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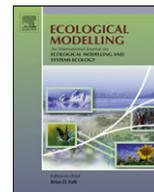
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Variance-based sensitivity analysis of a forest growth model

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ABSTRACT

Computer models are increasingly used to simulate and predict the behaviour of forest systems. Uncertainties in both parameter calibration and outputs co-exist in these models due to both the incomplete understanding of the system under simulation, and biased model structure. We used sensitivity analysis, including both screening and global variance-based methods, to explore these uncertainties. We applied these techniques to the widely used forest growth model Physiological Principles for Predicting Growth (3-PG2) using field data from 141 plots of *Corymbia maculata* and *Eucalyptus cladocalyx* in Australia. The screening method was used to select influential input parameters for the subsequent variance-based analysis and thereby reduce its computational cost. We assessed model outputs including biomass partitioning and water balance, and the sensitivities of the soil texture group, which includes 7 parameters. We also compared the screening and variance-based methods, and assessed the convergence of the variance-based method, and the change in sensitivities over time. Using these techniques, we quantified the relative sensitivities of each model output to each input parameter. The variance-based method exhibited good convergence and stable sensitivity rankings. The results indicated changes in input parameter sensitivities over longer simulation periods. The variance-based global sensitivity analysis can be very effective in calibration and identification of important processes within forest models.

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1. Introduction

Computer models are routinely used to understand forest systems (Battaglia et al., 2004; Landsberg and Waring, 1997; Pan et al., 2011; White et al., 2000). This is in part due to their ability to incorporate high levels of complexity that are characteristic of these systems. Although a computer model can be seen as series of mathematic functions that connect certain inputs and outputs, they are often complex, and this presents a barrier to the quantitative analysis of model performance (Morris, 1991). Another consequence of complex models is that the uncertainties in model structure, estimates of the model parameters, and the unexplained random variation in observed variables all increase greatly (Chatfield, 1995). For model parameters with high uncertainty and high sensitivity, a small perturbation in parameter values may have exaggerated effects on the outputs (Makler-Pick et al., 2011; Xu and Gertner, 2007). Thus, understanding the contribution of model structure and parameter estimation to the total model uncertainty is important in both model application and development (Cariboni et al., 2007;

Makler-Pick et al., 2011; Saltelli and Annoni, 2010; Saltelli et al., 2008).

To quantify the effect of different sources of uncertainty in forest model inputs on variability of model outputs, sensitivity analysis (SA) can be applied (Saltelli et al., 2008). SA evaluates the relative importance of each input parameter and can be used to identify the most influential parameters in determining the variability of model outputs. Uninfluential parameters can also be identified and be safely set in relatively wide ranges (Cariboni et al., 2007). In general, SA methods can be categorized as either local or global. Local SA is usually derivative-based and belongs to the class of “one-factor-at-a-time” (OAT). OAT methods involve changing one input parameter at a time whilst holding all others at their central values and variation in the outputs is measured. A critique of OAT methods is that they are only informative at the central point where the calculation is executed and do not cover the whole input parameter space. Thus, local SA methods are inadequate for analysing complex biophysical process models which may have many parameters, and may be high-dimensional and/or non-linear (Saltelli and Annoni, 2010; Yang, 2011). A model-independent global SA technique is preferable for these models (Helton et al., 2006; Nossent et al., 2011; Saltelli, 2000; Saltelli and Annoni, 2010).

Compared with local SA, global SA explores the full input parameter space, and the contribution of each input parameter to the

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Table 1
Input parameters of 3-PG2, including description, classification and values for *C. maculata*/*E. cladocalyx*, subjected to the sensitivity analysis.

Description	Name	Unit	Reference value	Description	Name	Unit	Reference value
<i>Allometric relationships and partitioning</i>				Specific leaf area for mature leaves	<i>SLA1</i>	m ² kg ⁻¹	5.365
Foliage:stem partitioning ratio @ DBH = 20 cm	<i>pFS20</i>	–	0.3	Age at which specific leaf area = (<i>SLA0</i> + <i>SLA1</i>)/2	<i>tSLA</i>	years	2.5
Constant in the stem mass vs. diameter relationship	<i>aWS</i>	–	0.1032	<i>Light interception & VPD attenuation</i>			
Power in the stem mass vs. diameter relationship	<i>nWS</i>	–	2.5447	Extinction coefficient for absorption of PAR by canopy	<i>k</i>	–	0.5
Maximum fraction of NPP to roots	<i>pRx</i>	–	0.8	Age at canopy cover	<i>fullCanAge</i>	years	3
Minimum fraction of NPP to roots	<i>pRn</i>	–	0.2	LAI for 50% reduction of VPD in canopy	<i>cVPD0</i>	–	5
Volume of soil accessed by 1 kg of root dry matter	<i>spRootVol</i>	m ³ kg ⁻¹	3.8	<i>Rainfall interception</i>			
<i>Litterfall and root turnover</i>				Maximum thickness of water retained on leaves	<i>tWaterMax</i>	mm	0.25
Maximum litterfall rate	<i>gammaF1</i>	month ⁻¹	0.008	Maximum proportion of rainfall evaporated from canopy	<i>MaxIntcptn</i>	–	0.15
Litterfall rate at t = 0	<i>gammaF0</i>	month ⁻¹	0.001	LAI for maximum rainfall interception	<i>LAImaxIntcptn</i>	–	3
Age at which litterfall rate has median value	<i>tgammaF</i>	months	10	<i>Production and respiration</i>			
Average monthly root turnover rate	<i>gammaR</i>	month ⁻¹	0.015	Maximum canopy quantum efficiency	<i>alphaCx</i>	mol mol ⁻¹	0.06
<i>Temperature modifier</i>				Conductance			
Minimum temperature for growth	<i>Tmin</i>	°C	10	Maximum canopy conductance	<i>MaxCond</i>	m s ⁻¹	0.02
Optimum temperature for growth	<i>Topt</i>	°C	30	LAI for maximum canopy conductance	<i>LAIgCx</i>	–	3.33
Maximum temperature for growth	<i>Tmax</i>	°C	40	Defines stomatal response to VPD	<i>CoeffCond</i>	mbar ⁻¹	0.05
<i>Fertility effects</i>				Canopy aerodynamic conductance	<i>gAc</i>	m s ⁻¹	0.22
Value of 'fNutr' when fertility rating = 0	<i>fN0</i>	–	0.6	<i>Branch and bark fraction (fracBB)</i>			
<i>Age modifier</i>				Branch and bark fraction at age 0	<i>fracBB0</i>	–	0.6
Maximum stand age used in age modifier	<i>MaxAge</i>	years	50	Branch and bark fraction for mature stands	<i>fracBB1</i>	–	0.41
Power in age modifier	<i>nAge</i>	–	4	Age at which $fracBB = (fracBB0 + fracBB1)/2$	<i>tBB</i>	years	4.3
Relative age at age modifier = 0.5	<i>rAge</i>	–	0.95	<i>Basic density</i>			
<i>Stem mortality and self-thinning</i>				Minimum basic density – for young trees	<i>rho0</i>	t m ⁻³	0.6
Max. stem mass per tree @ 1000 trees/ha	<i>wS</i> × 1000	kg tree ⁻¹	300	Maximum basic density – for older trees	<i>rho1</i>	t m ⁻³	0.6
Specific leaf area				Age at which whole tree basic density = (<i>rhoMin</i> + <i>rhoMax</i>)/2	<i>tRho</i>	years	4.5
Specific leaf area at age 0	<i>SLA0</i>	m ² kg ⁻¹	5.365				

variation in outputs is averaged over the variation of all input parameters, i.e. all input parameters are changed together (Saltelli et al., 1999). The most popular variance-based methods include the Fourier amplitude sensitivity analysis test (FAST) (Cukier et al., 1973; Xu and Gertner, 2007), the Sobol' method (Sobol', 1990), and Saltelli's method (Saltelli et al., 2010). In this study, we used Saltelli's method which has been demonstrated to be effective in identifying both the main sensitivity effects (first-order effects) (the contribution to the variance of the model output by each input) and the total sensitivity effects (the first-order effect plus interactions with other inputs) of input parameters. However, as it relies on Monte Carlo simulation to sample over the entire parameter space, a major barrier to the use of this method is its high computational cost (Nossent et al., 2011; Saltelli et al., 2010). Fortunately, the increasing availability of high-performance computing resources has reduced this barrier (Bryan, 2012).

Global SA methods can help to identify influential model parameters and processes which constitute the major portion of the

uncertainties in model outputs, understand model processes and enhance model reliability (Nossent et al., 2011). Global SAs have been applied to several different kinds of models in forestry (Wu et al., 2011; Xu et al., 2009), hydrology (Nossent et al., 2011; Tang et al., 2007), ecology (Makler-Pick et al., 2011) and biology (Marino et al., 2008). Here, we focus on the application of global SA to forest ecosystem models. Models of forest ecosystems are vulnerable to the impact of uncertainties originating from the limited understanding of forest ecological processes, biased model structure and complex environmental interactions (Dale et al., 2001; van Oijen et al., 2005). To improve the accuracy of prediction, a thorough understanding of the underlying model processes and the sensitivities of the major model outputs to the input parameters is needed.

We undertook the variance-based SA on a moderately complex, parameter-rich forest growth model—Physiological Principles for Predicting Growth (3-PG2). Most of the input parameters in 3-PG2 are typically adjusted manually through a combination of experience and field data from forest stands. The model has been subject

to several local sensitivity studies (Almeida et al., 2007; Esprey et al., 2004; Sands and Landsberg, 2002). Due to the high computational requirement of the variance-based SA for parameter-rich models, we used a 2-stage evaluation strategy (Campolongo and Saltelli, 1997). First, a low-cost screening method—elementary effects (EE) (Campolongo et al., 2007; Morris, 1991)—was used to screen out the least important parameters. As an extension of the EE method, we analysed the sensitivities of a group of soil-related parameters to model outputs (Campolongo et al., 2007; Saltelli et al., 2008). We then used the variance-based method to calculate the first-order and total sensitivities of the most influential input parameters on the major model outputs (Saltelli et al., 2010). To evaluate the distributions of the sensitivity metrics, a bootstrap technique was applied (Efron and Tibshirani, 1993). We also assessed the convergence in sensitivities as the model was run over a longer simulation period. The advantages of global SA for understanding forest model structure and sensitivities are discussed.

2. Materials and methods

2.1. Overview of the 3-PG2 model

The 3-PG2 model simulates the growth of even-aged, relatively homogeneous forests or plantations through the simplification of well-established and richly parameterized physiological processes. The model has been calibrated for a few specific tree species, such as *Eucalyptus* species (Almeida et al., 2004b, 2010; Paul et al., 2007; Sands and Landsberg, 2002), loblolly pine (*Pinus taeda* L.) (Landsberg et al., 2001) and Douglas-Fir (*Pseudotsuga menziesii* (Mirb.) Franco) (Coops et al., 2010). This model has been widely used to quantify forest stand growth for a range of applications including assessing land use trade-offs between carbon bio-sequestration and biodiversity conservation (Crossman et al., 2011), agricultural production (Paterson and Bryan, 2012) and bioenergy (Bryan et al., 2008, 2010).

2.1.1. Structure

The 3-PG2 model estimates both biomass production and partitioning between various components of trees (Coops et al., 1998; Landsberg et al., 2003). Biomass production is driven by the radiation intercepted by the canopy and canopy photosynthesis. The gross primary production (*GPP*) is determined by the canopy quantum efficiency (*QE*) and absorbed photosynthetically active radiation (*APAR*). *QE* is, in turn, determined by canopy conductance, air temperature, frost, vapour pressure deficit (*VPD*), available soil water, soil nutritional status and stand age. A constant ratio between net primary production (*NPP*) and *GPP* is assumed so that the calculation of respiration is not necessary (Landsberg and Waring, 1997). The *NPP* partitioning between roots and above-ground is determined by soil fertility rate and several growth modifiers (e.g. temperature, soil water and age). For the above-ground biomass, a series of allometric equations related to stem diameter at breast height (*DBH*) are applied to determine carbon allocation between foliage and stem (Landsberg and Waring, 1997; Landsberg et al., 2003).

2.1.2. Inputs and outputs

The 3-PG2 model runs in a monthly time step. The model requires climatic data specified as monthly average values for short wave solar radiation, mean maximum and minimum air temperature, *VPD*, frost days and rainfall. Other site-specific parameters include latitude, soil texture, maximum available soil water, soil fertility rating and initial number of stems per hectare. Species parameter values have usually been determined by field observations and combined with empirical formulas. Detailed parameterizations can be found in Paul et al. (2008), Sands and

Table 2

A list of the major 3-PG2 output variables used in the sensitivity analysis.

Output variable	Unit	Description
avDBH	cm	Stand-based mean diameter at breast height
BasArea	m ² ha ⁻¹	Stand basal area
LAI	m ² m ⁻²	Canopy leaf area index
StandVol	m ³ ha ⁻¹	Stand volume excluding bark and branch
ET	mm	Evapotranspiration rate in current period
fASW	mm	Plant available soil water under forest
Transp	mm	Monthly transpiration rate in current period
WF	t ha ⁻¹	Foliage biomass
WS	t ha ⁻¹	Stem biomass including branches and bark
WR	t ha ⁻¹	Root biomass

Landsberg (2002) and Almeida et al. (2004a). In this study, we used the input parameters listed in Table 1. This selection covers: (1) allometric parameters which determine the biomass allocation in different parts of a tree (e.g. *aWS*, *nWS*, *pRx*, *pRn* and *fracBB1*) and parameters which determine the stand volume (e.g. *rho1*); (2) leaf area, photosynthesis and water availability related parameters (e.g. *SLA1*, *k*, *alphaCx*, *MaxCond*, *LAIgcx*, *CoeffCond* and *tWaterMax*); and (3) biomass removal/turnover rates in above-ground components and root (e.g. *gammaF1*, *gammaR*).

Outputs can be specified as monthly or annual. We analysed the sensitivity of ten selected output variables from 3-PG2 (Table 2) to each of the input parameters (Table 1). For clarity below, we used normal font for output variables and *italics* for input parameters. The ten selected output variables can be classified into three categories: (1) growth/volume related variables (i.e. avDBH, BasArea, LAI and StandVol); (2) water related variables (i.e. ET, Transp, fASW); and (3) biomass allocation related variables (i.e. WF, WS, WR).

2.2. Data sources for sensitivity analysis

We used growth measurements for *Corymbia maculata* and *Eucalyptus cladocalyx* plantations from 141 sites across Australia to undertake SA for 3-PG2. These two species were grouped into one dataset due to their similarities in growth rates (Paul et al., 2008). For each site, stand data included planting date, initial and final stocking, initial available soil water and initial biomass (foliage, root and stem). Additionally, site factors including geographic location, climatic data and soil properties (e.g. depth, texture and fertility rating) were also recorded. The monthly average climatic data during the plant growth period were obtained and summarised from daily climatic data (Jeffrey et al., 2001). For all the sites during the study periods, the mean annual air temperature varied between 8 and 25 °C, and the mean annual rainfall varied between 100 and 2974 mm.

In 3-PG2, the available soil water is determined by soil depth and texture. A soil terrain analysis technique (MRVBF) was used to determine soil profiles deeper than 2 m. A total of 12 soil textural classes can be used in 3-PG2 (Almeida et al., 2007; Polglase et al., 2008). For each soil texture (*SoilTexture*) there are seven soil attributes, which include the critical soil water content (i.e. soil water content at saturation (*SWsat*), field capacity (*SWfcap*) and permanent wilting point (*SWwilt*)); the growth modifying effect of relative available soil water (*cTheta* and *nTheta*), and; *kDrain* and *kSCond* that define the diffusion of water from non-rooted to rooted zone of the soil profile (Almeida et al., 2007). Soil fertility rating (*FR*) is an empirical index between 0 and 1 to describe nutrient availability.

In addition to *FR* and soil texture group of parameters, another 38 species-specific input parameters were evaluated in the sensitivity analysis (Table 1). As there is no information about the prior probability distributions for each parameter, we assumed an

independent uniform distribution for each parameter with bounds varying 30% either side of its reference value (Esprey et al., 2004; van Oijen et al., 2005). Some of the more reliably estimated input parameters were kept constant, such as ambient CO₂ concentration and the ratio between NPP and GPP (Almeida et al., 2007; Landsberg and Waring, 1997).

2.3. Elementary effects (EE) method

To describe the EE method, let's assume a model with input X_i , $i = 1, \dots, k$, which varies across p selected levels. For a given value of X , the elementary effects of the i -th input parameter is defined as follows:

$$EE_i(X) = \frac{f(x_1, \dots, x_{i-1}, x_i + \Delta, \dots, x_k) - f(X)}{\Delta} \quad (1)$$

where Δ is a predefined value in $\{1/(p-1), \dots, 1-1/(p-1)\}$, and $X = (x_1, \dots, x_{i-1}, x_i, \dots, x_k)$ is any random sample in the input parameter space whilst the transformed point $(x_1, \dots, x_{i-1}, x_i + \Delta, \dots, x_k)$ is still in the input parameter space. A sampling strategy suggested by Morris (1991) was applied to randomly sample different X from the input parameter space, with each providing n elementary effects corresponding to each input parameter, forming a so-called trajectory, r . In practice, an even value for p is preferred and $\Delta = p/2(p-1)$. We adopted a scheme of $p=4$, $\Delta = 2/3$ and $r=10$ (Saltelli et al., 2008). For EE analysis with r trajectories and k input parameters, the total model simulation number is $r(k+1)$.

Morris (1991) proposed a statistic, μ to represent the mean of the distribution of the EE_i . A larger value for μ represents a higher overall influence of a certain input parameter on the output. However, for models that are non-monotonic or have interaction effects, it is common that some elementary effects may cancel out due to opposite signs, thus increasing the risk of failing to identify important parameters (Saltelli et al., 2008). Campolongo et al. (2007) proposed a revised version of μ , termed μ^* , which is the mean of the absolute value of the elementary effects, i.e. $|EE_i|$, to address the above problem with using μ .

We also adopted the extension to the EE method using the μ^* measure proposed by Saltelli et al. (2008) to address the cases in which groups of parameters need to be evaluated so as to produce overall sensitivity indices to each group. The basic idea is to move all the parameters belonging to the same group simultaneously. This differs with the original EE method in which only one parameter is incremented per elementary effect calculation (Campolongo et al., 2007).

2.4. Variance-based sensitivity analysis

The variance-based global sensitivity analysis approach can be used to quantify the first-order effect and total effect (which includes the interactions with other parameters) on the variance of model output (Nossent et al., 2011). More formally, given a model $Y = f(X)$, where Y is the model output, $X = (X_1, X_2, \dots, X_k)$ is the input parameter vector. A variance decomposition of f suggested by Sobol' (1990) is:

$$V(Y) = \sum_{i=1}^k V_i + \sum_{i=1}^k \sum_{j=i+1}^k V_{ij} + \dots + V_{1,\dots,k} \quad (2)$$

where X is rescaled to a k -dimensional unit hypercube Ω^k , $\Omega^k = \{X | 0 \leq X_i \leq 1, i = 1, \dots, k\}$; $V(Y)$ is the total unconditional variance; V_i is the partial variance or 'main effect' of X_i on Y and given by $V_i = V[E(Y|X_i)]$; V_{ij} is the joint impact of X_i and X_j on the total variance minus their first-order effects.

Here, the first-order sensitivity index S_i and total effect sensitivity index S_{Ti} are given as (Saltelli et al., 2008):

$$S_i = \frac{V_i}{V(Y)} = \frac{V[E(Y|X_i)]}{V(Y)} \quad (3)$$

$$S_{Ti} = S_i + \sum_{j \neq i} S_{ij} + \dots = \frac{E[V(Y|X_{-i})]}{V(Y)} \quad (4)$$

where X_{-i} denotes variation on all input parameters but X_i , S_{ij} is the contribution to the total variance by the interactions between parameters.

Following Saltelli et al. (2010), to compute S_i and S_{Ti} , we created two independent input parameter sampling matrices P and Q with dimension (N, k) , where N is the sample size and k is the number of input parameters. Each row in matrix P and Q represents a possible value of X . The Monte Carlo approximations for $V(Y)$, S_i and S_{Ti} are defined as follows (Jansen, 1999; Nossent et al., 2011; Saltelli et al., 2010):

$$\hat{f}_0 = \frac{1}{N} \sum_{j=1}^N f(P)_j \quad (5)$$

$$\hat{V}(Y) = \frac{1}{N} \sum_{j=1}^N (f(P)_j)^2 - \hat{f}_0^2 \quad (6)$$

$$\hat{S}_i = \frac{1}{N} \sum_{j=1}^N \frac{f(Q)_j f(P_Q^{(i)})_j - f(P)_j}{\hat{V}(Y)} \quad (7)$$

$$\hat{S}_{Ti} = \frac{1}{2N} \sum_{j=1}^N \frac{(f(P)_j - f(P_Q^{(i)})_j)^2}{\hat{V}(Y)} \quad (8)$$

where $\hat{\cdot}$ means the estimate; \hat{f}_0 is the estimated value of the model output; $P_Q^{(i)}$ represents all columns from P except the i -th column which is from Q , using a radial sampling scheme (Campolongo et al., 2011; Saltelli and Annoni, 2010).

We generated a quasi-random sequence matrix of size $(N, 2k)$, where P and Q are the left and right half of this matrix, respectively (Saltelli et al., 2010; Sobol', 1967; Tang et al., 2007). The quasi-random sequence helps to distribute the sampling points as uniformly as possible in the parameter space and avoid clustering, and to increase the convergence rate. To compute S_i and S_{Ti} simultaneously, a scheme suggested by Saltelli (2002) was used which reduced the model runs to $N(k+2)$.

2.5. Convergence and sensitivity ranking

For each plantation field site, the elementary effects and variance-based SA return a single value for each input/output parameter combination. To get the mean values of both of these measures, a bootstrap with replacement technique was applied (Efron and Tibshirani, 1993). For the i -th input parameter, the samples, which included the elementary effects and sensitivity indices S_i and S_{Ti} , were resampled with replacement 10,000 times—a large enough number to guarantee the frequency distributions of the means of EE_i , S_i and S_{Ti} calculated from each resampled set will approach their actual probability distributions. We then took the means of these three frequency distributions as the representative values of EE_i , S_i and S_{Ti} , respectively. We selected a sample size of 2^{14} (16,384) which was large enough to test the convergence of sensitivity indices and the stability of the rankings—for there is still no general-purpose method to estimate a sample size large enough to guarantee the convergence of parameter sensitivity indices. The large number of sampling points eliminated the chance of pseudo-convergence.

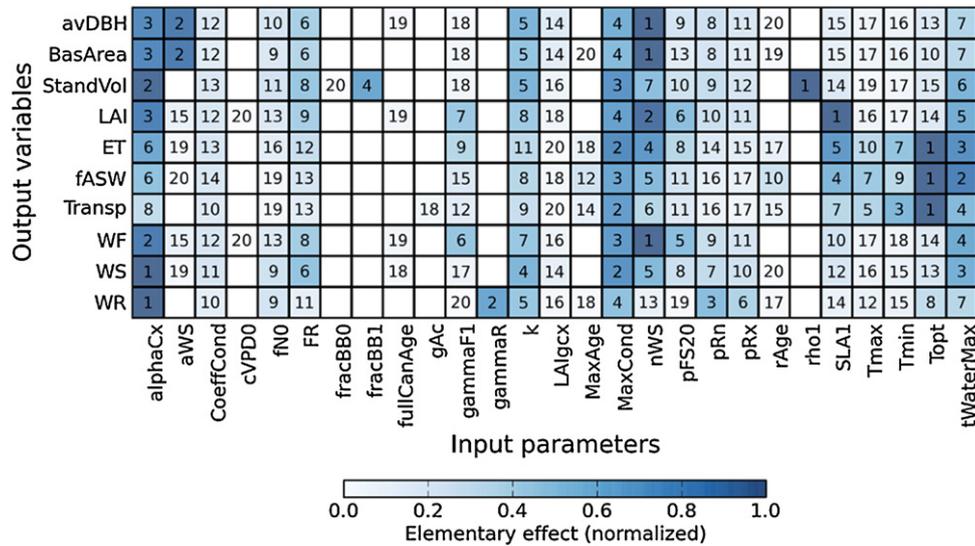


Fig. 1. Elementary effects of 3-PG2 input parameters on output variables. The 20 most influential input parameters are shown. Numbers indicate the rank of elementary effects μ^* for each input parameter on each output variable from highest (1) to lowest (20) by descending order of EE value. Colour shading is used to symbolize elementary effects using normalized values of μ^* . Note that for avDBH and BasArea, because the EE values of nWS are much higher than the other input parameters, the remaining input parameters were normalized between the range of 0 and 0.9.

2.6. Model simulation and comparison

The current version of 3-PG2 is a Microsoft Excel extension written in Visual Basic for Applications. We ported the 3-PG2 model into Python (<http://python.org>) for this analysis. The EE method was run for 400 ($=r(k+1)$) model simulations, where $r = 10$ and $k = 39$. For the variance-based method, 360,448 ($N(k+2)$) model simulations were run, where N is the sample size and equals 2^{14} , and k is the number of input parameters selected through EE analysis and equals 20. We ran the simulations in parallel on a Linux computer cluster. To check the consistency of the sensitivity ranking derived from EE and variance-based methods for each output variable, a Spearman's rank correlation method was used (Myers and Well, 2003).

3. Results

3.1. Elementary effects

Elementary effects of input parameters corresponding to each model output are presented in Fig. 1. The influential parameters and their rankings (as measured by the elementary effects) for the model outputs avDBH and BasArea, and for LAI and WF, were very similar. For the water-related model outputs ET, fASW and Transp, the parameters T_{opt} and $tWaterMax$ had the highest sensitivities. For the biomass-related output variables listed in Table 2, nWS was a relatively sensitive parameter. Much of the variation in biomass-related model outputs was determined by canopy parameters (e.g. alphaCx, MaxCond and k), limiting parameters to plant growth (e.g. tWaterMax and FR), and biomass partitioning parameters (e.g. pRn and pRx). For StandVol, both rho1 and fracBB1, which determine the density and biomass partitioning of the stem, had higher sensitivities.

3.2. EE method applied to group parameters

The SoilTexture group parameters had a strong influence on the variations in the water related output variables ET, fASW, Transp, and on root biomass WR (Fig. 2). Especially for fASW, the SoilTexture group parameters had a much higher sensitivity than the other

Table 3

Ranking correlations between corresponding input parameters used in elementary effects before (Section 3.1) and after (Section 3.2) adding soil texture as a group (EE vs. EE-SoilTexture), and elementary effects and variance-based methods (EE vs. SA) using Spearman's rank correlation.

Output variable	EE vs. EE-SoilTexture	EE vs. SA
avDBH	0.995	0.929
BasArea	0.998	0.980
StandVol	0.995	0.871
LAI	0.998	0.968
ET	0.998	0.955
fASW	0.995	0.913
Transp	0.996	0.913
WF	0.998	0.964
WS	0.998	0.902
WR	0.996	0.952

input parameters (more than 5 times as high as the next most sensitive parameter T_{opt}).

3.3. Sensitivity results for the variance-based method

For the first few hundred simulations, S_i exhibited large oscillations, but after which began to converge (Fig. 3). Most S_i values stabilized at around 8000–10,000 simulations. S_{Ti} oscillated less than S_i and stabilized more quickly.

Differences between S_i and S_{Ti} , as evaluated by the variance-based method (Fig. 4), indicated that interactions between model parameters were common. In particular, the total effect sensitivities were substantially greater for ET, fASW and Transp than their first-order sensitivities.

3.4. Comparing EE and variance-based sensitivities

The high Spearman's rank correlations illustrate that the most sensitive input parameters identified for each output variable were similar using the EE and variance-based methods (Table 3). Adding the SoilTexture group parameters into the input parameter list changed the sensitivity rankings only slightly. Despite the high rank correlations, differences occurred between sensitivities of some less influential input parameters calculated using the variance-based method (Fig. 4), compared with the EE results (Fig. 1). In the

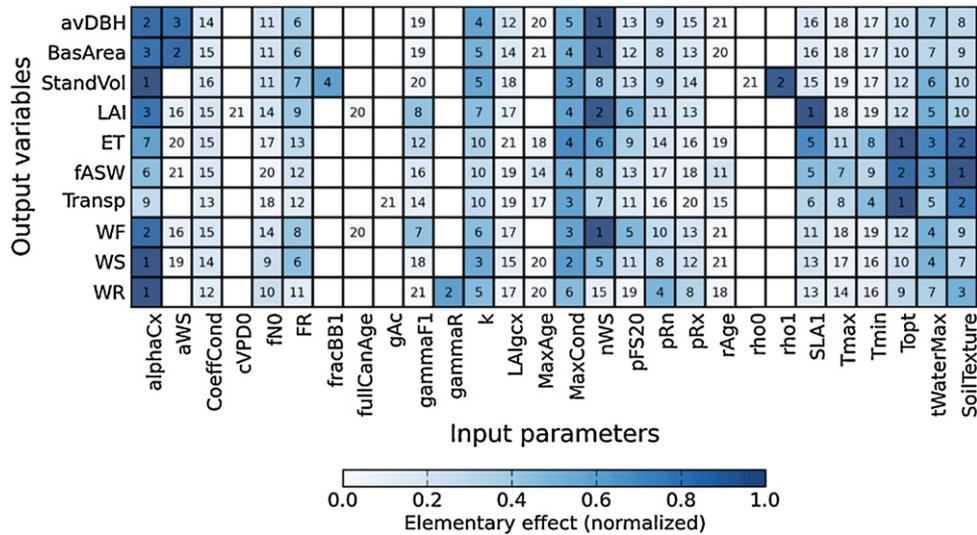


Fig. 2. Elementary effects whilst treating soil texture parameters (see Section 2.2) as a group factor (*SoilTexture*, the last column on the figure). For comparability with Fig. 1, the 20 input parameters with the highest EE values plus *SoilTexture* are shown.

variance-based method, for *fASW*, T_{min} was more sensitive than T_{max} , but this is reversed under the EE method. In addition, *fASW* and *Transp* were more sensitive to the forest stand age modifiers (e.g. *MaxAge*, *rAge*) under the variance-based method than under EE. *fracBB0* had some influence on *StandVol* under the variance-based method but negligible influence under EE (ranking is 20).

4. Discussion

We implemented a systematic sensitivity analysis for a process-based forest model 3-PG2, using a large set of field data for *C. maculata* and *E. cladocalyx* by sequentially combining a screening method (elementary effects) and a global variance-based method. Our assessment of the consistency of sensitivity rankings derived from the EE and variance-based methods (Table 3) suggested that the EE method can estimate the sensitivity rankings at a relatively high accuracy but with a much lower computational cost, although there were some discrepancies as mentioned in Section 3.4. However, due to its limited exploration of the input parameter space, it is not suitable for the accurate measurement of the contributions of

each input parameter to the variance in model outputs. An efficient and robust SA strategy could involve a screening of the most influential parameters using the EE method. Initial screening should not be too conservative but rather, a more inclusive parameter selection should be made to avoid missing some important parameters or details of the model structure. Influential parameters should then be input to the variance-based SA.

Our results showed commonalities with, and advances on, previous local SA studies on 3-PG and 3-PG2 (Almeida et al., 2004a, 2007; Esprey et al., 2004). These studies conducted local SAs for species including *Eucalyptus grandis*, *Pinus radiata* and *Eucalyptus globulus* for stand ages from less than 10 years to over 30 years. Consistent with our results, these studies identified that the soil water outputs (*ET*, *fASW* and *Transp*) were highly sensitive to *SoilTexture* and *tWaterMax* (Almeida et al., 2007). In Esprey et al. (2004), *StandVol* was found to be highly sensitive to *alphaCx*, *MaxCond* and *rho1*. For *LAI*, also consistent with our results, *SLA1*, *nWS*, *alphaCx*, *MaxCond*, *gammaF1* and *pFS20* were highly sensitive (*k* was moderately sensitive in *StandVol* and less sensitive in *LAI*), in which *gammaF1* and *pFS20* were also identified by Almeida et al. (2007)

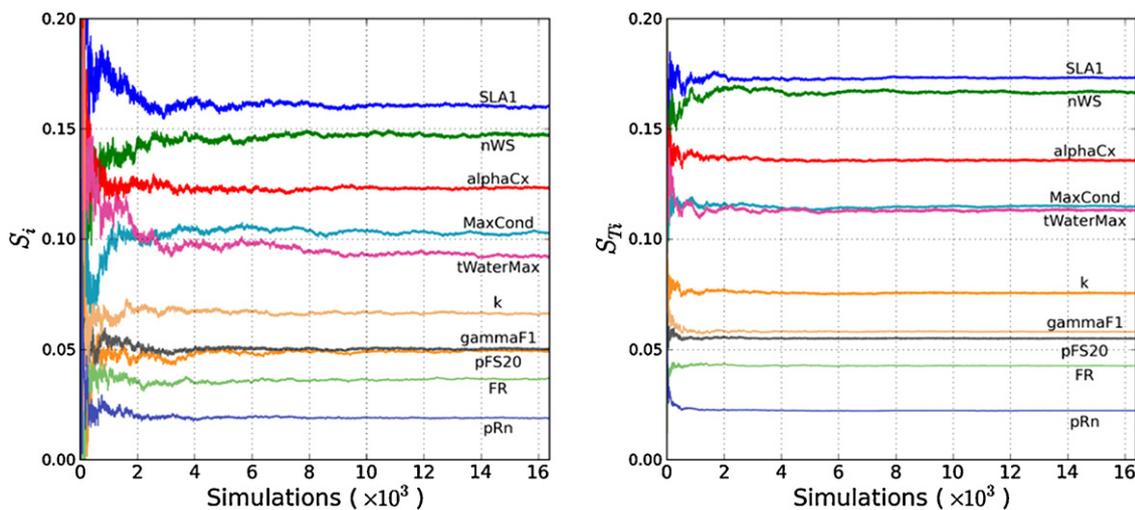


Fig. 3. Convergence of the first-order sensitivity indices S_i (left) and total effect sensitivity indices S_{Ti} (right). Above, *LAI* is presented as an example with its 10 most sensitive input parameters.

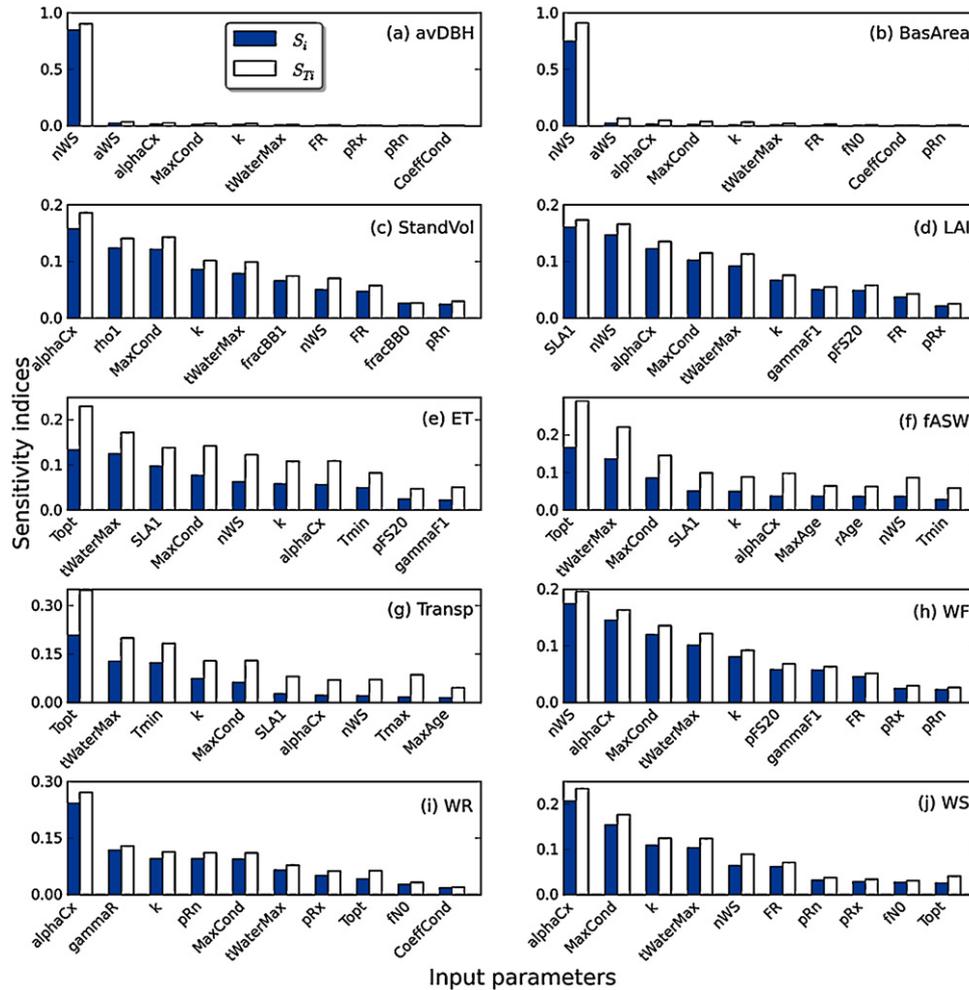


Fig. 4. First-order sensitivity index S_i and total effect sensitivity index S_{Ti} as evaluated by the variance-based method. Ranking of the 10 most sensitive input parameters for each model output variable listed in Table 2 according to their first-order sensitivity index values.

to be sensitive. Esprey et al. (2004) however, found the sensitivity of *fracBB1* in StandVol to be lower than our findings, and *fracBBO* was found to be insensitive. At the relatively young stand ages (4.5–11 years) analysed by Esprey et al. (2004), we found higher sensitivities of *fracBBO* but this decreased as the stands matured, and conversely, *fracBB1* showed the opposite effect (Song et al., submitted for publication). In concert with our results, Almeida et al. (2004a) found avDBH and WF to be highly sensitive to *nWS*; all the biomass related model outputs were sensitive to *alphaCx*; almost all model outputs were sensitive to *MaxCond*, and WR was sensitive to *pRn*.

To illustrate the effectiveness of global SA in revealing model structure and behaviour, some examples are discussed below. The similar sensitivity rankings of input parameters for both avDBH and BasArea are due to the allometric relationship for trees in 3-PG2, $BasArea = (avDBH/200)^2 \cdot \pi$. The higher sensitivities of avDBH and BasArea to *nWS* can be explained by the empirical power relationship $avDBH = (avWS/aWS)^{1/nWS}$, where *avWS* is the average stem biomass (Landsberg and Waring, 1997; Sands and Landsberg, 2002). Since the biomass allocation to stem and foliage depends on the DBH (Sands, 2004), both LAI and WF are also highly sensitive to *nWS*. LAI is a surrogate for WF due to the linear relationship $LAI = WF \cdot SLA1 \cdot c$, where *c* is a constant, and *SLA1*, as a scale factor in LAI, has the highest sensitivity. The similar rankings of input parameter sensitivity for LAI and WF (Fig. 4) are due to the close relationship of these variables. For the water-related model outputs ET, fASW and Transp, we observed that T_{opt} and *tWaterMax*

are the most sensitive parameters (Fig. 4). In 3-PG2, the canopy quantum efficiency is related to the monthly mean temperature T_a ($^{\circ}C$) and species-specific optimum temperature for growth T_{opt} (Landsberg and Waring, 1997). A dimensionless modifier f_T ($0 \leq f_T \leq 1$) is defined in 3-PG2 as follows:

$$f_T(T_a) = \left(\frac{T_a - T_{min}}{T_{opt} - T_{min}} \right) \left(\frac{T_{max} - T_a}{T_{max} - T_{opt}} \right)^{(T_{max} - T_{opt}) / (T_{opt} - T_{min})} \quad (9)$$

with $f_T = 0$ if $T_a \leq T_{min}$ or $T_a \geq T_{max}$. In the subsequent model logic, temperature modifier f_T is then involved in the calculation of canopy conductance, which in turn feeds into the Penman–Monteith equation in calculating transpiration (Transp) and evapotranspiration (ET). Whilst for fASW, the available soil water under forest is total rainfall minus the amount of water consumed by Transp and ET and intercepted by the canopy (*tWaterMax*). The relatively higher sensitivities of ET, fASW and Transp to *tWaterMax* imply that rainfall is a major limiting factor for most of the sites under study. Due to the nature of the temperature modifier (Eq. (9)), water-related model outputs are more sensitive to T_{opt} than the other two temperature parameters T_{min} and T_{max} . The implication of this model structure is that the sensitivities of model outputs may change with input data (e.g. the monthly average temperature).

The results from the elementary effects analysis of the *Soil-Texture* suggested that soil texture properties may have direct influence on soil water status and biomass allocation to roots

(WR), which may also be affected by the soil water stress partially determined by soil properties (Landsberg and Waring, 1997). In addition, the *SoilTexture* indirectly influenced other model outputs by altering the soil water and carbon allocations. Input parameters that have direct functional relationships with certain model outputs will exhibit greater sensitivities (e.g. LAI to *SLA1*, *avDBH* to *nWS*, and *ET*, *fASW* and *Transp* to T_{opt}), whilst interactions with other model parameters may attenuate the influence of the parameter itself to some extent.

Input parameter sensitivities also varied with model simulation time (stand age). This is partly due to the existence of time-dependent state variables (e.g. daily/monthly climate data, plant physiological processes such as the growth of trees) and some non-linear processes, a common feature of process-based ecological models. For example, many key physiological processes in 3-PG2 are age-dependent and are implemented through an age modifier f_{age} (Landsberg and Waring, 1997; Sands, 2004). f_{age} is a dimensionless variable which declines with age and acts as a scale factor to the physiological modifier *physMod*. *physMod*, in turn, modifies canopy conductance and quantum efficiency (Almeida et al., 2004a). The rationale is that the stomatal conductance of trees is sensitive to their hydraulic conductance, which declines as the tree ages (Landsberg and Waring, 1997). As a result, f_{age} has an indirect influence on almost all physiological processes in 3-PG2. Although some parameters did not exhibit obvious patterns due to the interference of site-specific data (e.g. soil properties and climate data), many sensitivities changed with model simulation time. For LAI, for example, the sensitivities of *SLA1*, *alphaCx*, *FR*, *LAIgcx* and *fullCanAge* decline as trees age, whilst the sensitivity of *gammaF1* increases. A full investigation of time-dependence of sensitivities is presented in Song et al. (submitted for publication).

Our results showed that the global SA technique can quantitatively identify influential input parameters for specific model output variables. To the best of our knowledge, this is the first use of a global SA technique to explore a forest growth model. We have demonstrated the physiological significance of influential input parameters for particular model output variables. This information can aid in both the elucidation of model structure and model calibration. Whilst previous local SA studies have qualitatively identified the influential input parameters, they fall well short of quantifying the contribution of variance of each input parameter to model output variables under an integrated framework. An accurate assessment, ranking, or comparison of sensitivities is impossible using local SA, which in turn clouds the understanding of model structure, especially for parameter-rich models (Saltelli and Annoni, 2010). From the perspective of model calibration, one great challenge in the application of ecological models is that sometimes we have to set default values for some input parameters which may be too expensive to quantify empirically. Global SA techniques, which quantify the relative importance of each input parameter to model outputs, can help set safe default values for those less influential input parameters. Global SA can also greatly simplify model calibration through enabling the most influential parameters to be targeted for data acquisition and refinement.

5. Conclusions

In this paper, we provided a systematic methodology for sensitivity analysis of a process-based forest growth model 3-PG2. Both screening and variance-based sensitivity analysis methods were adopted, and a comprehensive sensitivity analysis of the model outputs to model input parameters was presented. The sensitivity rankings from both methods were highly correlated, suggesting

that the elementary effects method can provide a reasonable estimate of model sensitivities with lower computational costs, but are not suitable for the precise quantification of model sensitivities. The results of our two-stage implementation showed that model parameters exhibit both direct effects on model outputs and indirect effects, through their influence on other parameters. We also demonstrated that for the process-based model 3-PG2, sensitivities of input parameters may change with input data or change with model simulation time although the latter requires further investigation and is examined in more detail elsewhere. We demonstrated that quantitative, global, variance-based sensitivity analyses were able to uncover model sensitivities not found through previous local SAs, and are essential for the thorough exploration of model structure and behaviour.

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