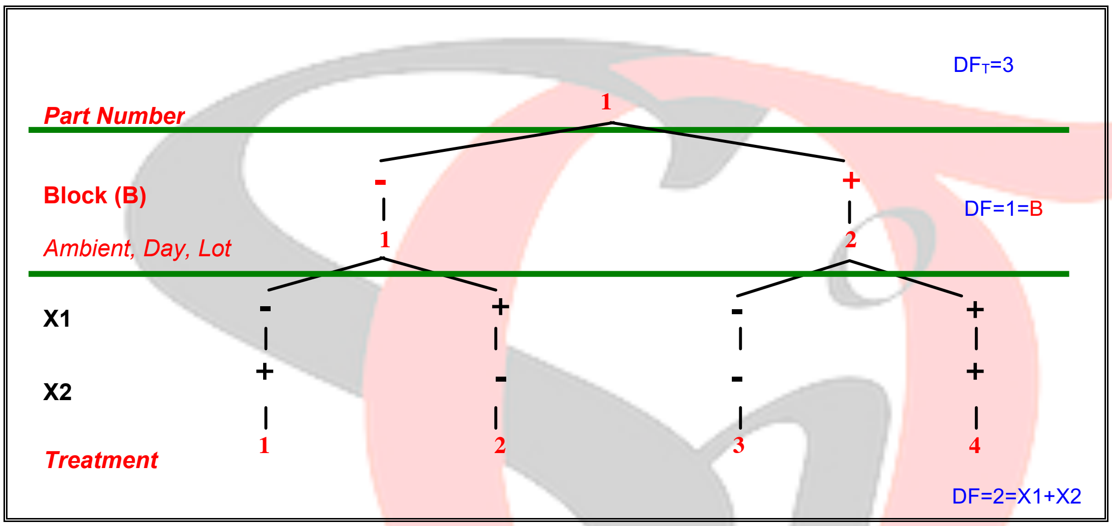
[8] Aka, Balanced Incomplete Block (BIB)

order term (the same procedure for aliasing a main effect with an interaction to fractionate a factorial design). When this is done, the result is additional confounding (in this case block-by-factor interactions are aliased with other factor effects). This would be useful in quantifying the effect of the chunk of unit structure variables for potential disaggregation later. In other words, future opportunities for improvement of the model.

Model:

y = X1+X2+B

Figure 7: FRD for RIBD



In this situation, the benefits of increased inference space and increased precision are obtained, but the noise-by-factor interactions are now aliased with design factor effects (in this case the X1*X2 interaction). This strategy is most useful for manufacturing processes where the block is a surrogate for a set of x's currently not managed. If the block effect is large, the block is studied (disaggregated) to identify x's that will likely improve the process.

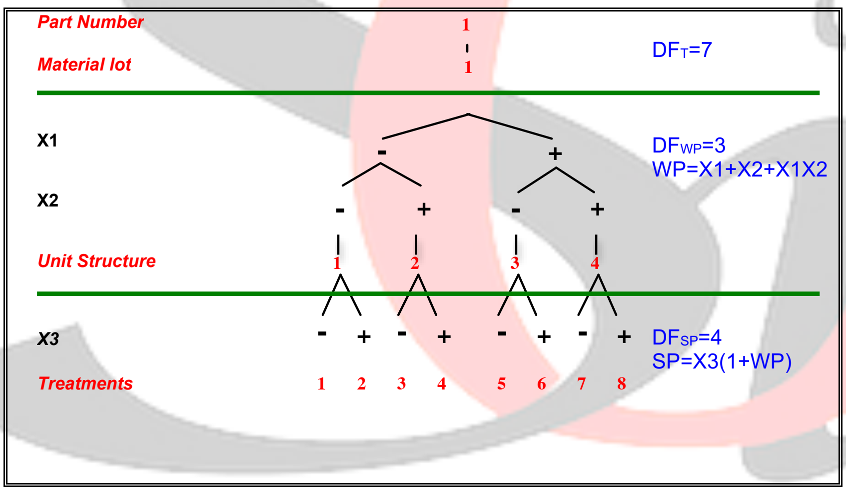
Strategy 3: Split-plots

Split-plot experiments can be seen as a practical and appropriate way to deal with certain situations that preclude randomization. The run order is executed by changing the subplot (SP) factor(s) while the whole plot (WP) factor(s) remains constant. Then the WP factor(s) is changed and the treatments replicated. This minimizes the number of times the whole plot factor(s) is changed.

There are two distinctly different reasons to restrict randomization. The runs of an experiment might be made in a split-plot fashion due to:

1. The desire to partition the unit structure and therefore manage the precision of the design structure (i.e., Efficiency Split-plot, figure 8):
    - Desired precision of design structure varies (e.g., design factors may interact with the noise and there is interest in noise-by-factor interactions)
    - Unit structure needs to be partitioned to increase design factor precision. The partitioning of the unit structure allows for greater precision when evaluating the design structure.

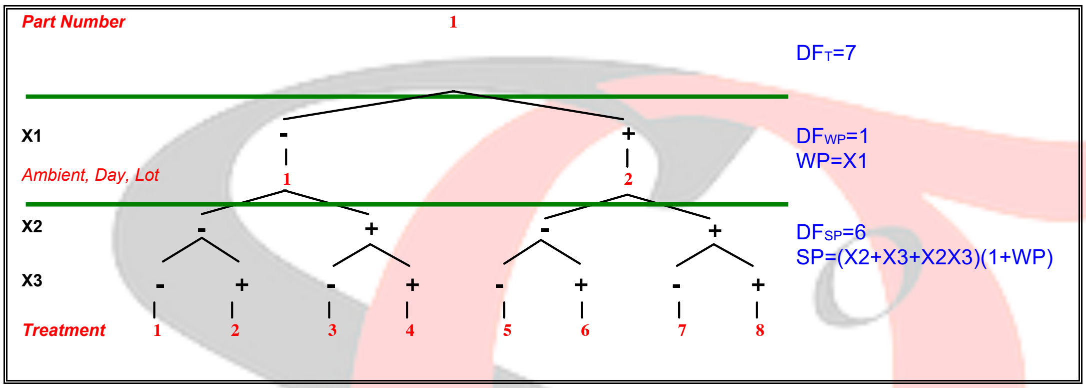
Figure 8: FRD for Hypothetical Efficiency Split-plot Design



2. Physical or economic reasons (i.e., Convenience Split-plot, figure 9):
    - One (or more) of the factors to be investigated is **hard or expensive** to change (it is not noise). This 'hard to change" factor(s) will be designated the **WP**. The other factors will make up the **SP**.
    - The experimenter wishes to make the experiment easier to execute.
    - If the unit structure is significantly partitioned, there will be a negative effect on the precision of the whole plot. In this case, it would NOT be desired to have a significant partitioning of the unit structure thereby not diminishing the precision of the whole plot.

For convenience split-plots, the information about the whole plot factor(s) is subject to **different** unit structure due to the minimized number of changes. For quantitative analysis, the whole plot error (rather than the mean square error of the subplot) should be used to determine whether the whole plot factor had a statistically significant effect. This requires replication. Replication of the runs at the whole plot level is necessary to have any information about the whole plot error and therefore a **quantitative** test of significance.

Figure 9: FRD for Hypothetical Convenience Split-plot Design

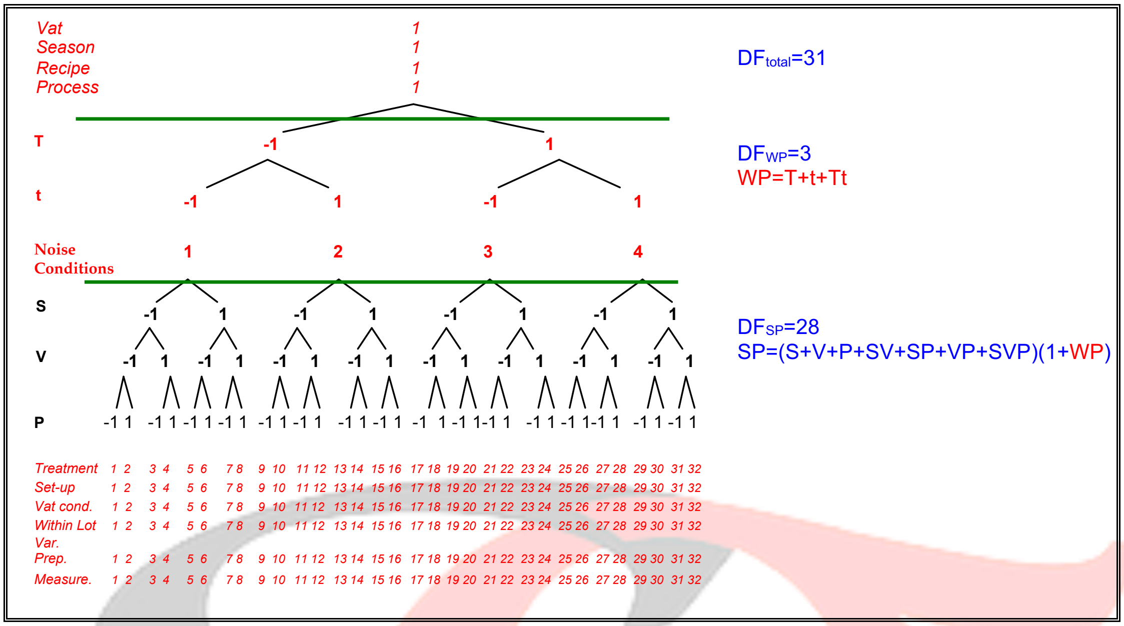


Example scenarios[9]: For the following illustration, consider a situation where the product is a sauce added to food. The response variable of interest is the taste of the sauce. The design factors for the recipe include amount of **V**inegar, variety of **P**eppers and type of **S**ecret ingredient. The unit structure includes **T**ime and **t**emperature of the cooking process (these are noise to the designer of the recipe).

1. A split-plot arrangement where the Noise matrix is the WP and the design factors create the subplots (SP factors), figure 10.

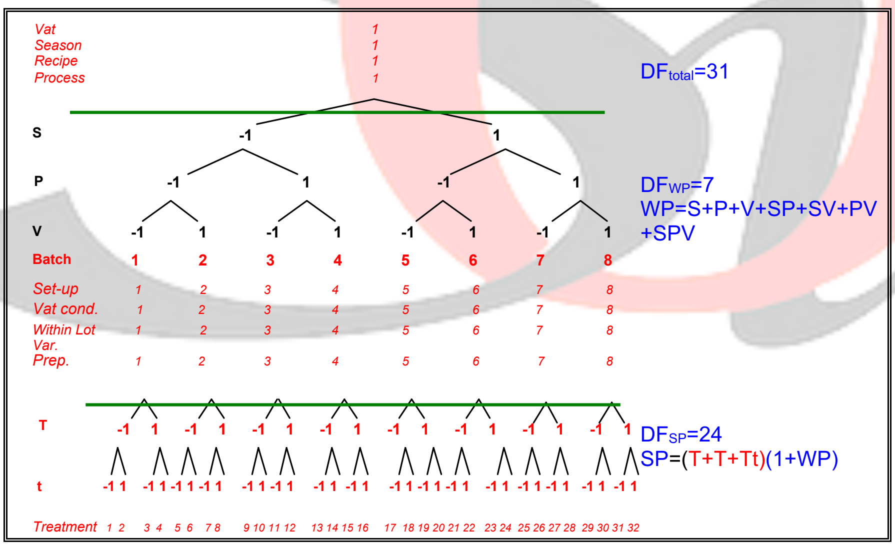
[9] Cross-product or Inner-Outer arrays

Figure 10: FRD Unit Structure in the Whole Plot



2. A split-plot arrangement where the design factors are in the WP and the Noise variables create the SP, figure 11.

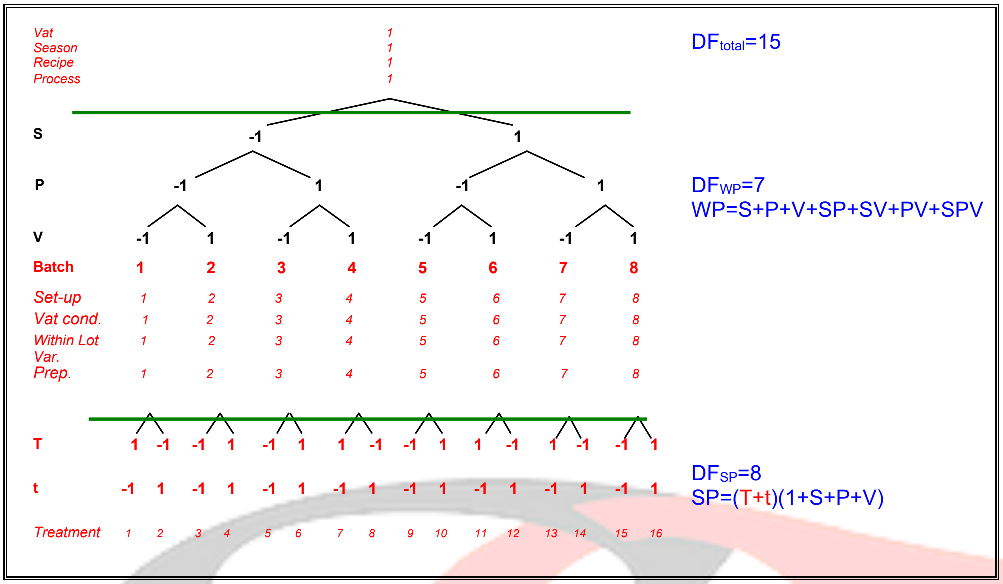
Figure 11: FRD Noise in the SP



3. Fractional split-plots using split-plot confounding[10] – an arrangement where the unit structure "matrix" (SP) is fractionated and two orthogonal fractions are run. The fractional SP's are confounded with the higher order interaction in the WP (SPV=Tt), figure 12.

[10] Split-plot confounding occurs when a fractional split-plot is run using two different fractions.

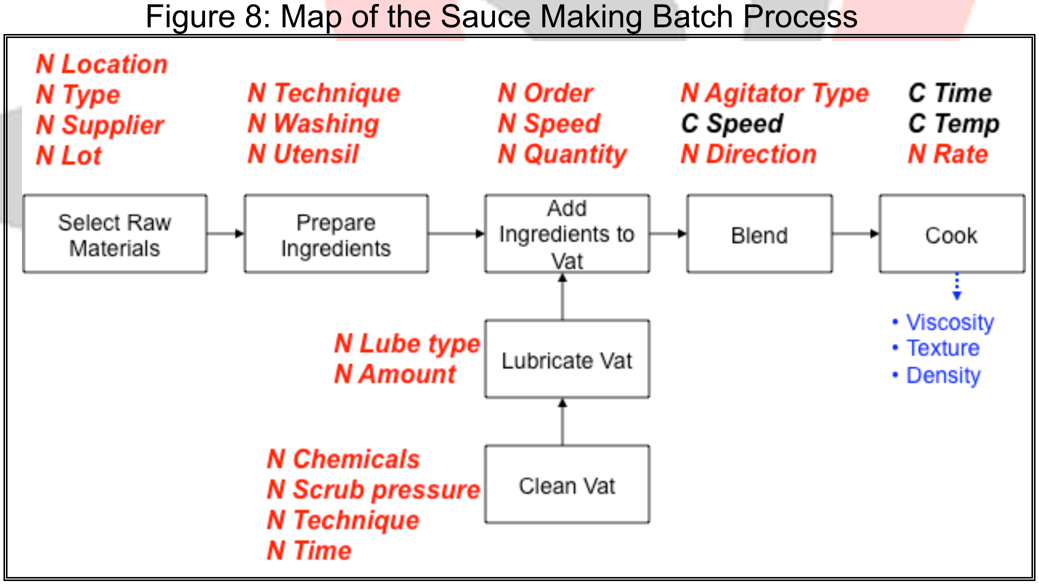
Figure 12: FRD Noise is fractionated in the SP



Case Study Example: Viscosity of a Sauce

The following example will illustrate the differences in the strategies previously considered. In this example an experiment is run to understand the effect of three design factors on the viscosity of a sauce. A process map used to identify the noise and design factors of the batch process is shown in figure 8.

Figure 8: Map of the Sauce Making Batch Process



The factors to be manipulated in the experiment are:

S: Speed of agitator
T: Time for cooking the batch
H: Temperature of the batch

The Y of interest in this study is the viscosity, measured in centipoise. Table 1 contains the matrix for the factors showing two experimental units for each treatment for a total of 16 experimental units. These results will be evaluated using several of the scenarios already discussed.

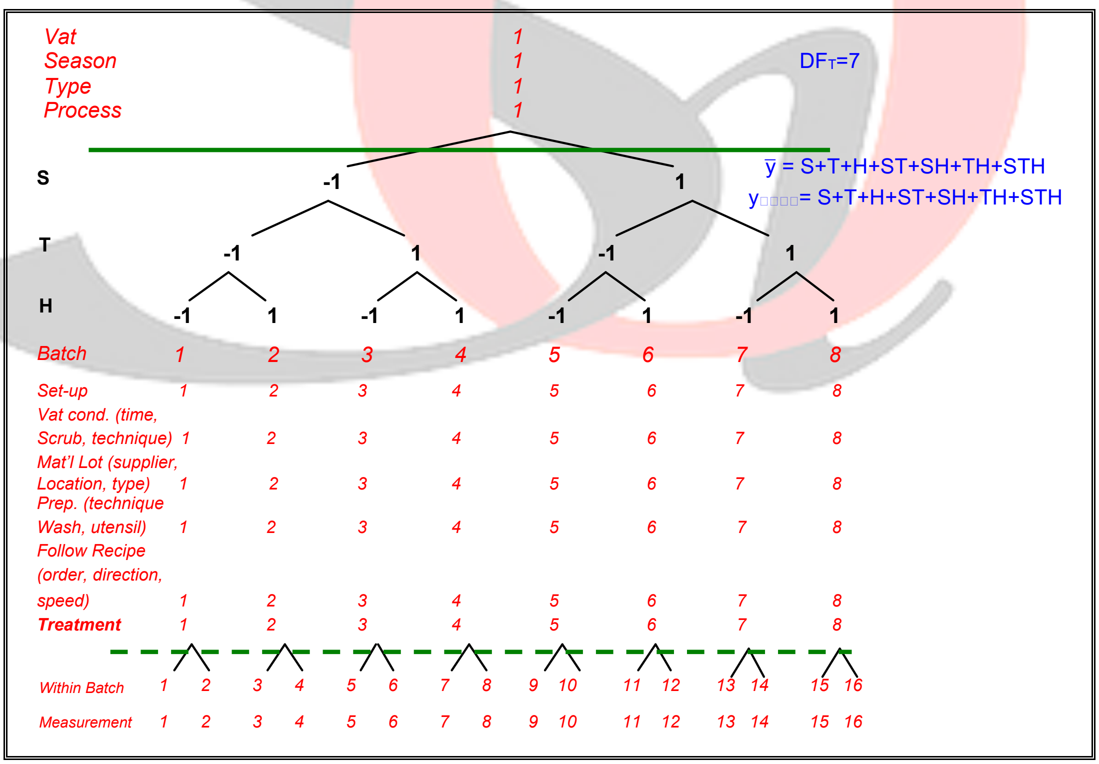
Table 1: The Design Factor Matrix and the Results of the Experiment

S	T	H	Y	Y
-1	-1	-1	6391	6429
1	-1	-1	6329	6395
-1	1	-1	6276	6310
1	1	-1	6245	6305
-1	-1	1	6090	6124
1	-1	1	6067	6106
-1	1	1	6058	6095
1	1	1	6044	6097

Scenario 1: Repeats

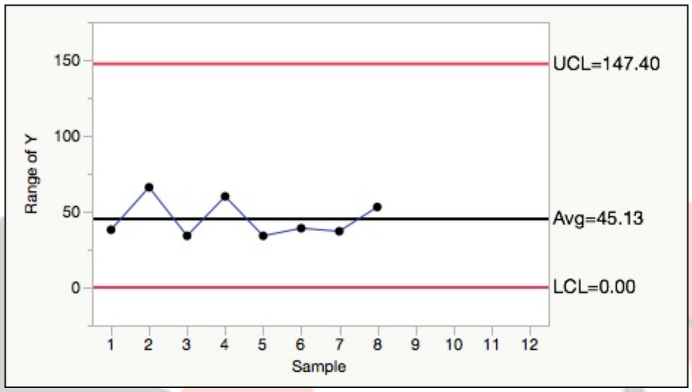
The first scenario is the strategy where the two data points are two samples of the batch created by the 8 treatments.

Figure 10: FRD Repeats



For repeats, the within batch and measurement system components of variation are partitioned (separated) from the treatments. As a note, the repeated measures could each be measured multiple times to further separate and assess measurement uncertainty. Without repeats, those sources of variation would be confounded with the treatments resulting in an increase in the unit structure being compared to the design factors. When averaged, the variation is reduced thereby increasing the precision of the design factors. When repeats are available, a test for consistency and outliers (special causes) within treatment is possible and is done using a range chart (figure 11).

Figure 11: Range chart of Y



As shown by the range chart, the variation within treatment is consistent so the repeated measures may be used to calculate estimates of central tendency and dispersion (i.e., it is appropriate to calculate the average and standard deviation within treatment shown in table 3).

Table 3: Statistics Calculated from the Within Treatment Data

S	T	H	Mean(Y)	Std Dev(Y)
-1	-1	-1	6410	26.87
-1	-1	1	6107	24.04
-1	1	-1	6293	24.04
-1	1	1	6076.5	26.16
1	-1	-1	6362	46.67
1	-1	1	6086.5	27.58
1	1	-1	6275	42.43
1	1	1	6070.5	37.48

The effects for both Y's are plotted on Normal plots to determine significance of factor effects (figure 12 & 13).

Figure 12: Normal Plot of Effects for Means, T & H are significant

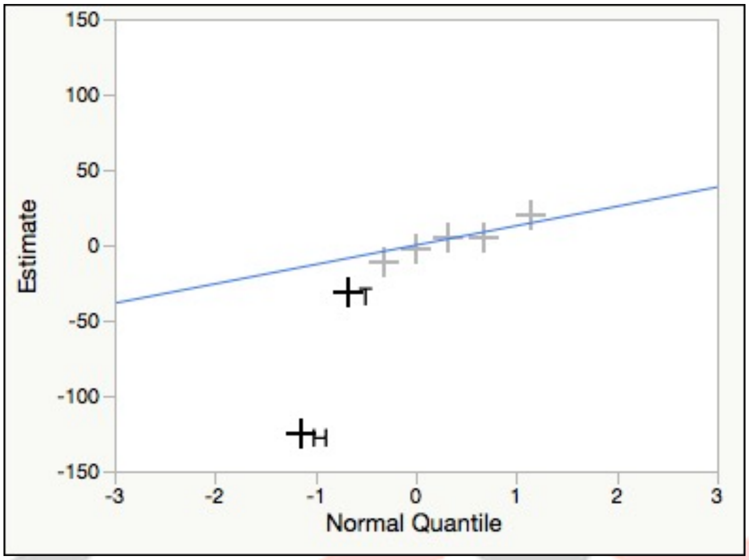
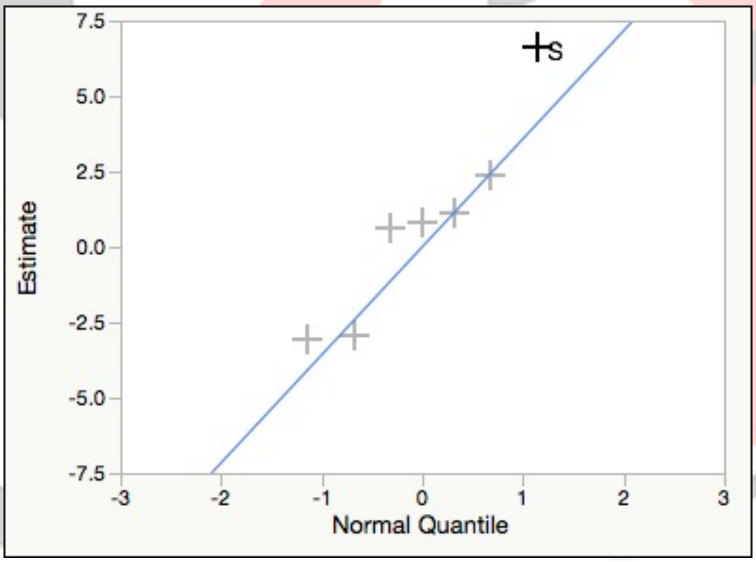


Figure 13: Normal Plot of Effects for Standard Deviation, S is significant

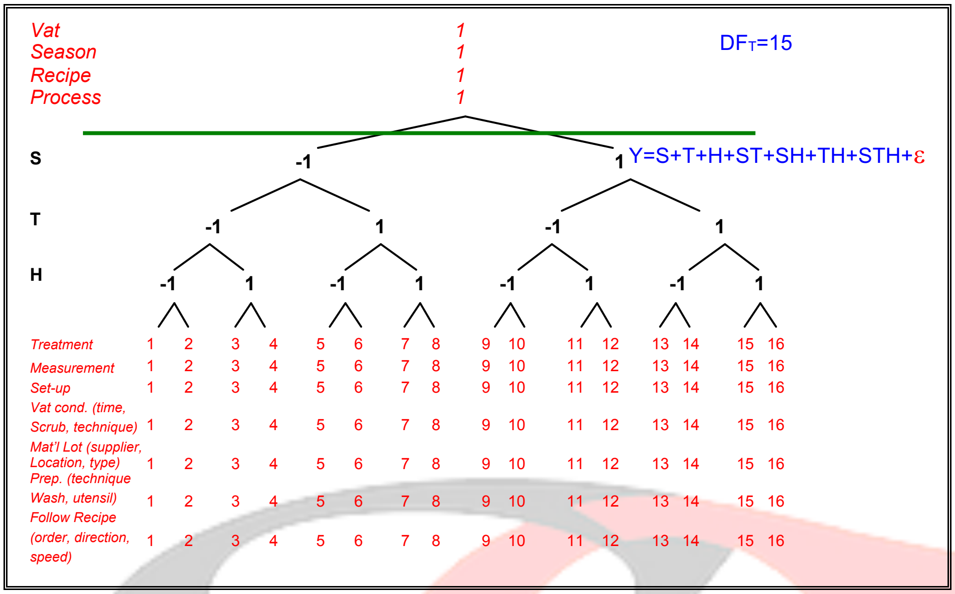


The effect of S on the variation within batch is discovered.

Scenario 2: **Completely Randomized Replicates**

This scenario is the typically advocated completely randomized replicates with the FRD shown in figure 9. Each EU for each treatment combination is created in random order.

Figure 9: FRD for CRR



In this scenario, the only partitioning of the unit structure is the unit structure in the inference space. The design factor effects will be compared to the unit structure confounded with treatments. This includes variation due to the measurement system, batch, etc. The Y columns from table 1 are stacked for analysis. Table 2 shows the analysis summary.

Table 2: ANOVA for CRR

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	7	274723.44	39246.2	36.0657
Error	8	8705.50	1088.2	**Prob > F**
C. Total	15	283428.94		<.0001

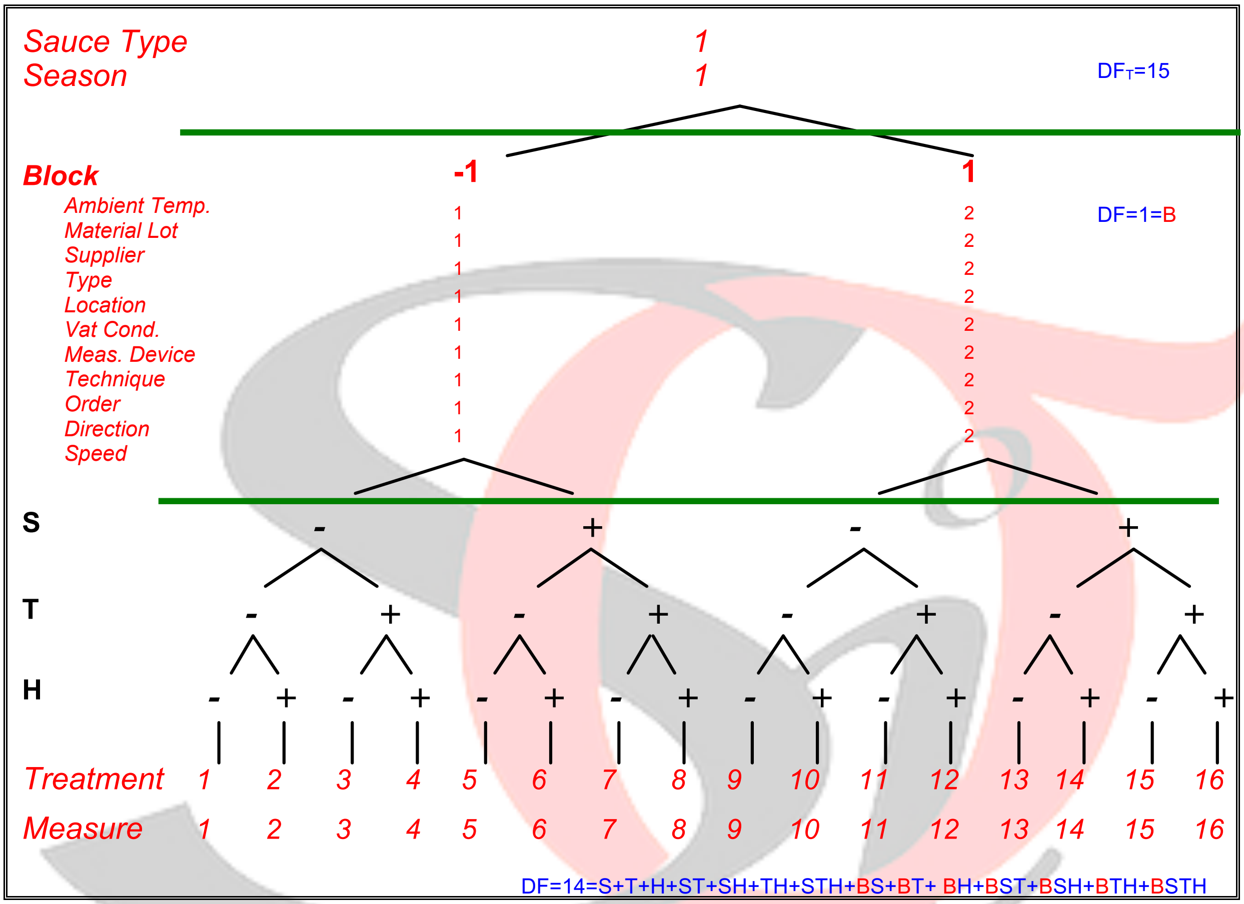
Source	DF	Sum of Squares	F Ratio	Prob > F
S	1	2139.06	1.9657	0.1985
T	1	15687.56	14.4162	0.0053
S*T	1	495.06	0.4549	0.5190
H	1	249750.06	229.5101	<.0001
S*H	1	390.06	0.3585	0.5659
T*H	1	6201.56	5.6990	0.0440
S*T*H	1	60.06	0.0552	0.8202

The results of this analysis indicate significant main effects of H & T and possibly a T*H interaction effect. These are significant based on the comparison of the factor effects to the Mean Square Error (un-assignable unit structure). Note the effect of **S** on variation is not exposed with this strategy.

Scenario 3: **Randomized Complete Block Design**

In the third scenario all 8 of the treatment combinations are run once (first EU) in the first block and then replicated in the second block (second EU). This results in a total of 16 treatments.

Figure 14: FRD for RCBD

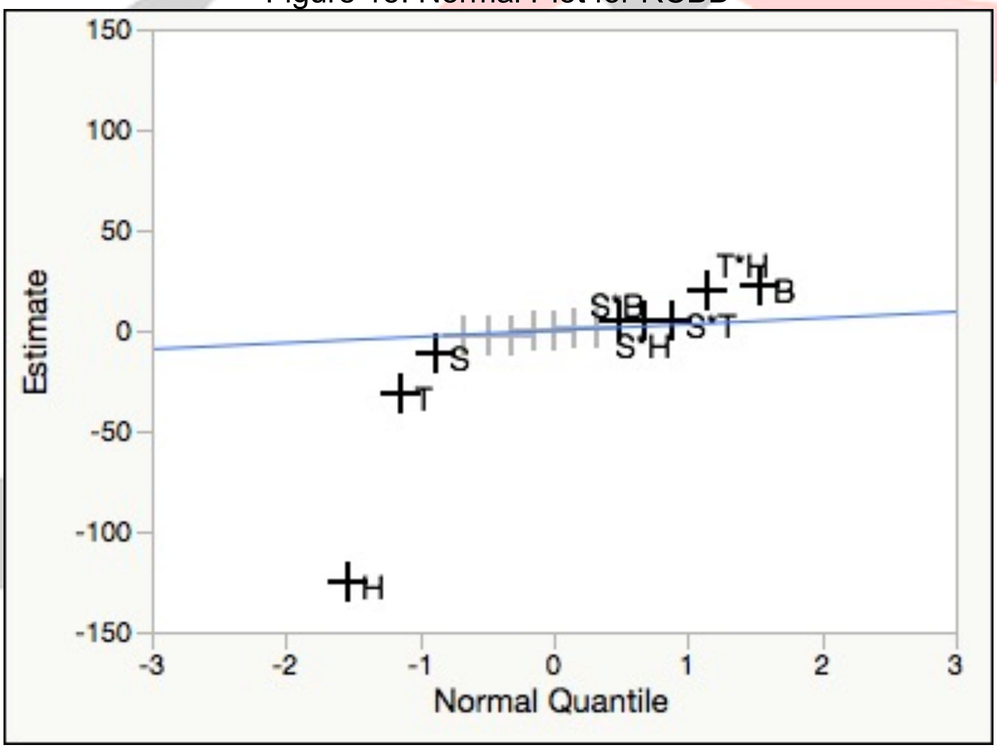


Unit structure previously in the inference space and unit structure previously confounded with treatments is allocated to the blocks. This has the effect of increasing the inference space and increasing the precision **simultaneously**. Analysis allows for assigning the degrees of freedom of both block and block-by-factor interactions. For this analysis the column B (for block) is added to the data set (table 4).

Table 4: Data Set Arranged for RCBD

B	S	T	H	Y
-1	-1	-1	-1	6391
-1	1	-1	-1	6329
-1	-1	1	-1	6276
-1	1	1	-1	6245
-1	-1	-1	1	6090
-1	1	-1	1	6067
-1	-1	1	1	6058
-1	1	1	1	6044
1	-1	-1	-1	6429
1	1	-1	-1	6395
1	-1	1	-1	6310
1	1	1	-1	6305
1	-1	-1	1	6124
1	1	-1	1	6106
1	-1	1	1	6095
1	1	1	1	6097

Figure 15: Normal Plot for RCBD



The ability to detect factor effects (precision) increases and the block effects are estimable. The main effects of H, T & S and the interaction of T*H as well as the block effect are all significant (as well as some other possible effects).

Disclaimer: While the same data has been used to illustrate the differences in the strategies, the correct analysis will always depend on how the data was actually acquired.

Conclusion

Table 5 provides a summary comparison of the strategies.

Table 5: Summary Comparison

Strategy	Pro	Con	Application
Repeats	• Test for outliers • Two models (mean and variation) • Increases precision	• Resources for multiple EU's	• Measurement error • Y in the form of variation
CRR	• Unbiased estimate of error • Unassigned DF's available • Increased inference space	• Noise un-assignable • "Unknown" tests of significance • Doubles the size of the experiment	• Unknown noise • Situations where the noise has not been identified
RCBD	• Block & block-by-factor interactions estimable • Increased precision • Increased inference space	• Doubles the size of the experiment	• Robust design • Design engineering
RIBD	• Block effect estimable • No additional treatments • Increased precision • Increased inference space	• Block-by-factor interactions aliased	• Manufacturing • Post design
Split-plots	• Increased precision • Increased inference space • Noise-by-factor interactions estimable	• Possible reduced precision of the WP	• Robust design • Design engineering

The most effective designed experiments include noise in the experiment. The idea is for the experiment to represent, as closely as possible, reality and in reality noise changes. The challenge is as the experiment more closely approximate reality, the noise increases and the ability to detect the factor effects decreases. Repeats, replicates and split-plots are three of a number of strategies to handle the noise effectively to both increase the precision without negatively affecting the inference space. This increases the likelihood the results of the experiment will be applicable in the future.

"*Block What You Can, Randomize What You Canno*t." (G.E.P. Box[11])

[11] Box, George, Hunter, William, and Hunter, J. Stuart (1978) *Statistics for Experimenters: An Introduction to Design, Data Analysis, and Model Building*, Wiley.

Definitions

ANOVA (ANalysis Of VAriance): a general term referring to a calculational procedure for allocating the amount of variation due to each effect in a factorial experiment. The usual objective is to test for differences among factor levels and/or treatment combinations.

Blocks: in industrial experimentation, blocks are frequently a frame where *noise* (x's not explicitly manipulated in the experiment), can reasonably be expected to remain constant (or are held constant) while that part of the experiment takes place. Subsequent replicates are selected so *that noise* changes *between* those blocks. In this manner, there is increased precision for the design factors and information regarding design factors is acquired across changing noise (e.g., environmental conditions, variation in raw materials, and other known and unknown noise)

Completely Randomized Replicates (CRR): the selection of all experimental units, the order of the application of the treatment combinations to the experimental units, and the order of measurement are all done randomly.

Cross Product Arrays: factorials of design factors run inside of factorial treatments of unit structure factors (also called inner and outer arrays)

Degrees of Freedom (DF): the number of independent pieces of information that can be used to estimate a statistic. Degrees of freedom can be thought of as the number of paired comparisons available to learn about a statistic.

Experimental Unit (EU): the independent output of a treatment combination from a designed experiment. (e.g., multiple parts, multiple measures of the same batch, multiple batches, etc.)

Factor Relationship Diagram (FRD): a graphical description of an experiment showing the relationship between manipulated factors and unit structure. It consists of design structure, unit structure and line(s) of restriction that depict partitioning of the unit structure and degrees of freedom.

Inference Space: the totality of material, conditions and processing techniques to which the data analysis results will apply.

Nested: a condition where one layer (set of x's) is contingent (dependent) upon another.

Noise: the set of x's one is unwilling to manage (for reasons of cost, difficulty or convenience).

Precision: the ability to detect effects.

Robust: the condition where performance and functionality is consistent over changing conditions (i.e., the absence of noise-by-factor interactions).

Scientific Method: the iterative process of induction and deduction.

Split-plot Designs: a method of handling restrictions on randomization for factorial designs.

Statistics: the science of extracting information from data. This *science* includes the collection, analysis, interpretation and communication of information based on data.

Treatment or Treatment Combination: a unique experimental condition in a factorial design defined by the specific level of each factor.

References

1. Fisher, R.A. (1926) “The Arrangement of Field Experiments”, *Journal of the Ministry of Agriculture of Great Britain*, 33: 503-513.
2. Fisher, R.A. (1931) “Principles of Plot Experimentation in Relation to the Statistical Interpretation of the Results”, *Conference on the Technique of Field Experiments*, Rothemsted, p 11-13.
3. Bergerud, Wendy A. (1996) “Displaying Factor Relationships in Experiments”, *The American Statistician*, August 1996, 50(3).
4. Sanders, Doug, and Cheryl Hild (modified by Ross, W., 2001) “Evaluation of Experimental Plans Using Factor Relationship Diagrams”, *Sigma Science Inc. Internal Paper*.
5. Hild, Cheryl, D. Sanders (2000) “The Thought Map”, *Quality Engineering*, Vol. 12, No. 1.
6. Box, G.E.P., Stephen Jones (1992), “Split-plot designs for robust product experimentation”, *Journal of Applied Statistics*, Vol. 19, No. 1
7. Sanders, Doug, W. Ross, and J. Coleman (2000), “The Process Map”, *Quality Engineering*, Vol. 11, No. 4.
8. Hild, Cheryl and Doug Sanders (2008), “Factor Relationship Diagrams: A Tool for Experimenters”, *Wiley’s Encyclopedia of Reliability and Statistics.*
9. Daniel, Cuthbert (1976) *Applications of Statistics to Industrial Experiments*, Wiley.
10. Ross, William (2009) “Considerations for Project Selection”, *Sigma Science Inc. Internal Paper*
11. Sanders, D., Leitnaker M., and McLean R. (2002) “Randomized Complete Block Designs in Industrial Studies”, *Quality Engineering*, Vol. 14, No. 1.
12. Box, George, Hunter, William, and Hunter, J. Stuart (1978) *Statistics for Experimenters: An Introduction to Design, Data Analysis, and Model Building*, Wiley.
13. Hwan, Marilyn (2000), “To Randomize or Not to Randomize, That is the Question”, *ASQ Statistics Division Newsletter*, Vol. 17, No. 1, 26

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