JMP 2021 Discovery Summit

Reliability Defect Models

David C. Trindade, Ph.D.
STAT-TECH

Analysis of Reliability Data

- The common assumption in the analysis of reliability data is that **all** units on stress will eventually fail for specific failure mechanisms.

- However, how do we treat reliability data that doesn't seem to follow that assumption?

Reliability Stress Test Example

- A stress test of **100 units** is run for **1,000 hours**.
- There are **30 failures** by 500 hours, but **no additional failures** in the next 500 hours up to the test end.
- ***Question:*** Had the surviving **70 units continued** on the stress test beyond **1,000 hours**, would we have seen additional, similar failures or would there be no further failures?
- ***Question:*** Are we dealing with **two different, mixed populations** or is the data just behaving randomly?

Second Example: Major Computer Manufacturer's Incoming Inspection Reliability Data

Readout Stress Test Results for Gate Oxide Fails

Time (Hours)	0	24	48	168	500	1000
Rejects	0	201	23	1	1	1
Sample Size	58,133	58,133	57,932	10,000	2,000	1,999
Censored Units	0	0	47,909	7,999	0	1,998

Censored units are **surviving** devices removed from stress following a readout time.

Company assumed failure distribution was lognormal.

Analysis Assuming a Lognormal Distribution

We have **multi-censored, interval** data. The JMP data table is created as shown.

Time (Hours)	0	24	48	168	500	1000
Rejects	0	201	23	1	1	1
Sample Size	58,133	58,133	57,932	10,000	2,000	1,999
Censored Units	0	0	47,909	7,999	0	1,998

Units surviving after readout
24 Hrs: 58,133 – 201 = 57,932
48 Hrs: 57,932 – 23 = 57,909
57,909 – 10,000 = 47,909 Censored
168 Hrs: 10,000 - 1 = 9,999
9,999 – 2,000 = 7,999 Censored
500 Hrs: 2,000 – 1 = 1,999
1000 Hrs: 1,999 – 1 = 1,998 Censored

Blank cells • under the **End** column indicate a censoring time.

	Start	End	Freq
1	0	24	201
2	24	•	0
3	24	48	23
4	48	•	47909
5	48	168	1
6	168	•	7999
7	169	500	1
8	500	•	0
9	500	1000	1
10	1000	•	1998

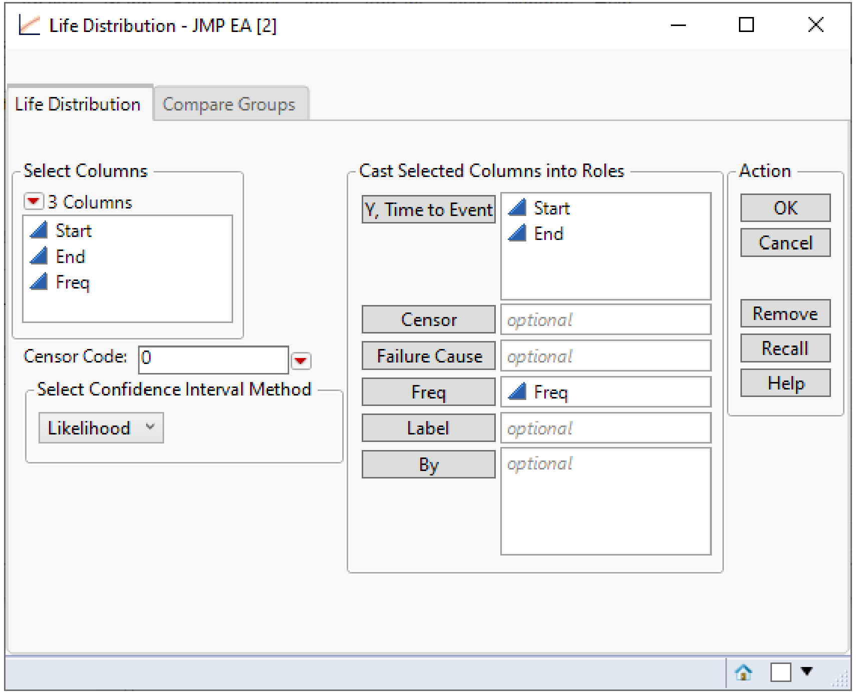
JMP's Life Distribution Launch Window

We select **Analyze > Reliability and Survival > Life Distribution** and provide the input shown.

Note **Start** and **End** columns entered for **Y, Time to Event**.

Censor Code is 0.

Confidence Interval Method selected is **Likelihood.**



JMP Life Distribution Report: CDF

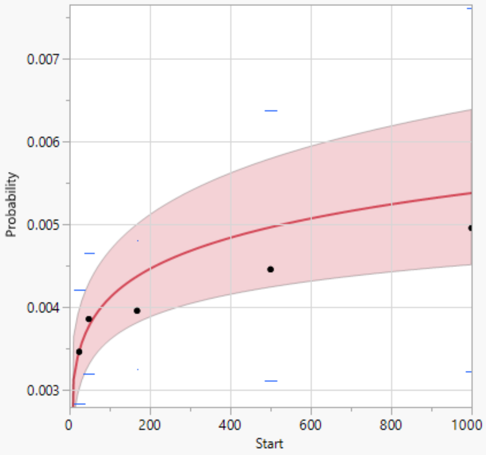
Select the **Lognormal Distribution** box option to display assumed model fit to data.

The suitability of the lognormal model fit is questionable.

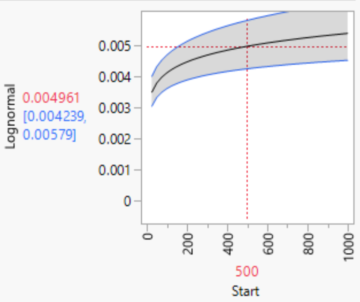
Parameter Estimates for the fitted Lognormal distribution are provided.

Compare Distributions

Distribution	Scale
☑ Nonparametric	◉
☑ Lognormal	○
☐ Weibull	○
☐ Loglogistic	○
☐ Frechet	○
☐ Normal	○
☐ SEV	○
☐ Logistic	○



Distribution Profiler



Statistics

Model Comparisons

Distribution	AICc	-2Loglikelihood	BIC
Lognormal	3152.7491	3148.7489	3170.6899

Summary of Data

Nonparametric Estimate

Parametric Estimate - Lognormal

Parameter	Estimate	Std Error	Lower 95%	Upper 95%
location	70.6501493	12.6246803	51.1388684	103.34941
scale	24.9890321	4.73551271	17.6695357	37.2534408
T50(Median)	4.8191e+30	6.084e+31	1.6193e+22	7.6574e+44
Mean (Wald CI)	1.91e+166	2.501e+168	6.5551e+54	5.563e+277

Criterion	
-2*LogLikelihood	3148.7489
AICc	3152.7491
BIC	3170.6899

JMP Life Distribution Report: Probability Plot

Select the **Scale** button for the **Lognormal Distribution** to display the probability plot.

Few points are falling near or on the lognormal model (straight line), indicating again a questionable fit.

Note parameter estimates.

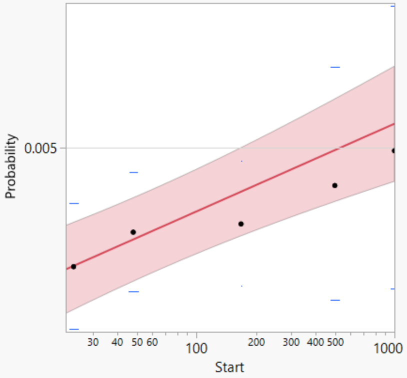
Life Distribution

Frequency=zero encountered 2 times in table

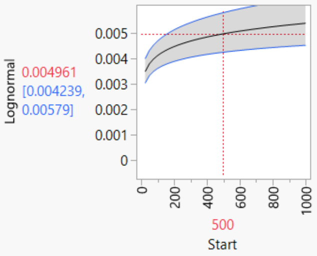
Event Plot

Compare Distributions

Distribution	Scale
☑ Nonparametric	○
☑ Lognormal	◉
☐ Weibull	○
☐ Loglogistic	○
☐ Frechet	○
☐ Normal	○
☐ SEV	○
☐ Logistic	○



Distribution Profiler



Statistics

Model Comparisons

Distribution	AICc	-2Loglikelihood	BIC
Lognormal	3152.7491	3148.7489	3170.6899

Summary of Data

Nonparametric Estimate

Parametric Estimate - Lognormal

Parameter	Estimate	Std Error	Lower 95%	Upper 95%
location	70.6501493	12.6246803	45.9062307	95.394068
scale	24.9890321	4.73551271	15.7075977	34.2704664
T50(Median)	4.8191e+30	6.084e+31	8.6461e+19	2.6861e+41
Mean	1.91e+166	2.501e+168	6.5551e+54	5.563e+277

Criterion	
-2*LogLikelihood	3148.7489
AICc	3152.7491
BIC	3170.6899

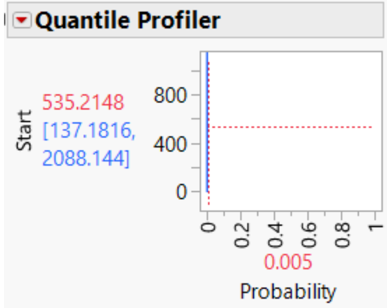
Lognormal Parameter Estimates,
Assumes All Units Can Eventually Fail

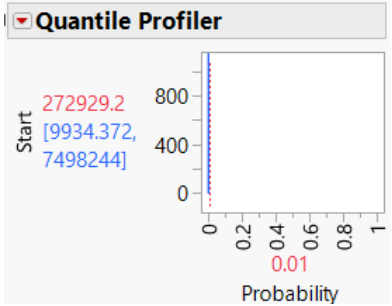
T_{50} = 4.82E+30 hours **sigma** = 24.99

4.82E+30 hours = 5.5E26 years!

(For comparison, the age of the universe is estimated to be ~1.4E+10 years.)

The lognormal **Quantile Profiler** estimates 535 hours to reach 0.5% failures, but 272,929 hours (~31 years) to reach 1% failures.





Defect Models: Mortals Vs. Immortals

- In contrast to the usual assumption that **all** units on stress can eventually fail, if a ***defective subpopulation*** exists, only the **fraction** of the units containing the defect may be susceptible to failure. These are called ***mortals***.

- Units without the fatal flaw **are not susceptible to failure** for the observed cause. These are called ***immortals***. Immortals can eventually fail but very, very far out in time and most likely for other reasons.

Let’s Consider a Defect Model for the Data

Time (Hours)	0	24	48	168	500	1000
Rejects	0	201	23	1	1	1
Sample Size	58,133	58,133	57,932	10,000	2,000	1,999
Censored Units	0	0	47,909	7,999	0	1,998

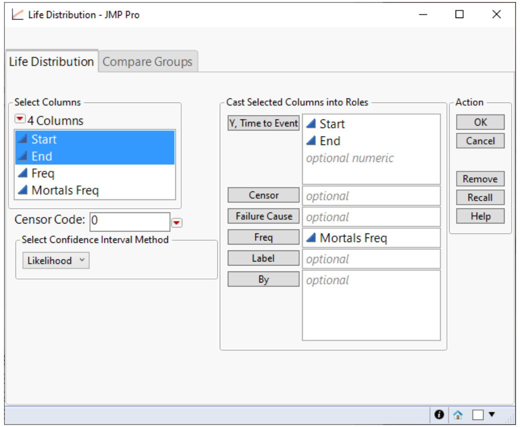
- For the incoming inspection data, we **assume** that **nearly 99%** of the failures occurring **by 48 hours** were **mortal failures**, implying a mortal sample size of $(201+23)/0.99 \approx 227$.
- Practically, 100% of the mortal failures occur by 168 hours. Any failures thereafter are **not likely related** to the **defective subpopulation** and could, for example, be handling induced.

Modeling with Defective Subpopulations

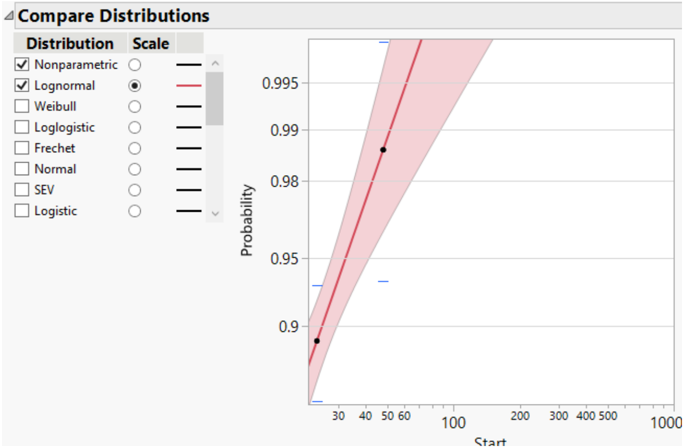
- Then, a possible model implies a fraction **mortal subpopulation** in the time zero sample size of 227/58,133 = **0.39%.**
- We now add a Mortals column to the data table as shown and run JMP's Lifetime Distribution using the **Mortal Freq** column for the **Freq** entry.
- Note that the model estimates that 2 (~1%) of the 227 mortals are included with the 47,909 censored at 48 hours.

	Start	End	Freq	Mortals Freq
1	0	24	201	201
2	24	•	0	0
3	24	48	23	23
4	48	•	47909	2
5	48	168	1	1
6	168	•	7999	0
7	169	500	1	0
8	500	•	0	0
9	500	1000	1	0
10	1000	•	1998	0

Life Distribution of the Mortals



Fitting the **lognormal distribution** to the failure times of the **mortals** provides estimates **T_{50} = 10.6 hours** and **sigma = 0.68**, which are more reasonable and realistic values.



Parametric Estimate - Lognormal

Parameter	Estimate	Std Error	Lower 95%	Upper 95%
location	2.359028	0.1991693	1.968663	2.749392
scale	0.680983	0.1432282	0.400261	0.961705
T50(Median)	10.580659	2.1073429	7.161095	15.633131
Mean	13.341729	1.5241647	10.665237	16.689900

Criterion	
-2*LogLikelihood	180.17806
AICc	184.23163
BIC	191.02796

Defective Subpopulation Models

- This example shows if we don’t consider **mortals** vs. **immortals** in the analysis, an incorrect assumption can strongly affect the results.

- **Projections of field reliability** can be **biased** unless the possible existence of a **limited** number of defective units is recognized and taken into consideration.

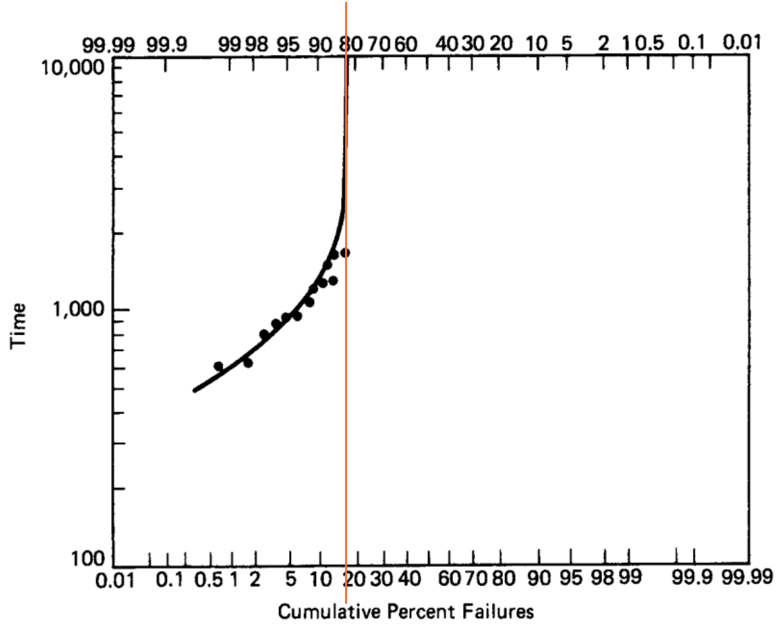
Spotting a Defective Subpopulation

Graphical Analysis

- Assume that a specified failure mode follows a lognormal distribution.
- Plot the data using a lognormal probability plot. If instead of following a straight line, the points seem to **curve away from the cumulative percent axis**, it's a **signal** that a **defective subpopulation** may be present.
- If test is run long enough, expect the plot to bend over **asymptotic** to the cumulative percent line that represents the **proportion of defectives** in the **sample**.

Spotting a Defective Subpopulation

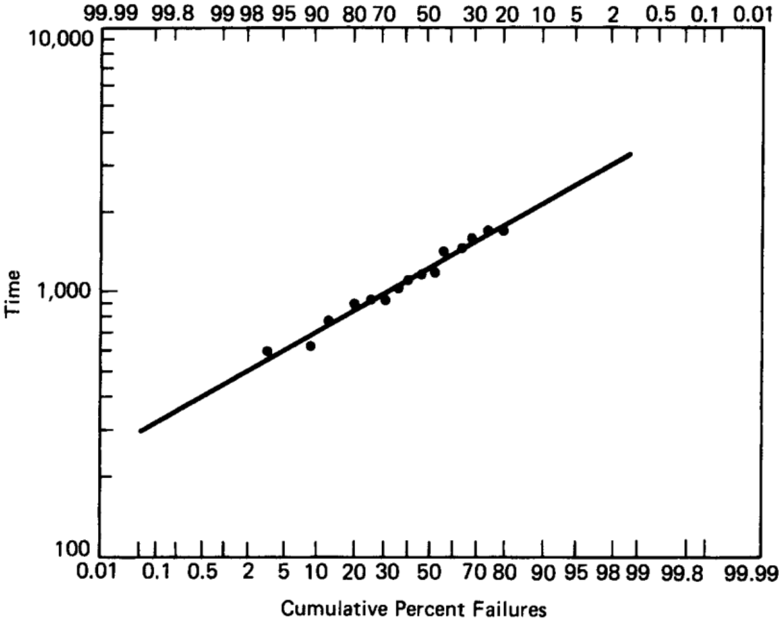
The probability plot is based on the **total** sample (mortals and immortals).



The curve asymptotically approaches a cumulative percent line that represents the **proportion of defective units in the sample.**

Plotting the Defective Subpopulation Data

The probability plot is based only on the **mortal** subpopulation.



The probability plot supports the fitted distribution model.

Defect Model: Observed CDF

- The **observed CDF** $F_{obs}(t)$ is

$$F_{obs}(t) = p\, F_m(t)$$

  where $F_m(t)$ is the CDF of the **mortals** and p is the **fraction of mortals** (units with a fatal defect) in the total sample size.

- For example, if there are 25% mortals in the population, and the **mortal** CDF at time t is 40%, then we would expect to observe about

$$0.25 \times 0.40 = 0.10$$

  or 10% failures in the total random sample at time t.

Example 9.5 From *Applied Reliability, 3rd ed.*[1]

For a type of semiconductor module, it is known that a small fraction of hermetically sealed modules can have moisture trapped within the seal which increases the likelihood of a failure mechanism that causes these units to fail early. It is desired to fit a suitable life distribution model to these failures.

Test parts were specially made to greatly increase the chance of enclosing moisture typical of the defects in the normal manufacturing process. 100 randomly selected parts were stress tested for 2,000 hours.

15 failures occurred at the following times (in hours): 597, 623, 776, 871, 914, 917, 1021, 1117, 1170, 1182, 1396, 1430, 1565, 1633, and 1664.

By the end of test, 85 units had survived (censored observations).

Example 9.5 Data Table of Failure and Censor Times

Since we have **exact failure times** and a **single censoring time**, the times are entered into a JMP data table as shown.

85 surviving units are singly censored at 2,000 hours.

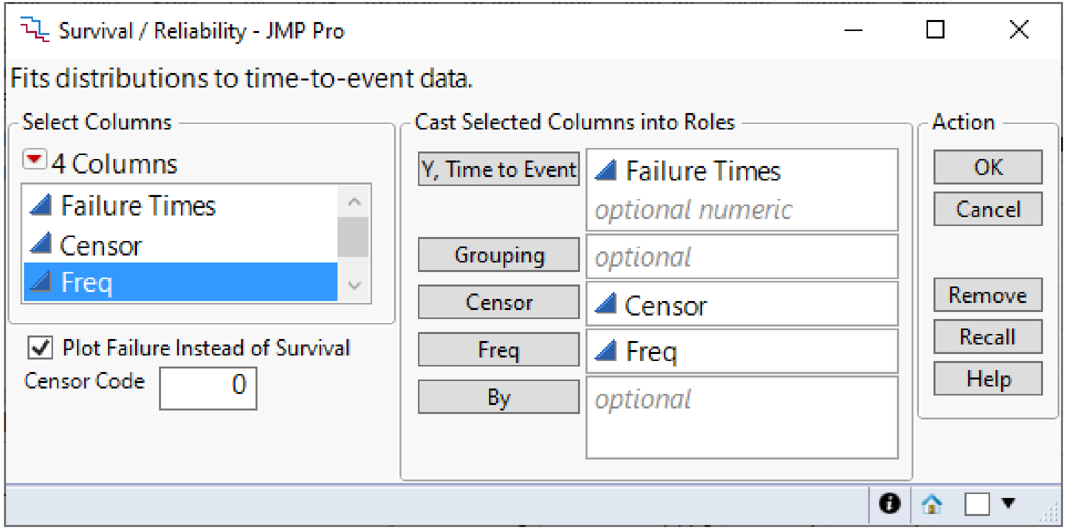
	Failure Times	Censor	Freq
1	597	1	1
2	623	1	1
3	776	1	1
4	871	1	1
5	914	1	1
6	917	1	1
7	1021	1	1
8	1117	1	1
9	1170	1	1
10	1182	1	1
11	1396	1	1
12	1430	1	1
13	1565	1	1
14	1633	1	1
15	1664	1	1
16	2000	0	85

JMP's Survival Platform Launch Window

We select **Analyze>Reliability and Survival>Survival** and cast the columns as shown.

Check the box to plot **Failure Instead of Survival**.

Enter "0" for the Censor Code.



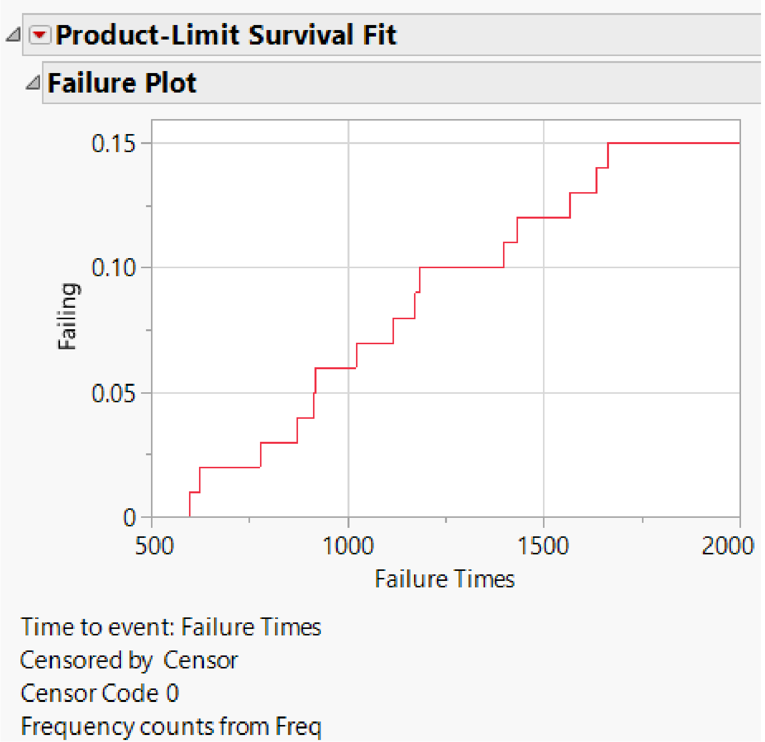
Nonparametric CDF Failure Plot

Adjust vertical scale and add gridlines to both axes.

Note that the fraction failing is **flat** from the last failure to 2000 hours, a period of 336 hours.

Yet, in the same time period prior to the last failure, there were five failures.

We suspect a possible defective subpopulation.



Maximum Likelihood Estimation

Solving for the parameters in the defect model using the method of maximum likelihood (ML) is a simple extension of ML theory for censored data.

The basic building blocks of the ML equations are the **PDF *f* (*t*)** and the **CDF *F*(*t*)**. If, however, only a fraction *p* of the population is susceptible to the failure mechanism modeled by *F*(*t*), then ***pF*(*t*)** is the probability a randomly chosen component fails **by time *t***. Similarly, the "likelihood" of a randomly chosen component failing **at the exact instant *t*** is ***pf*(*t*)**.

The rule for writing likelihood equations for the defect model is to substitute *pf* and *pF* wherever *f* and *F* appear in the likelihood equation.

MLE Equations

The standard likelihood equation for **Type I censored data** (n on test, r fails at exact times t_1, t_2, . . . , t_r, and (n - r) units censored at time T = the end of test) is given by

$$LIK = \left[\prod_{i=1}^{i=r} f(t_i)\right]\left[1 - F(T)\right]^{n-r}$$

If only a fraction p of the population is susceptible to failure, the ML equation for the defect model becomes

$$LIK = p^r \left[\prod_{i=1}^{i=r} f(t_i)\right]\left[1 - pF(T)\right]^{n-r}$$

ML **estimates** are the values of ***p*** and the **population parameters** that maximize *LIK*, or equivalently, minimize L = -log *LIK*, an easier equation to work with.

JMP’s Life Distribution Platform

JMP’s **Life Distribution** platform under **Reliability and Survival** uses the MLE method to fit defect models.

To illustrate, we use the original Example 9.5 data table.

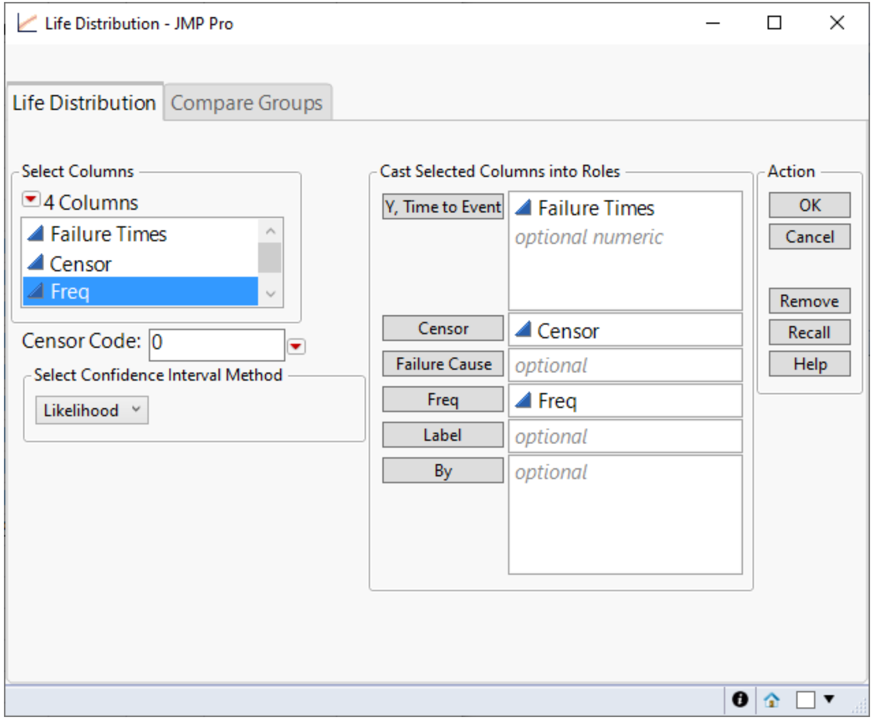
	Failure Times	Censor	Freq
1	597	1	1
2	623	1	1
3	776	1	1
4	871	1	1
5	914	1	1
6	917	1	1
7	1021	1	1
8	1117	1	1
9	1170	1	1
10	1182	1	1
11	1396	1	1
12	1430	1	1
13	1565	1	1
14	1633	1	1
15	1664	1	1
16	2000	0	85

Life Distribution Column Roles

In launch window, cast columns into roles as shown.

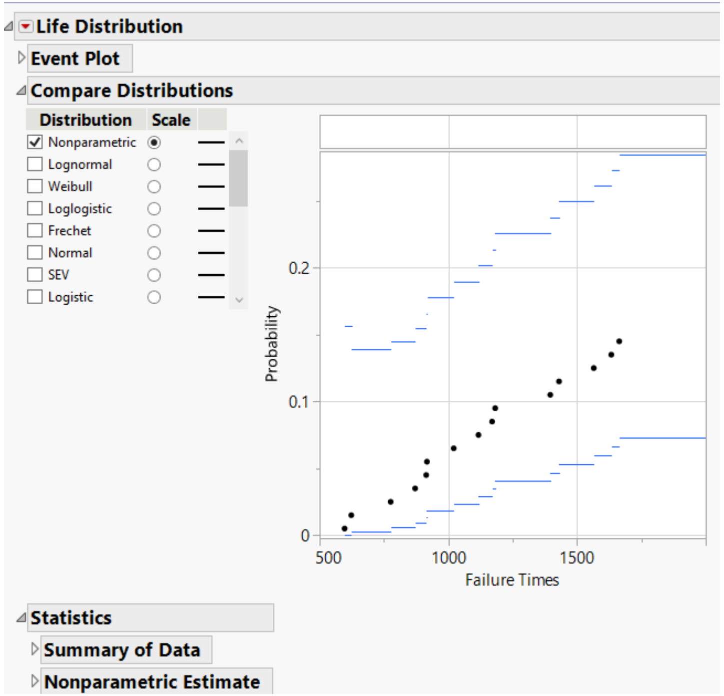
We select **Likehood** for the **Confidence Interval Method**.

Censor Code is 0.



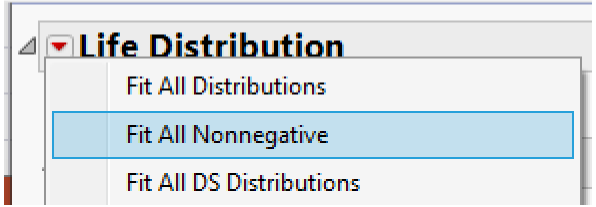
Life Distribution Report Window

The first plot displayed is the **Nonparametric** CDF estimate of the cumulative fraction failures probability versus failure times.

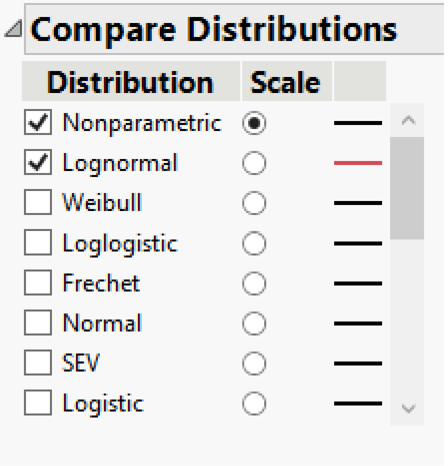


Fitting Lognormal Distribution

Under **Life Distribution** hotspot select **Fit All Nonnegative**, which assumes all units can fail.



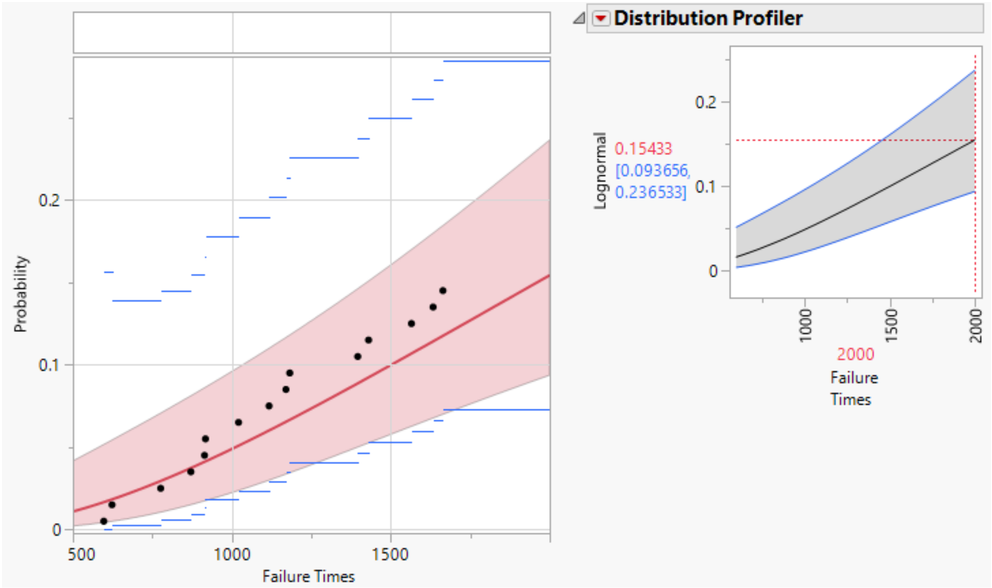
Click only the **Lognormal** box to display the lognormal CDF plot.



Life Distribution Report (CDF)

The report includes the lognormal CDF plot, **Profilers**, and **Parametric Estimates**.

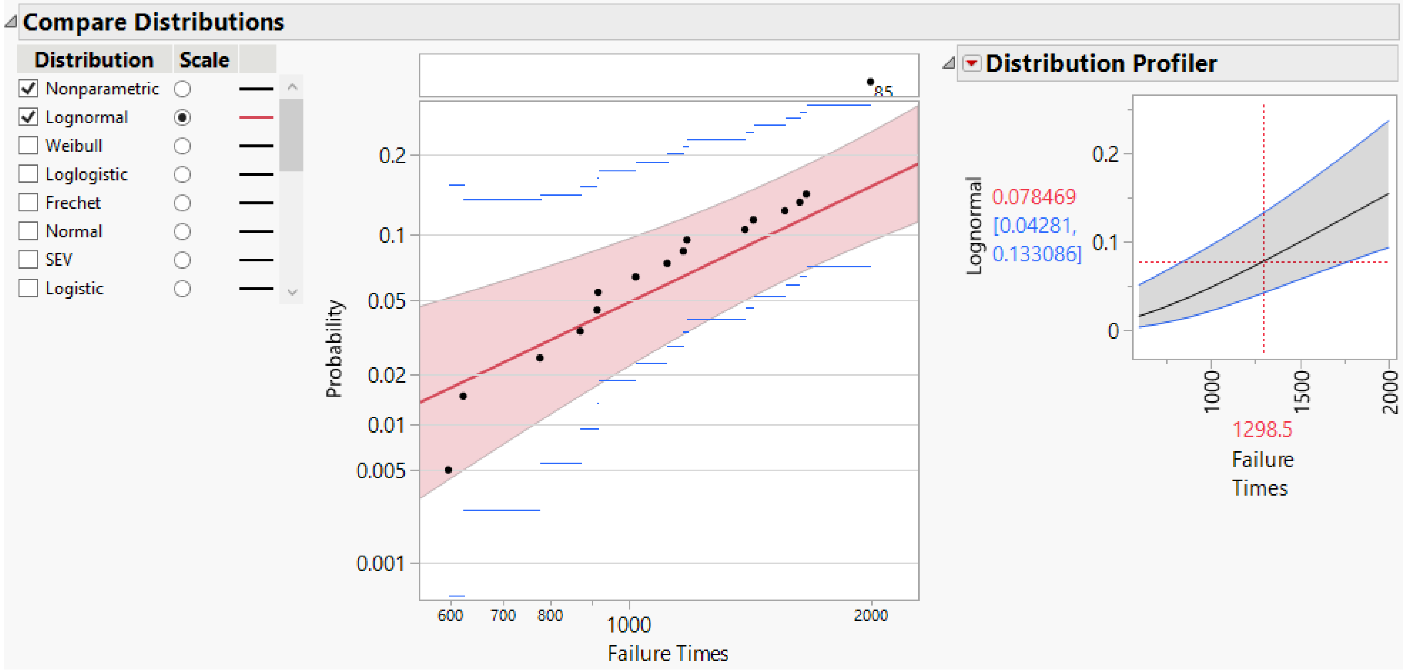
The lognormal fit is poor.



Lognormal Probability Plot

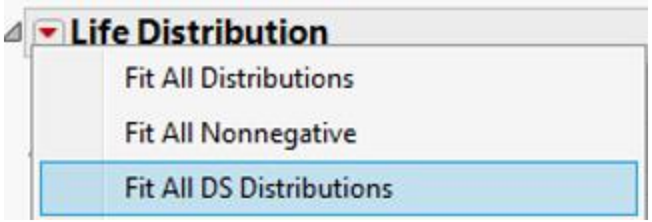
Click the **Lognormal Scale** to display the lognormal probability plot.

The lognormal fit is poor.

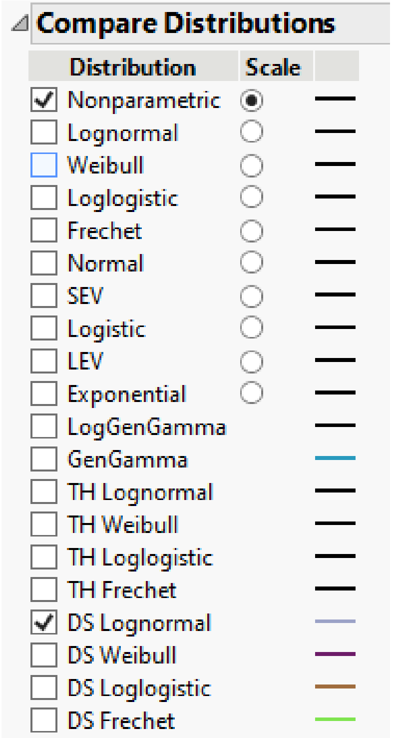


Fitting DS Lognormal Distribution

Under Life Distribution hotspot select **Fit All DS Distributions**, which fits a **D**efective **S**ubpopulation model.



DS Lognormal is selected.



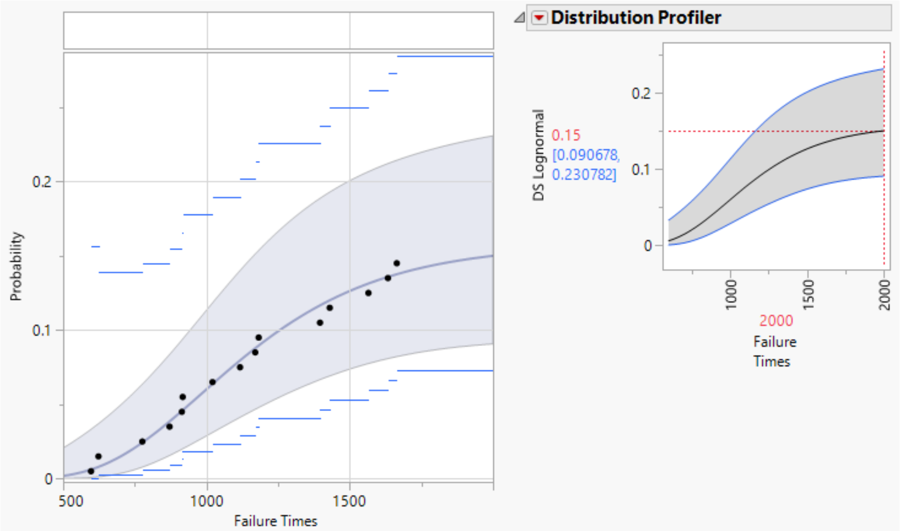
Life Distribution Report for DS Lognormal

Click **Nonparametric Scale**. The fit is very good.

DS Lognormal parameter estimates now include an estimate for the fraction defective parameter *p* = 0.158.

T_{50} = exp(7.014…) = 1,112.

Sigma = 0.353



Parametric Estimate - DS Lognormal

Parameter	Estimate	Std Error	Lower 95%	Upper 95%
location	7.0141224	0.11038028	6.8188691	7.5936306
scale	0.3531441	0.08697588	0.2396830	0.7360846
p	0.1576123	0.03884920	0.0929089	0.3126496

Criterion	
-2*LogLikelihood	301.07812
AICc	307.32812
BIC	314.89363

Hypothesis Test (Requested Addition)

If ML estimates have been calculated for a suspected defect model, we can test the hypothesis $p = 1$, (i.e., there is **no defect subpopulation**) versus the alternative **defect model**.

Let L1 be the minimum log likelihood for the standard (non-defect) model and let L2 be the minimum log likelihood for the defect model. The **likelihood ratio (LR) test** statistic is $\lambda = 2(L1 - L2)$.

If the hypothesis $p = 1$ is true, λ will have ***approximately*** a chi-square distribution with 1 degree of freedom. If λ is larger than, say, $\chi^2_{1;95} = 3.84$, then we reject the standard model and accept the defect model at the 95% confidence level.

Hypothesis Test on Example 9.5 Data

The LR test statistic is $\lambda = 2(L1 - L2)$.

The -2*LogLikehood estimates are provided under the **Criterion** column.

Parametric Estimate - Lognormal

Parameter	Estimate	Std Error	Lower 95%	Upper 95%
location	8.707	0.3363	8.2131	9.656
scale	1.087	0.2454	0.7298	1.783
T50(Median)	6047.443	2033.8705	3689.1069	15618.770
Mean (Wald CI)	10916.720	6383.7665	3470.0471	34343.851

Criterion	
-2*LogLikelihood	307.18419
AICc	311.30790
BIC	316.39453

Parametric Estimate - DS Lognormal

Parameter	Estimate	Std Error	Lower 95%	Upper 95%
location	7.0141224	0.11038028	6.8188691	7.5936306
scale	0.3531441	0.08697588	0.2396830	0.7360846
p	0.1576123	0.03884920	0.0929089	0.3126496

Criterion	
-2*LogLikelihood	301.07812
AICc	307.32812
BIC	314.89363

$2(L1) = 307.18419$
$2(L2) = 301.07812$
Test Statistic $\lambda = 2(L1 - L2) \approx 307.2 - 301.1 = 6.1$

Since λ is larger than $\chi^2_{1;95} = 3.84$, we reject the standard model and accept the defect model at ≈95% confidence level.

Defect Models in the Reliability Literature

- Google "**defective subpopulations in reliability data**" for further information.
- For a detailed case study involving accelerated testing in the presence of defective subpopulations, see reference **2.**
- In the reliability literature, such models are also called **Limited Failure Population (LFP) Models**.[3]

Summary

- It is important in the analysis of reliability data to recognize and factor in the presence of **defective subpopulations** (**DS**) for unbiased results.
- JMP's Life Distribution platform has the capability to analyze DS data using both visual and MLE methods.

References

1. Tobias, P.T., and D.C. Trindade, 2012. *Applied Reliability*, *3rd ed*., CRC Press, Boca Raton, FL
2. Trindade, D.C., 1991. "Can burn-in screen wearout mechanisms? Reliability models of defective subpopulations – A case study." In *29th Annual Proceedings of the Reliability Physics Symposium*, Las Vegas, NV, 260-263. (Available at https://www.trindade.com/publications.html.)
3. Meeker, W.Q., and L.A. Escobar, 1998. *Statistical Methods for Reliability Data,* John Wiley & Sons, New York

Thank You