consistency in a model, and many statistical model builders rigorously follow the principle. However, hierarchy is not always a good idea, and many models actually work better as prediction equations without including the nonsignificant terms that promote hierarchy. For more information, see the supplemental text material for this chapter.

The computer output also gives model coefficient estimates and a final prediction equation for battery life in coded factors. In this equation, the levels of temperature are $A = -1, 0, +1$, respectively, when temperature is at the low, middle, and high levels (15, 70, and 125°C). The variables $B[1]$ and $B[2]$ are coded **indicator variables** that are defined as follows:

	Material Type		
	1	**2**	**3**
$B[1]$	1	0	−1
$B[2]$	0	1	−1

There are also prediction equations for battery life in terms of the actual factor levels. Notice that because material type is a qualitative factor there is an equation for predicted life as a function of temperature for each material type. Figure 5.18 shows the response curves generated by these three prediction equations. Compare them to the two-factor interaction graph for this experiment in Figure 5.9.

If several factors in a factorial experiment are quantitative a **response surface** may be used to model the relationship between y and the design factors. Furthermore, the quantitative factor effects may be represented by single-degree-of-freedom polynomial effects. Similarly, the interactions of quantitative factors can be partitioned into single-degree-of-freedom components of interaction. This is illustrated in the following example.

EXAMPLE 5.5

The effective life of a cutting tool installed in a numerically controlled machine is thought to be affected by the cutting speed and the tool angle. Three speeds and three angles are selected, and a 3^2 factorial experiment with two replicates is performed. The coded data are shown in Table 5.16. The circled numbers in the cells are the cell totals $\{y_{ij.}\}$.

Table 5.17 shows the JMP output for this experiment. This is a classical ANOVA, treating both factors as categorical. Notice that both design factors tool angle and speed as well as the angle–speed interaction are significant. Since the factors are quantitative, and both factors have three levels, a **second-order model** such as

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \varepsilon$$

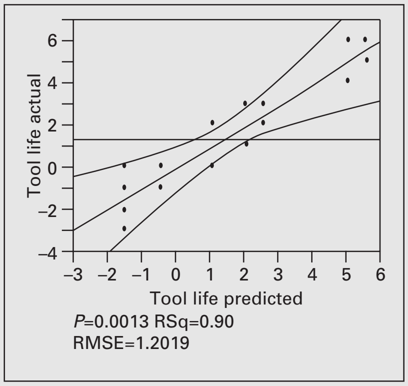
where x_1 = angle and x_2 = speed could also be fit to the data. The JMP output for this model is shown in Table 5.18. Notice that JMP "centers" the predictors when forming the interaction and quadratic model terms. The second-order model

■ **TABLE 5.16**
Data for Tool Life Experiment

Total Angle (degrees)	Cutting Speed (in/min) 125		150		175		$y_{i..}$
15	−2, −1	(−3)	−3, 0	(−3)	2, 3	(5)	−1
20	0, 2	(2)	1, 3	(4)	4, 6	(10)	16
25	−1, 0	(−1)	5, 6	(11)	0, −1	(−1)	9
$y_{.j.}$	−2		12		14		$24 = y_{...}$

■ **TABLE 5.17**
JMP ANOVA for the Tool Life Experiment in Example 5.5

Response Tool Life
Whole Model
Actual by Predicted Plot



Summary of Fit

RSquare	0.895161
RSquare Adj	0.801971
Root Mean Square Error	1.20185
Mean of Response	1.333333
Observations (or Sum Wgts)	18

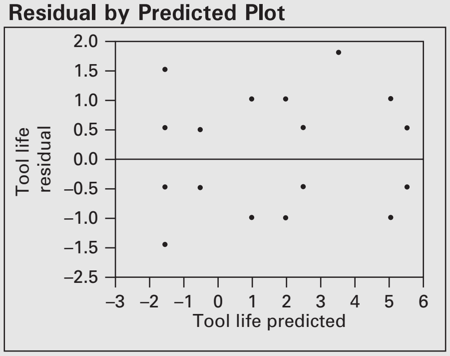
Analysis of Variance

Source	DF	Sum of Squares	Mean Square	*F* Ratio
Model	8	111.00000	13.8750	9.6058
Error	9	13.00000	1.4444	Prob > *F*
C. Total	17	124.00000		0.0013

Effect Tests

Source	Nparm	DF	Sum of Squares	*F* Ratio	Prob > *F*
Angle	2	2	24.333333	8.4231	0.0087
Speed	2	2	25.333333	8.7692	0.0077
Angle*Speed	4	4	61.333333	10.6154	0.0018

Residual by Predicted Plot

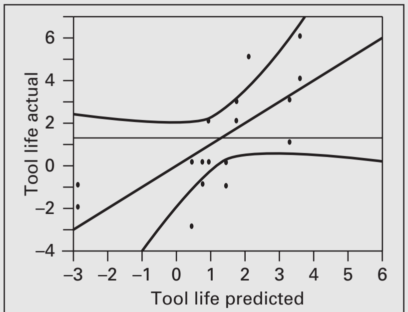


doesn't look like a very good fit to the data; the value of R^2 is only 0.465 (compared to $R^2 = 0.895$ in the categorical variable ANOVA) and the only significant factor is the linear term in speed for which the P-value is 0.0731. Notice that the mean square for error in the second-order model fit is 5.5278, considerably larger than it was in the classical categorical variable ANOVA of Table 5.17. The JMP output in Table 5.18 shows the **prediction profiler**, a graphical display showing the response variable life as a function of each design factor, angle and speed. The prediction profiler is very useful for optimization. Here it has been set to the levels of angle and speed that result in maximum predicted life.

Part of the reason for the relatively poor fit of the second-order model is that only one of the four degrees of freedom for interaction are accounted for in this model. In addition to the term $\beta_{12}x_1x_2$, there are three other terms that could be fit to completely account for the four degrees of freedom for interaction, namely $\beta_{112}x_1^2x_2$, $\beta_{122}x_1x_2^2$, and $\beta_{1122}x_1^2x_2^2$.

■ **TABLE 5.18**
JMP Output for the Second-Order Model, Example 5.5

Response Tool Life
Actual by Predicted Plot



P = 0.1377 RSq = 0.47
RMSE = 2.3511

Summary of Fit

RSquare	0.465054
RSquare Adj	0.242159
Root Mean Square Error	2.351123
Mean of Response	1.333333
Observations (or Sum Wgts)	18

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	5	57.66667	11.5333	2.0864
Error	12	66.33333	5.5278	Prob > F
C. Total	17	124.00000		0.1377

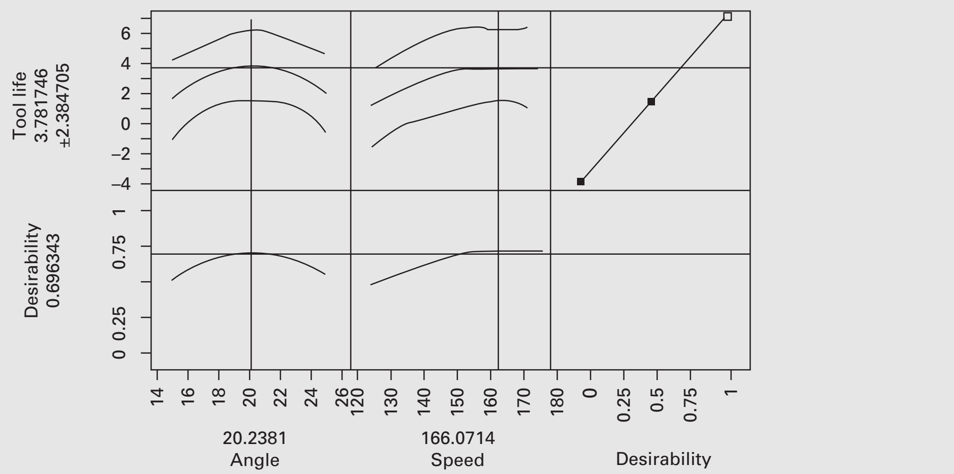
Parameter Estimates

Term	Estimate	Std. Error	t Ratio	Prob > |t|
Intercept	−8	5.048683	−1.58	0.1390
Angle	0.1666667	0.135742	1.23	0.2431
Speed	0.0533333	0.027148	1.96	0.0731

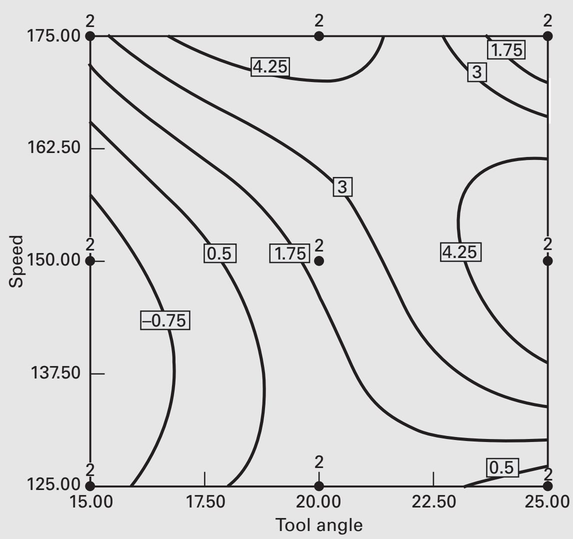
■ **TABLE 5.18** (*Continued*)

(Angle-20)*(Speed-150)	−0.008	0.00665	−1.20	0.2522
(Angle-20)*(Angle-20)	−0.08	0.047022	−1.70	0.1146
(Speed-150)*(Speed-150)	−0.0016	0.001881	−0.85	0.4116

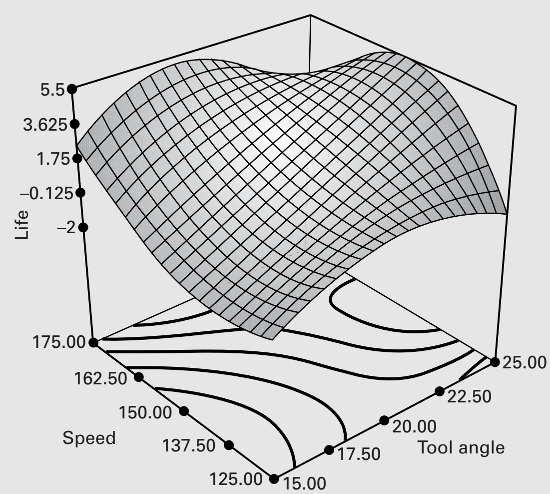
Prediction Profiler



JMP output for the second-order model with the additional higher-order terms is shown in Table 5.19. While these higher-order terms are components of the two-factor interaction, the final model is a reduced quartic. Although there are some large *P*-values, all model terms have been retained to ensure hierarchy. The prediction profiler indicates that maximum tool life is achieved around an angle of 25 degrees and speed of 150 in/min.



■ **FIGURE 5.19**
Two-dimensional contour plot of the tool life response surface for Example 5.5

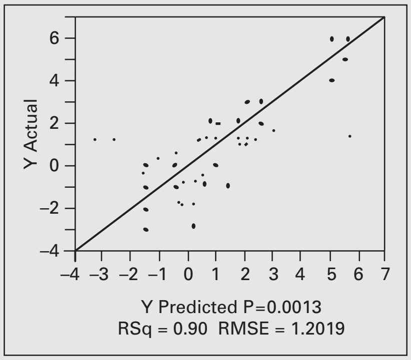


■ **FIGURE 5.20**
Three-dimensional tool life response surface for Example 5.5

Figure 5.19 is the contour plot of tool life for this model and Figure 5.20 is a three-dimensional response surface plot. These plots confirm the estimate of the optimum operating conditions found from the JMP prediction profiler. Exploration of response surfaces is an important use of designed experiments, which we will discuss in more detail in Chapter 11.

■ **TABLE 5.19**
JMP Output for the Expanded Model in Example 5.5

Response Y
Actual by Predicted Plot



Summary of Fit

RSquare	0.895161
RSquare Adj	0.801971
Root Mean Square Error	1.20185
Mean of Response	1.333333
Observations (or Sum Wgts)	18

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	8	111.00000	13.8750	9.6058
Error	9	13.00000	1.4444	**Prob > F**
C. Total	17	124.00000		0.0013*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	−24	4.41588	−5.43	0.0004*
Angle	0.7	0.120185	5.82	0.0003*
Speed	0.08	0.024037	3.33	0.0088*
(Angle-20)*(Speed-150)	−0.008	0.003399	−2.35	0.0431*
(Angle-20)*(Angle-20)	2.776e-17	0.041633	0.00	1.0000
(Speed-150)*(Speed-150)	0.0016	0.001665	0.96	0.3618
(Angle-20)*(Speed-150)*(Angle-20)	−0.0016	0.001178	−1.36	0.2073
(Speed-150)*(Speed-150)*(Angle-20)	−0.00128	0.000236	−5.43	0.0004*
(Angle-20)*(Speed-150)*(Angle-20)*(Speed-150)	−0.000192	8.158a-5	−2.35	0.0431*

■ **TABLE 5.19** (*Continued*)

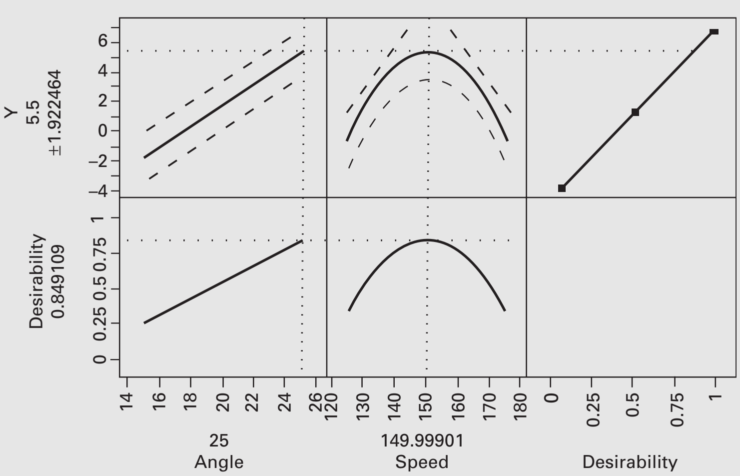
Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Angle	1	1	49.000000	33.9231	0.0003*
Speed	1	1	16.000000	11.0769	0.0088*
Angle*Speed	1	1	8.000000	5.5385	0.0431*
Angle*Angle	1	1	6.4198e-31	0.0000	1.0000
Speed*Speed	1	1	1.333333	0.9231	0.3618
Angle*Speed*Angle	1	1	2.666667	1.8462	0.2073
Speed*Speed*Angle	1	1	42.666667	29.5385	0.0004*
Angle*Speed*Angle*Speed	1	1	8.000000	5.5385	0.0431*

Sorted Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Angle	0.7	0.120185	5.82	0.0003*
(Speed-150)*(Speed-150)*(Angle-20)	−0.00128	0.000236	−5.43	0.0004*
Speed	0.08	0.024037	3.33	0.0088*
(Angle-20)*(Speed-150)*(Angle-20)*(Speed-150)	−0.000192	8.158a-5	−2.35	0.0431*
(Angle-20)*(Speed-150)	−0.008	0.003399	−2.35	0.0431*
(Angle-20)*(Speed-150)*(Angle-20)	−0.0016	0.001178	−1.36	0.2073
(Speed-150)*(Speed-150)	0.0016	0.001665	0.96	0.3618
(Angle-20)*(Angle-20)	2.776e-17	0.041633	0.00	1.0000

Prediction Profiler



5.6 Blocking in a Factorial Design

We have discussed factorial designs in the context of a completely randomized experiment. Sometimes, it is not feasible or practical to completely randomize all of the runs in a factorial. For example, the presence of a nuisance factor may require that the experiment be run in blocks. We discussed the basic concepts of blocking in the context of a single-factor experiment in Chapter 4. We now show how blocking can be incorporated in a factorial. Some other aspects of blocking in factorial designs are presented in Chapters 7, 8, 9, and 13.