

A Graphical Tool for Detection of Outliers in Completely Randomized, Unreplicated 2^k And 2^{k-p} Factorials

Tony Cooper (SAS, Inc.),

Doug Sanders, Cheryl Hild developed and published the idea



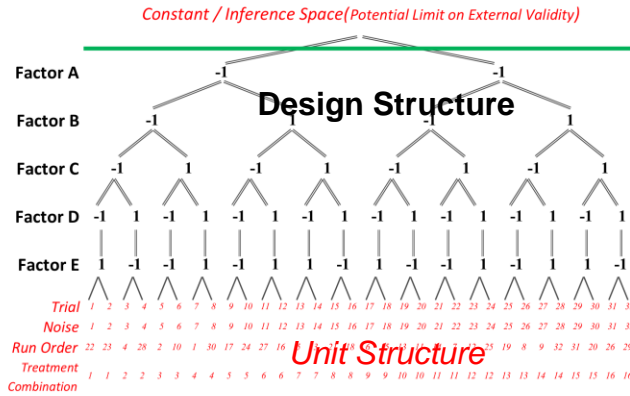
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Outline of Presentation

- Stating the Issue
 - Unreplicated 2^k & 2^{k-p} Designs
 - » Common in Industry
 - » Provide useful information using limited resources
- Addressing the Issue
 - Mechanics
 - » Developing intuition on the information in the plot
 - Using the technique on various data sets.
 - » Simple cases
 - » Extension to finding more subtle information
- Questions and Reference

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Replicated Design



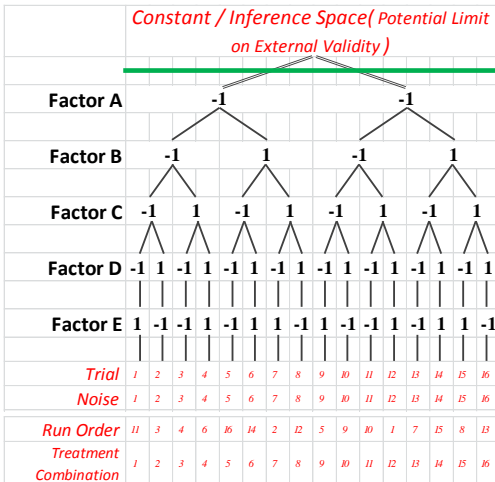
Run Order	Treatment
1	4
2	3
3	7
4	2
5	7
6	9
7	11
8	13
9	14
10	3
11	10
12	12
13	10
14	11
15	9
16	6
17	5
18	8
19	13
20	15
21	8
22	1
23	1
24	5
25	12
26	16
27	6
28	2
29	16
30	4
31	15
32	14

$$\text{Residual} = y_i - \hat{y}_i \quad \text{Mean Square Error} = \frac{\sum_i (y_i - \hat{y}_i)^2}{df}$$

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But What if Unreplicated?



Run Order	Treatment
1	12
2	7
3	2
4	3
5	9
6	4
7	13
8	15
9	10
10	11
11	1
12	8
13	16
14	6
15	14
16	5

Pseudo Standard Error Methods such as Lenth's can be used to evaluate statistical significance

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Rank Order Outlier (ROO) Plot in Unreplicated

- Proposed method to evaluate fit at each point when the design is unreplicated
- Compliments other techniques
 - Torres comparison of analysis of ranks to actual Y
 - Box (& Daniel) consideration of the shape of the probability plot.
- An iterative, interactive plotting procedure of effects after data is placed in rank order:
 - Develop a feel for the data and order of effects
 - Outliers and noise shifts often highlighted
 - Non-constant error structures sometimes found
 - Does not identify all unit structure issues

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A Plot in Rank Order – How might it look?

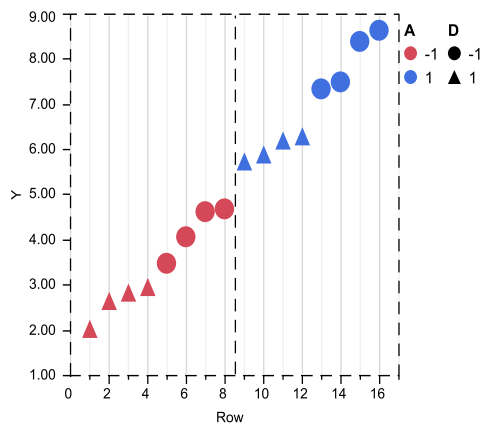
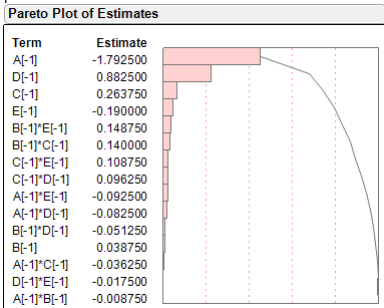
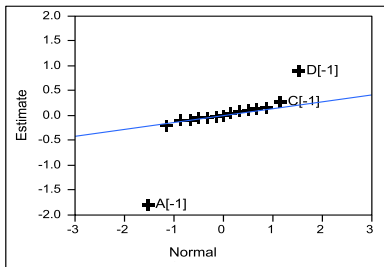
- If there was a single large effect, how would the plot look?
 - say A has a strong positive effect

A	B	C	D	E	Sort by Y	
-1	1	1	1	-1	2.01	$\mu - A + \epsilon_i$
-1	-1	1	1	1	2.64	
-1	1	-1	1	1	2.82	
-1	-1	-1	1	-1	2.95	
-1	-1	1	-1	-1	3.46	
-1	1	-1	-1	-1	4.07	
-1	1	1	-1	1	4.6	
-1	-1	-1	-1	1	4.69	$\mu + A + \epsilon_i$
1	-1	1	1	-1	5.74	
1	1	-1	1	-1	5.87	
1	1	1	1	1	6.2	
1	-1	-1	1	1	6.29	
1	1	1	-1	-1	7.33	
1	-1	1	-1	1	7.49	
1	1	-1	-1	1	8.37	Tend to be larger
1	-1	-1	-1	-1	8.63	

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Example 1: Effects of Design Structure



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Detecting outliers

- Rank Order Outlier Plots are a tool that primarily evaluates information in the unit structure.
 - Use residuals in replicated designs
- Definition of outlier
 - Stefansky *“An outlying observation is one that does not fit in with the pattern of the remaining observations. In linear models, an observation which differs very much from its fitted values is considered to be an outlier.”*
 - An inconsistency in the pattern of the (modeled) active factors
 - » Includes a “special event”

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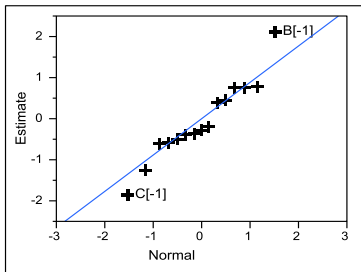
Example 2

A	B	C	D	ab	ac	ad	bc	bd	cd	abc	acd	abd	bcd	abcd	Y	Rank
-1	1	-1	-1	-1	1	1	-1	-1	1	1	-1	1	1	-1	43.13	1
1	1	-1	1	1	-1	1	-1	1	-1	-1	-1	1	-1	-1	44.45	2
-1	1	-1	1	-1	1	-1	-1	1	-1	1	1	-1	-1	1	44.83	3
1	1	1	-1	1	1	-1	1	-1	-1	1	-1	-1	-1	-1	46.1	4
1	1	-1	-1	1	-1	-1	-1	-1	1	-1	1	-1	1	1	46.31	5
-1	-1	-1	1	1	1	-1	1	-1	-1	-1	1	1	1	-1	46.76	6
-1	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1	1	-1	47.02	7
-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	1	47.46	8
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	47.9	9
1	-1	1	-1	-1	1	-1	-1	1	-1	-1	-1	1	1	1	48.49	10
1	-1	-1	1	-1	-1	1	1	-1	-1	1	-1	-1	1	1	48.56	11
-1	1	1	-1	-1	-1	1	1	-1	-1	-1	1	1	-1	1	49.34	12
1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	49.62	13
1	-1	1	1	-1	1	1	-1	-1	1	-1	1	-1	-1	-1	51.33	14
-1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1	51.47	15
-1	-1	1	1	1	1	-1	-1	-1	1	1	-1	1	1	1	59.15	16

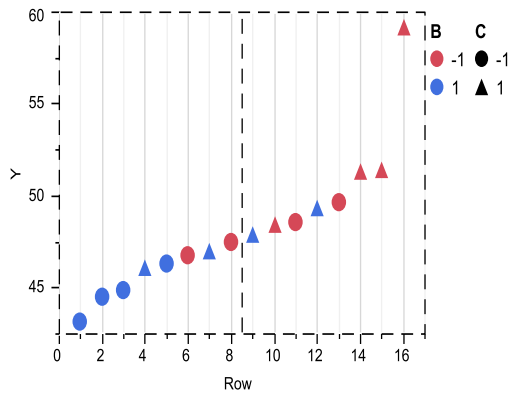
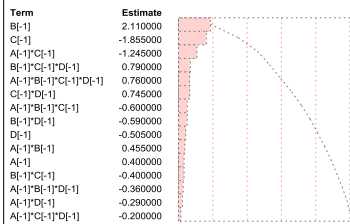
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Example 2: Output



Pareto Plot of Estimates



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Example 3 (Daniel)

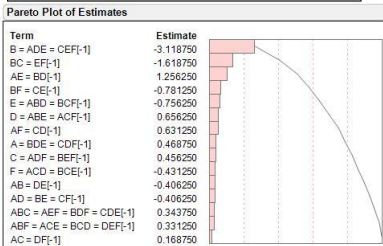
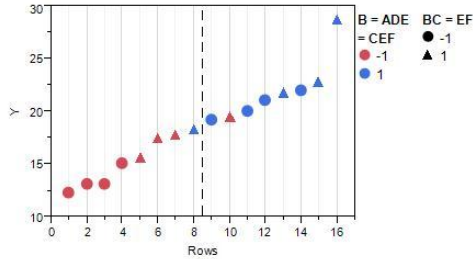
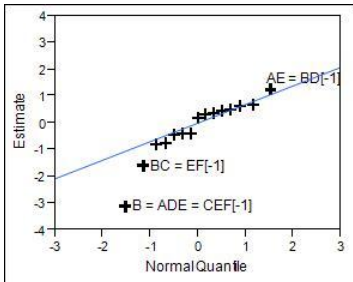
A = BDE = CDF	B = ADE = CEF	C = ADF = BEF	D = ABE = ACF	E = ABD = BCF	F = ACD = BCE	AB = DE	AC = DF	AD = BE = CF	AE = BD	AF = CD	BC = EF	BF = CE	ABF = ACE = BCD = DEF	ABC = AEF = BDF = CDE	Y	Rank
1	-1	1	1	-1	1	-1	1	1	-1	1	-1	-1	-1	-1	12.3	1
1	-1	1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	-1	13.1	2
-1	-1	1	-1	-1	1	1	-1	1	1	-1	-1	-1	1	1	13.1	3
-1	-1	1	1	1	-1	1	-1	-1	-1	1	-1	1	-1	1	15.1	4
1	-1	-1	-1	1	1	-1	-1	-1	1	1	1	-1	-1	1	15.6	5
1	-1	-1	1	-1	-1	-1	-1	1	-1	-1	1	1	1	1	17.4	6
-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	-1	-1	17.7	7
-1	1	1	1	-1	-1	-1	-1	-1	1	1	1	-1	1	-1	18.2	8
-1	1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	19.2	9
-1	-1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	-1	19.5	10
1	1	-1	1	1	-1	1	-1	1	1	-1	-1	-1	-1	-1	20	11
-1	1	-1	-1	1	-1	-1	1	1	-1	1	-1	-1	1	1	21	12
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	21.8	13
1	1	-1	-1	-1	1	1	-1	-1	-1	1	-1	1	1	-1	22	14
1	1	1	-1	-1	-1	1	1	-1	-1	-1	1	-1	-1	1	22.8	15
-1	1	1	1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	28.7	16

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Example 3: Output



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Remaining Examples

- Outliers that fall far from the bulk of the data are easy to find; inconsistent patterns within the expected range of experimental results are more difficult to discover and, hence, are often missed. The risk is erroneous conclusions regarding the effect of factors on the experimental response.
- The remainder of the presentation considers “more interesting / subtle” information in the unit structure.

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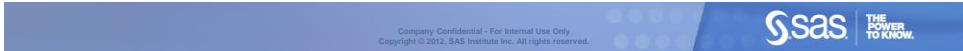
 **THE
POWER
TO KNOW.**



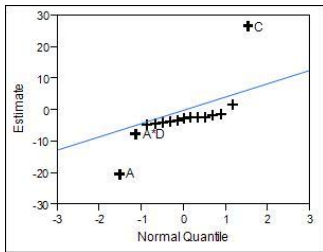
Example 4a

A	B	C	D	AB	AC	AD	BC	BD	CD	ABC	ABD	ACD	BCD	ABCD	Y	Rank
1	1	-1	1	1	-1	1	-1	1	-1	-1	1	-1	-1	-1	-35	1
1	-1	-1	1	-1	-1	1	1	-1	-1	1	-1	-1	1	1	-22	2
1	1	-1	-1	1	-1	-1	-1	-1	1	-1	-1	1	1	1	-21	3
1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-16	4
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-12	5
-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	-1	1	-1	-1	6
-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	1	12	7
-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	16	8
-1	-1	-1	1	1	1	-1	1	-1	-1	-1	1	1	1	-1	18	9
1	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	1	1	32	10
1	-1	1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	-1	34	11
1	1	1	-1	1	1	-1	1	-1	-1	1	-1	-1	-1	-1	42	12
-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	55	13
-1	1	1	-1	-1	-1	1	1	-1	-1	-1	1	1	-1	1	70	14
-1	-1	1	1	1	-1	-1	-1	-1	1	1	1	-1	-1	1	70	15
-1	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1	1	-1	85	16

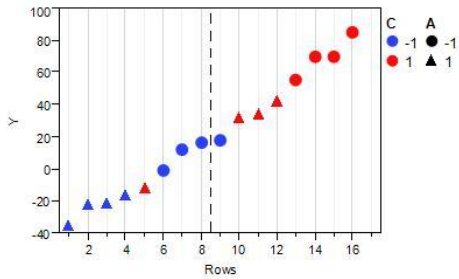
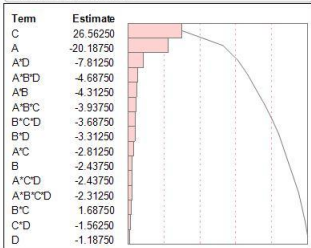
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Example 4a: Output

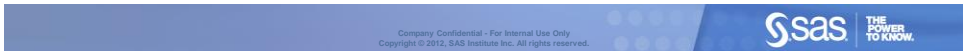


Pareto Plot of Estimates



Does not conform to the model. May represent an additional opportunity.

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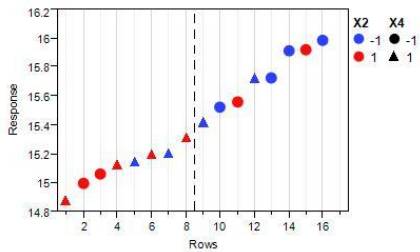
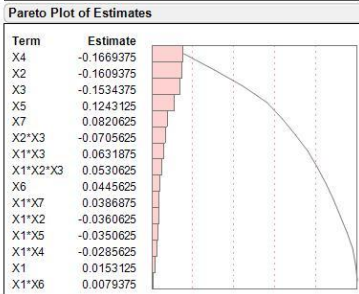
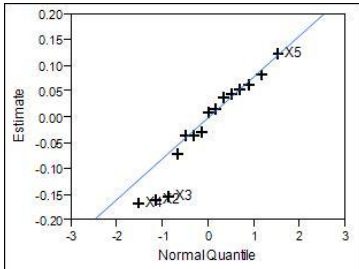
Example 4b

Run Order	X1	X2	X3	X4	X5	X6	X7	Response	X1*X2	X1*X3	X1*X4	X1*X5	X1*X6	X1*X7	X2*X3	X1*X2*X3
16	-1	1	1	1	1	-1	-1	14.878	-1	-1	-1	-1	1	1	1	-1
11	-1	1	1	-1	-1	1	1	14.996	-1	-1	1	1	-1	-1	1	-1
12	1	1	1	-1	-1	-1	-1	15.061	1	1	-1	-1	-1	-1	1	1
9	1	1	-1	1	-1	-1	1	15.127	1	-1	1	-1	-1	1	-1	-1
6	-1	-1	1	1	-1	-1	1	15.143	1	-1	-1	1	1	-1	-1	1
8	1	1	1	1	1	1	1	15.195	1	1	1	1	1	1	1	1
10	1	-1	1	1	-1	1	-1	15.204	-1	1	1	-1	1	-1	-1	-1
3	-1	1	-1	1	-1	1	-1	15.313	-1	1	-1	1	-1	1	-1	1
15	1	-1	-1	1	1	-1	-1	15.423	-1	-1	1	1	-1	-1	1	1
14	-1	-1	-1	-1	-1	-1	-1	15.519	1	1	1	1	1	1	1	-1
2	1	1	-1	-1	1	1	-1	15.56	1	-1	-1	1	1	-1	-1	-1
5	-1	-1	-1	1	1	1	1	15.721	1	1	-1	-1	-1	-1	1	-1
4	-1	-1	1	-1	1	1	-1	15.725	1	-1	1	-1	-1	1	-1	1
7	1	-1	1	-1	1	-1	1	15.91	-1	1	-1	1	-1	1	-1	-1
1	-1	1	-1	-1	1	-1	1	15.922	-1	1	1	-1	1	-1	-1	1
13	1	-1	-1	-1	-1	1	1	15.982	-1	-1	-1	-1	1	1	1	1

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Example 4b: Output



Possible interpretations?

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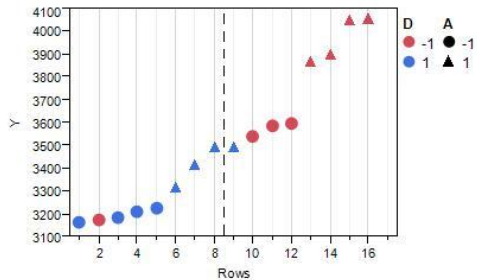
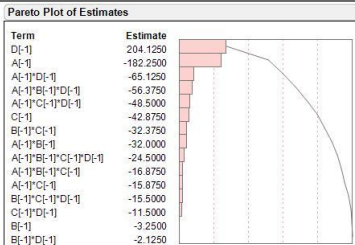
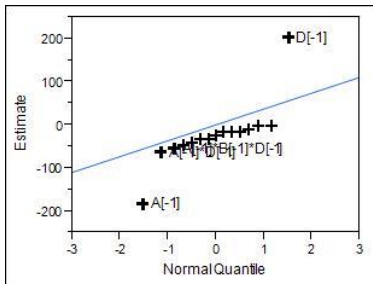
Example 4c

A	B	C	D	AB	AC	AD	BC	BD	CD	ABC	ABD	ACD	BCD	ABCD	Y	Rank
-1	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1	1	-1	3161	1
-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	1	3171	2
-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	3182	3
-1	-1	-1	1	1	1	-1	1	-1	-1	-1	1	1	1	-1	3210	4
-1	-1	1	1	1	-1	-1	-1	-1	1	1	1	-1	-1	1	3226	5
1	-1	-1	1	-1	-1	1	1	-1	-1	1	-1	-1	1	1	3315	6
1	1	-1	1	1	-1	1	-1	1	-1	-1	1	-1	-1	-1	3415	7
1	-1	1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	-1	3492	8
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3494	9
-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	-1	1	-1	3537	10
-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	3587	11
-1	1	1	-1	-1	-1	1	1	-1	-1	-1	1	1	-1	1	3596	12
1	1	1	-1	1	1	-1	-1	-1	-1	1	-1	-1	-1	-1	3868	13
1	1	-1	-1	1	-1	-1	-1	-1	1	-1	-1	1	1	1	3901	14
1	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	1	1	4047	15
1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	4054	16

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Example 4c: Output



Does not conform to the model. May represent an additional opportunity.

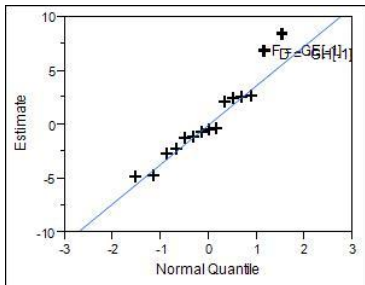
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Example 4d

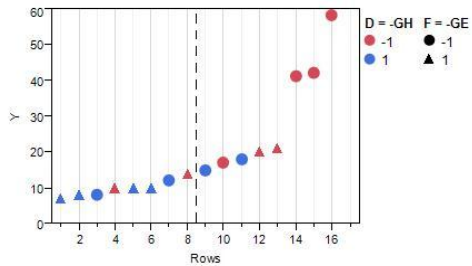
A = -BG	B = -AG	C = -GJ	D = -GH	E = -GF	F = -GE	G = -AB = -CI = - DH = -FE	H = - DG	J = -CG	AJ = BC = DE = HF	AH = BD = CE = JF	AC = BJ = DF = HE	AD = BH = CF = JE	AE = BF = CH = DJ	AF = BE = CD = JH	Y	Rank
-1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1	7	1
-1	-1	-1	1	1	1	-1	1	-1	1	-1	1	-1	-1	-1	8	2.5
-1	-1	1	1	-1	-1	-1	1	1	-1	-1	-1	-1	1	1	8	2.5
-1	-1	1	-1	1	1	-1	-1	1	-1	1	-1	1	-1	-1	10	5
1	-1	1	1	-1	1	1	-1	-1	-1	-1	1	1	-1	1	10	5
1	1	1	1	1	1	-1	1	1	1	1	1	1	1	1	10	5
-1	1	1	1	1	-1	1	-1	-1	1	1	-1	-1	-1	1	12	7
-1	1	1	-1	-1	1	1	1	-1	1	-1	-1	1	1	-1	14	8
1	1	-1	1	-1	-1	-1	1	-1	-1	1	-1	1	-1	-1	15	9
-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	17	10
1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	1	-1	18	11
1	1	-1	-1	1	1	-1	-1	-1	-1	-1	-1	-1	1	1	20	12
1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	1	21	13
-1	1	-1	-1	1	-1	1	1	1	-1	-1	1	1	-1	1	41	14
1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	42	15
1	1	1	-1	-1	-1	-1	-1	1	1	-1	1	-1	-1	-1	58	16

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Example 4d: Output



Term	Estimate
D = -GH[-1]	8.437500
F = -GE[-1]	6.937500
A = -BG[-1]	-4.812500
AC = BJ = DF = HE[-1]	-4.687500
B = -AG[-1]	-2.687500
AH = BD = CE = JF[-1]	2.687500
AD = BH = CF = JE[-1]	2.562500
AE = BF = GH = DJ[-1]	2.437500
J = -CG[-1]	-2.187500
AF = BE = CD = JH[-1]	2.062500
G = -AB = -CJ = -DH = -FE[-1]	-1.187500
C = -GJ[-1]	-1.062500
E = -GF[-1]	-0.687500
H = -DG[-1]	-0.437500
AJ = BC = DE = HF[-1]	-0.312500



What would it take to “clean this plot up”?

The ties indicate issues with the measurement system.

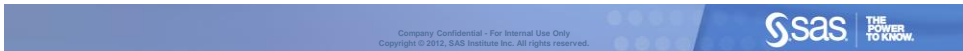
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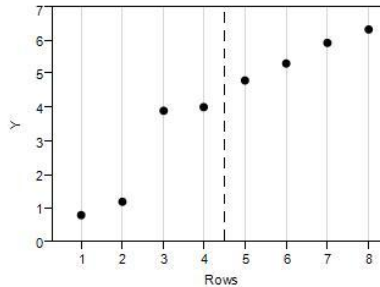
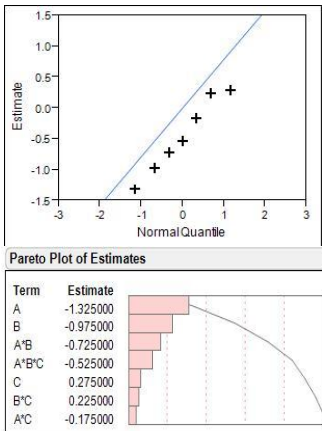
Example 5 (Hillstrand)

A	B	C	AB	AC	BC	ABC	Y	Rank
1	1	1	1	1	1	1	0.8	1
1	1	-1	1	-1	-1	-1	1.2	2
-1	1	-1	-1	1	-1	1	3.9	3
1	-1	-1	-1	-1	1	1	4	4
1	-1	1	-1	1	-1	-1	4.8	5
-1	-1	1	1	-1	-1	1	5.3	6
-1	-1	-1	1	1	1	-1	5.9	7
-1	1	1	-1	-1	1	-1	6.3	8

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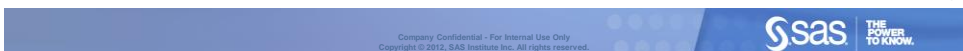
Example 5: Output



This may be a critical mix.

A critical mix is when a response occurs only at a particular mix of the factors

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Importance of the technique

- Outliers are responses that do not fit the proposed model. These discrepant points are of great interest.
- Possibilities:
 - Unit Structure
 - » Special Cause
 - » Background Noise Shift
 - Design Structure
 - » Wrong Model
 - » Metric
 - » Critical Mix
- Philosophies:
 - *A discrepant value should not just be thrown away-it might be trying to tell us something – George Box*
 - Parallel paths in the subsequent investigation should consider opportunities in the unit structure as well as the design structure

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Mechanics

- Add Additional Columns To Spreadsheet (DF = # Columns)
- Sort The Data by Y (smallest to largest)
- Use An Overlay Plot To Graph The Data
- Note Obvious Discontinuities In The Data Plot
 - » Used in synergy with the Normal Probability Plot
- Iteratively & Graphically Explore X Columns (Investigation For Subtle Discontinuities)

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Thank You

- **Accepted for Publication in Quality Engineering:**
 - **A Graphical Tool for Detection of Outliers in Completely Randomized, Unreplicated 2^k And 2^{k-P} Factorials**
 - By Doug Sanders and Cheryl Hild
 - » Publication 29 August 2012 (online), September 2012 (print)

- Questions?

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