Exploring Measurement System Analysis Sample Size and the Power to Detect Production Process Shifts

Laura Lancaster, Sr. Research Statistician Developer,
and Christopher Gotwalt, Director of Statistical Research and Development
JMP Division, SAS Institute Inc., Cary, NC 27513

Research Questions

1. How does the Measurement Systems Analysis (MSA) study design and sample size affect our ability to estimate shift detection probabilities?
2. How does the strength of the measurement system affect our ability to assess its impact on the Statistical Process Control (SPC)?

Measurement Systems

- Model for a given measurement: $X = P + E$ where X is the observed product measurement, P is the product value, and E is the measurement error. P and E are assumed independent with $P \sim N(\mu_p, \sigma^2_p)$ and $E \sim N(0, \sigma^2_e)$.
- Intraclass Correlation Statistic, $\hat{\rho}$, is the ratio of product variance to total variance. It is the proportion of variation that comes from the product stream.

$$\hat{\rho} = \frac{\sigma_p^2}{\sigma_p^2 + \sigma_e^2}$$

- EMP Classification are based on the Intraclass Correlation Statistic (ICS).

Classification	$\hat{\rho}$	Prob $3\sigma_p$ Shift & Warning Rule 1
First Class	0.80 – 1.00	0.99 – 1.00
Second Class	0.50 – 0.80	0.88 – 0.99
Third Class	0.20 – 0.50	0.40 – 0.88
Fourth Class	0.00 – 0.20	0.03 – 0.40

Method for Calculating Prior Confidence Limits

- Obtain *a priori* variance components for the MSA study.
- Compute prior variance of the ICS based on the *a priori* variance components.
  - Find the variance matrix of the variance components for random effects model.
  - Normalize the ICS using the logit transformation.
  - Use delta method to calculate the variance of the logit transformed ICS.
- Compute the Prior Confidence Limits (PCL) for the ICS and the probabilities of detecting process shifts.
  - Compute *95%* Wald PCL of the logit transformed ICS.
  - Use the antilogit to get the PCL for the ICS.
  - Get the PCL for each shift detection probability by computing the probability that corresponds to the PCL endpoints of the ICS.

Design and Sample Size Study

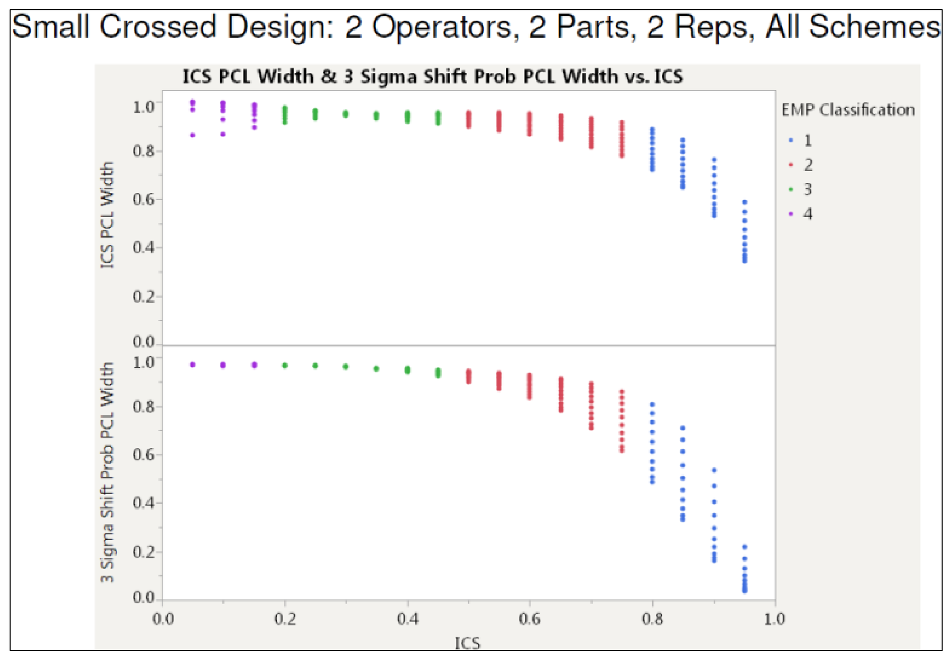
- **Two Factors Crossed:** $y_{ijk} = o_i + p_j + o_{pij} + e_{ijk}$, where y_{ijk} is the k^{th} measurement made by the i^{th} operator on the j^{th} part, and $o_i \sim N(0,\sigma^2_o)$ is the operator, and $p_j \sim N(\mu_p,\sigma^2_p)$ is the part, and $op_{ij} \sim N(0,\sigma^2_{po})$ is the operator by part, and $e_{ijk} \sim N(0,\sigma^2_e)$ is the within error. Study assumes $\sigma^2_p=1$ and $\sigma^2_{po}=0$.
- **Two Factors Nested:** $y_{ijk} = o_i + p(o)_{j(i)} + e_{ijk}$, where y_{ijk} is the k^{th} measurement made by the i^{th} operator on the j^{th} part, and $o_i \sim N(0,\sigma^2_o)$ is the operator, and $p(o)_{j(i)} \sim N(\mu_{p(o)},\sigma^2_{p(o)})$ is the part, and $op_{ij} \sim N(0,\sigma^2_{po})$, and $e_{ijk} \sim N(0,\sigma^2_e)$ is the within error. Study assumes $\sigma^2_{p(o)}=1$.
- **Sample Sizes:** Number of Operators from 2 to 10, Number of Parts from 2 to 10, Number of Reps from 2 to 5.
- **Intraclass Correlation Statistic values and schemes**
  - ***ICS:*** *0.05, 0.1, 0.15, 0.2, 0.25, 0.3,…, 0.95*

$$\hat{\rho} = \frac{\sigma_p^2}{\sigma_p^2 + \sigma_x^2} = \frac{1}{1+\sigma_x^2} \quad \text{and} \quad \sigma_x^2 = \sigma_o^2 + \sigma_e^2$$

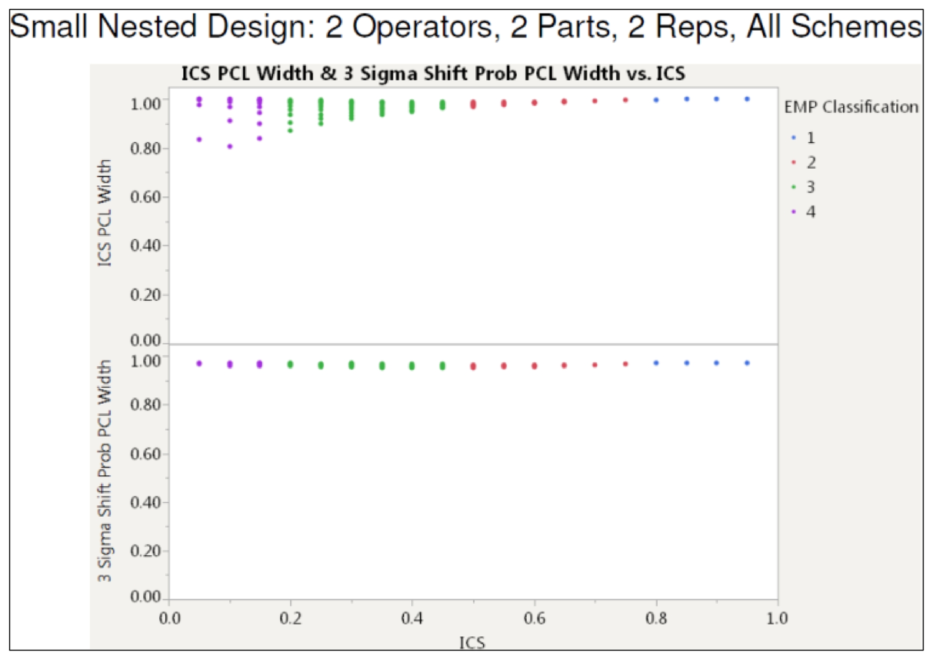
  - ***ICS Schemes:*** *0.0, 0.1, 0.2, …, 0.9*, determine linear combinations of σ^2_o and σ^2_e that give specific ICS values. Select $\lambda \in [0,1]$ where $\sigma^2_o = \lambda\sigma^2_x$, $\sigma^2_e = (1-\lambda)\sigma^2_x$.
- **Assumptions:** Subgroup sample size is 1. Probabilities detect warning within 10 subgroups after the shift.
- **Coding:** Scripts written in JSL in JMP 10.

Results

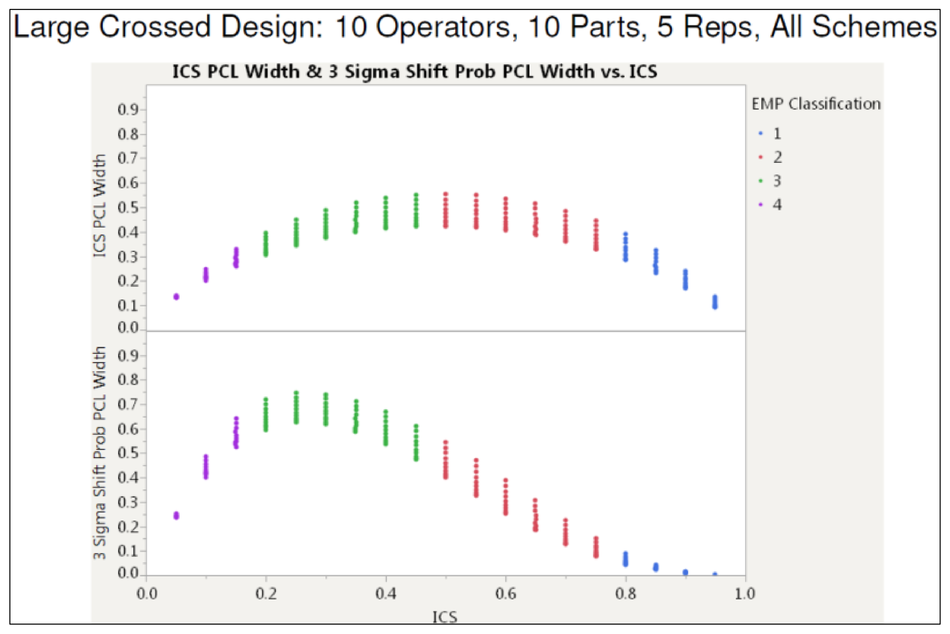
Small Crossed Design: 2 Operators, 2 Parts, 2 Reps, All Schemes



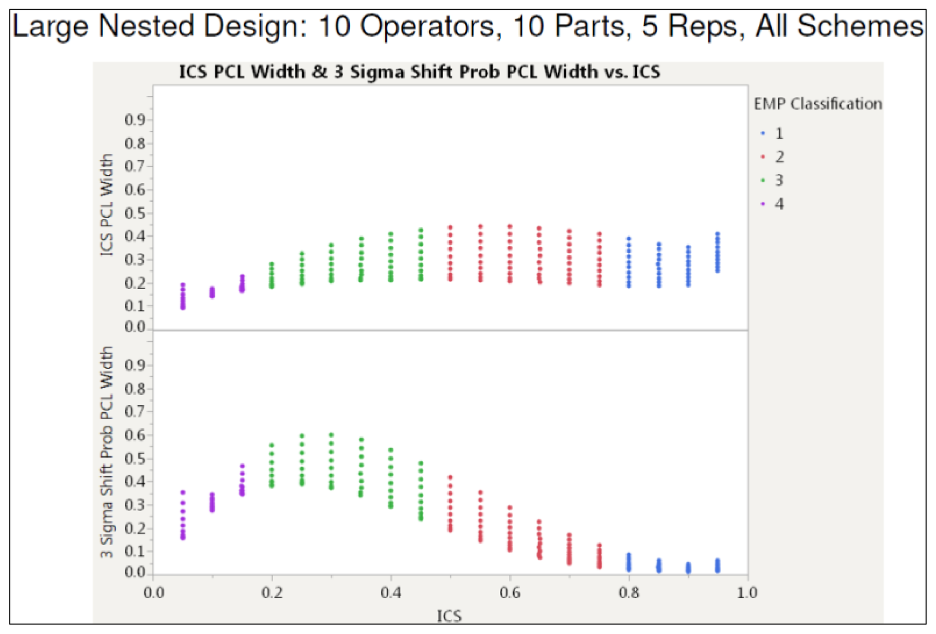
Small Nested Design: 2 Operators, 2 Parts, 2 Reps, All Schemes



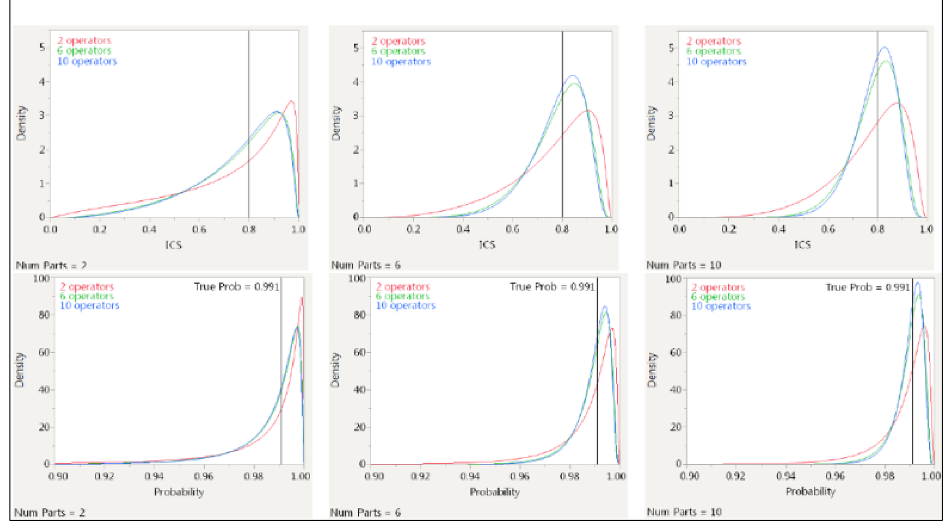
Large Crossed Design: 10 Operators, 10 Parts, 5 Reps, All Schemes



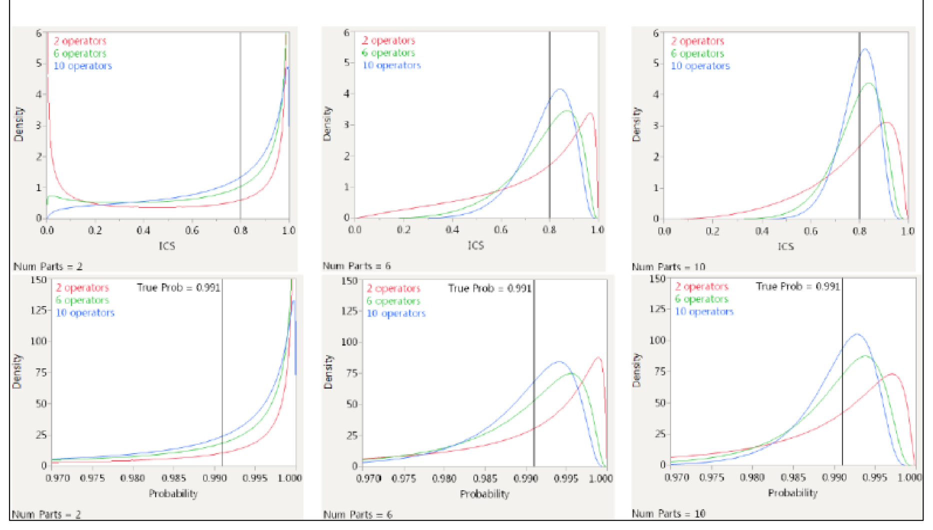
Large Nested Design: 10 Operators, 10 Parts, 5 Reps, All Schemes



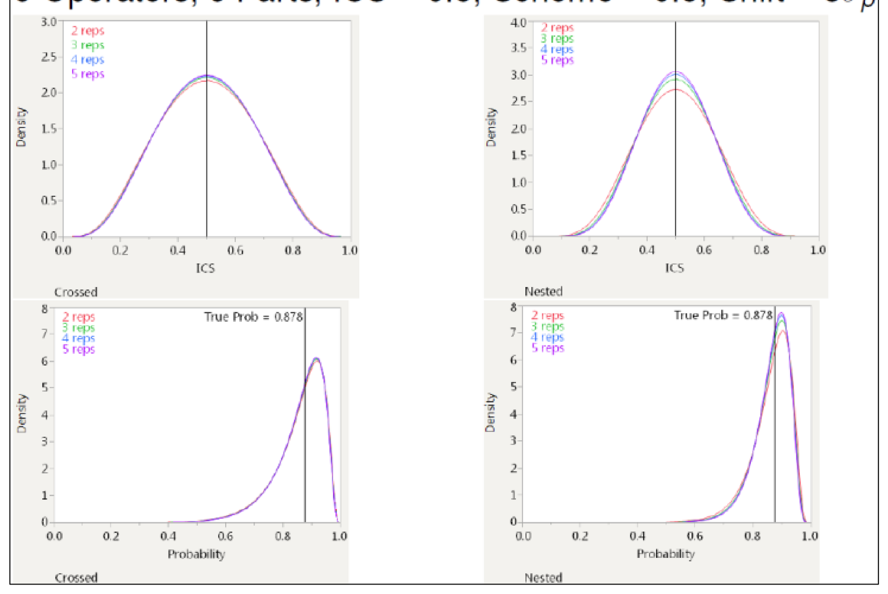
Operator and Part Size for Crossed Design
ICS = 0.8, Scheme = 0.5, Reps = 2, Shift = $3\sigma_p$
2, 6, and 10 Operators by 2, 6, and 10 Parts



Operator and Part Size for Nested Design
ICS = 0.8, Scheme = 0.5, Reps = 2, Shift = $3\sigma_p$
2, 6, and 10 Operators by 2, 6, and 10 Parts



Replication Size for Crossed and Nested Designs
6 Operators, 6 Parts, ICS = 0.5, Scheme = 0.5, Shift = $3\sigma_p$



Study Conclusions

- Number of replicates does not have a large impact on estimation accuracy.
- Number of operators and parts does have a huge impact on the estimation accuracy. Increasing the number of parts has the most impact.
- Except for extremely good or bad measurement systems, large nested designs estimate better than large crossed designs.
- Estimates are better for ICS schemes with higher within error and lower operator error variances.
- Estimate accuracy is poor for small designs.
- Even the larger designs do not estimate ICS accurately for most measurement systems. However, since EMP classification ranges are wide we can still be confident in classifications for larger designs.
- It is harder to estimate accurately with marginal measurement systems.

Reference

Wheeler, Donald J. (2006), *EMP III: Evaluating the Measurement Process & Using Imperfect Data*, SPC Press, Knoxville, TN.