HOPE Theory and JMP Software for Robust Design

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Abstract

Nowadays, robust design is an indispensable tool for manufacturing. Until recently, only Taguchi method has been mainly used for it. I introduce Hierarchical-structure Optimization for Prospective Engineering (HOPE) in this paper. HOPE is more flexible and powerful than traditional Taguchi's method. Signal to Noise Ratio (SNR) solution in Taguchi method is a special case solution in HOPE theory. By using the same Crossed Array Experiment (CAE) data as Taguchi method uses, multiple reasonable solutions are found by HOPE theory while only one SNR solution is obtained in Taguchi method. All solutions in HOPE are candidates for the final solution. The reproducibility has to be examined by confirmation runs. HOPE software is a JSL application developed on JMP8. It enables us to perform robust design and the confirmation of reproducibility easily and quickly. HOPE software is developed in cooperation with Takahashi and SAS Institute Japan.

Many advantages of HOPE theory are demonstrated by examples of design for paper helicopters, which are used as educational exercise worldwide. Education and implementations of HOPE theory are held in many Japanese companies. HOPE theory and HOPE software are explained in this paper.

Keywords

Robust Design, Regression Analysis, Process Capability Index, Hierarchical Structure , Mathematical Programming, Crossed Array Experiment

1. Introduction

Taguchi's robust parameter design finds an optimal factor setting based on Signal-to-Noise Ratios (SNR). SNR are calculated from Crossed Array Experiment (CAE) or direct product layout experiment. Taguchi's Robust Design (RD) is very popular as Quality Engineering (QE). But we should be careful for using Taguchi's RD since the optimum solutions based on SNR are not always reasonable.

This paper discusses HOPE proposed by Takahashi. HOPE is a method based on hyper-regression and mathematical programming for RD with many nuisance factors (or noise factors in QE terminology), many design factors (or control factors in QE terminology) and many items to be considered such as cost, delivery, safety and environment.

CAE is designed by combining two kinds of arrays. The array of design factors is called an inner array and the one of nuisance factors is called an outer array. The layout consisting of all combinations of the two arrays is called crossed arrays or direct product layout. The experiment conducted based on the crossed arrays is called CAE. The design factor is a factor for which you can control or select each level. However, the nuisance factor is a factor for which you cannot specify or select each level. The main feature of Taguchi's RD is that you can find desirable outputs by controlling the design factors. Accordingly, it is better to use more design factors since it allows for ensuring more favorable results. It is also better to take into account various management elements (cost, productivity, delivery, safety, environment etc.) in addition to the quality assurance achieved by RD.

The strategy of RD is to choose design factor settings that make a system less sensitive to nuisance variation. If there are many design factors, and if hyper-regressions are obtained, then HOPE has the following advantages.

(A) A solution with smaller nuisance variations can be obtained.

(B) More management items can be taken into account.

(C) A solution can be improved flexibly.

HOPE classifies a lot of terms in hyper-regression into three parts: (1) average part, (2) dispersion part and (3) error part. Hope optimizes the average part with constraining dispersion part variation to a small value. It also considers error part and other many items such as cost, delivery, safety, environment etc. Details of HOPE are shown in Takahashi (2007).

2. Inspection, Process Control and Development for Quality Assurance

Quality assurance has mainly three stages: (1) inspection, (2) process control and (3) development.

2.1 Retrospective Engineering

At inspection stage, engineers face to “far past” or “historical” results of manufacturing. Products were already manufactured at this stage. The inspection just selects good products for quality assurance. If an engineer finds bad products, he or she adjusts or scraps them.

At process control stage, engineers face to “near past” or “present” results of manufacturing. Products have already been manufactured at this stage. Products have already been manufactured or are being manufactured at this stage. I call both of inspection and process control “retrospective engineering”. Problem solving is the essential point of retrospective engineering because we should fix defects quickly and prevent recurrence of problems without changing the current system.

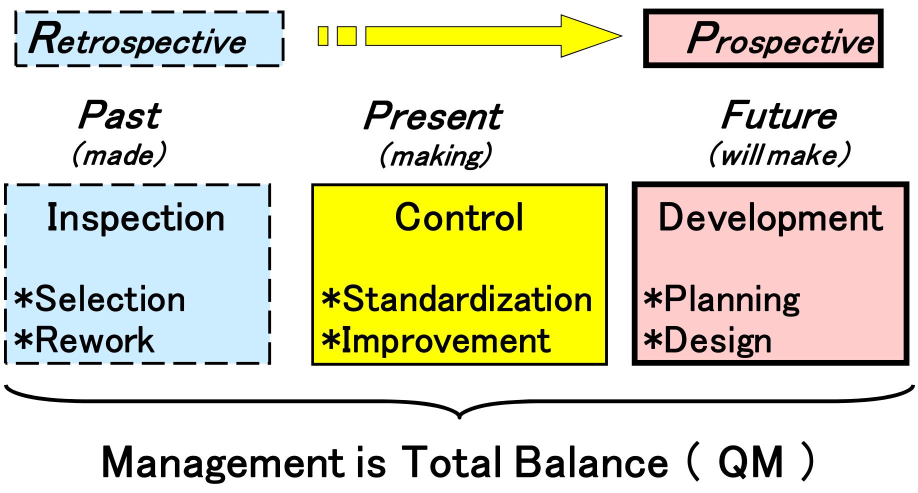


Figure 1: Paradigm Shift in Quality Management

2.2 Prospective Engineering

At development stage, engineers think about the future of manufacturing. They design new products which will be manufactured in the future. They image and estimate future situations, and then prepare countermeasures for problems in the future. The essential point of prospective engineering is task achieving because engineers should prevent occurrence of assumed problems based on the new system.

A reasonable design has three features as follows;

(1) It satisfies customers' requests.
(2) It satisfies stakeholders' requests.
(3) It prevents occurrence of assumed (predicted) problems.

One of the major assumed problems is disturbance by nuisance factors. Robust design is also called prospective engineering because it aims to reduce the future problems that are the inferences caused by nuisance factors.

3. HOPE (Hierarchical-structure Optimization for Prospective Engineering)

3.1 Product is Production- facility for User

Users buy products as means (=facilities, devices or equipments) to get (=generate) outputs they want. Form this viewpoint, we can think products for users as production facilities for manufactures, and the usage of products as manufacturing process in engineering. So they can be evaluated by the concept of the process capability index Cp as we evaluate facility and process in manufacturing. The detail about capability of product is discussed in 3.4.

The mechanism to generate requested outputs is important for users. The mechanism expressed by the input-output regression in HOPE. The relationship between an output y and an input factor (or signal factor in QE terminology) m can be modeled by

$$y=f(m),$$

We can say that customer's main interest lies in the input-output relationship f. If m is an effective input factor, it should have a significant effect on y. In many cases, the relationship f is a monotone function and it is approximated by linear regression or quadratic regression.

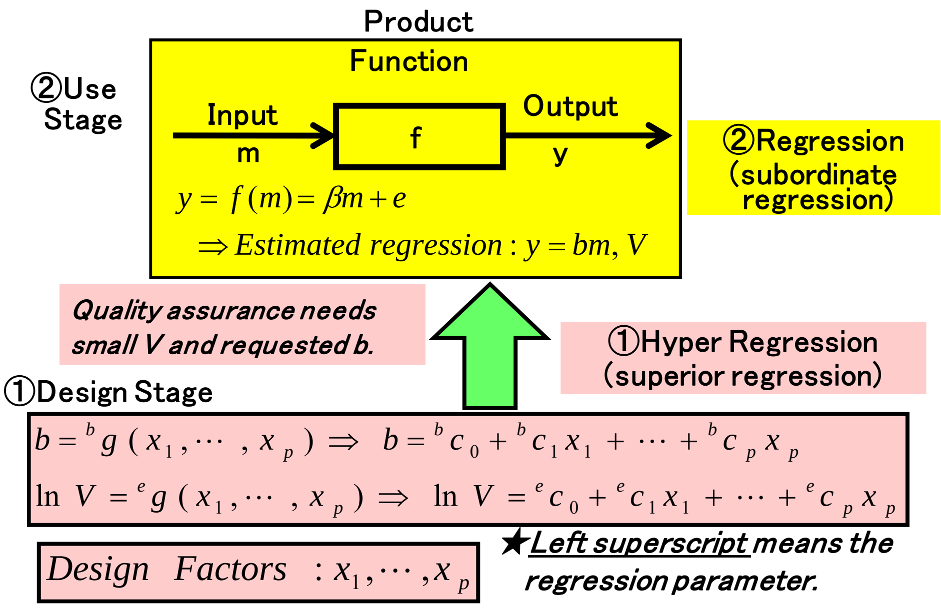


Figure 2: Hierarchical- Regression (for Design & Use)

3.2 Two Kinds of Disturbances against Quality Assurance

There are two kinds of disturbance against QA activities.

(1)Error: Random Variable

Disturbances caused by errors are obstructive to engineering. Errors are random variables, and the size of their influence depends on chance. There are two kinds of errors in RD. One is the error in input-output regression, and the other is the error in manufacturing. Those variances should be reduced by the quality management activity.

(2)Nuisance Factor: Population Parameter

Disturbances caused by nuisance factors are also obstructive. An nuisance factor in HOPE is called "an effect" in terminology of standard regression analysis. A nuisance factor is not a random variable but a population parameter. So that it is not called as "noise factor" in this paper in order to distinguish it from error. Its influence (bad effect) needs to be reduced by RD.

3.3 Hyper-regression Model for Robust Design

There are two kinds of regressions in HOPE:

* Input-output regression for customers to use the product.
* Hyper-regression for engineers to design the product.

The input-output regression generates estimated values of model parameters (intercept, slope and variance etc.). Response variables in the hyper-regression are intercepts, slopes and variances of error of input-output regressions. Hyper-regression means upper-level regression for parameters in subordinate regression (lower-level regression). In robust design, the former regression is related to "design" phase, and the latter regression is related to "use" phase.

Input-output regression and hyper-regression are described as 1) and 2) when the i-th condition of design factors is $(x_{1i},\cdots,x_{pi})$. The left side subscript "n" means the level of a nuisance factor.

1) Input-output regression (simple regression as for example)

Population regression

$$ {}_n y_{ij} = {}_n\beta_{0i} + {}_n\beta_{1i} m_j + {}_n e_{ij}, \quad {}_n e_{ij} \sim N(0, {}_n\sigma_{ij}^2) \tag{1} $$

Estimated regression

$$ {}_n y_{ij} = {}_n b_{0i} + {}_n b_{1i} m_j, \quad {}_n RMSE_{ij} = \sqrt{{}_n V_i} \tag{2} $$

2) Three Models in Hyper-regression

Denote θ a general expression about population parameters of input-output regression such as β_0, β_1, β_2 (for quadratic regression), and σ^2. The estimated parameters which are corresponding to them are b_0, b_1, b_2 and Ve (RMSE=$\sqrt{V_e}$). "t" is a general expression about estimated parameters of input-output regression such as b_0, b_1, b_2 and V.

There are three models in hyper-regression as follows.

(1) Population hyper regression

$$ Model1: {}_n\theta_i = {}_n\gamma_0 + \sum_i^p {}_n\gamma_i x_i + \sum_i^p \sum_{i<j}^p {}_n\gamma_{ij} x_i x_j + \sum_i^p {}_n\gamma_{ii} x_i^2 + {}_n\varepsilon_i \tag{3} $$

$$ Model2: {}_n\theta_i = {}_n\gamma_0 + \sum_i^p {}_n\gamma_i x_i + \sum_i^p \sum_{i<j}^p {}_n\gamma_{ij} x_i x_j + {}_n\varepsilon_i \tag{4} $$

$$ Model3: {}_n\theta_i = {}_n\gamma_0 + \sum_i^p {}_n\gamma_i x_i + {}_n\varepsilon_i \tag{5} $$

$$ {}_n\varepsilon_i \sim N(0, {}_n\sigma_i^2) $$

(2) Estimated hyper-regression

$$Model1: {}_nt_i = {}_nc_0 + \sum_i^p {}_nc_i x_i + \sum_i^p \sum_{i<j}^p {}_nc_{ij} x_i x_j + \sum_i^p {}_nc_{ii} x_i^2 \quad (6)$$

$$Model2: {}_nt_i = {}_nc_0 + \sum_i^p {}_nc_i x_i + \sum_i^p \sum_{i<j}^p {}_nc_{ij} x_i x_j \quad (7)$$

$$Model3: {}_nt_i = {}_nc_0 + \sum_i^p {}_nc_i x_i \quad (8)$$

I recommend you use the logarithm transformation for V (RMSE= $\sqrt{V}$)

Model 3 is the simplest model. However, Model 3 may not be fit well. We should be careful for the lack of fit, because we have considerably wide range experimental conditions in experiments for RD. We should include product terms and /or second-order terns in the model in order to fit the model well

3.4 Function Capability Index Cf

Process capability Cp is generally evaluated based on the target and the width of specification limits. Function capability Cf should be evaluated based on the target and the width of acceptance limits as shown in Figure 3.

*Cp $=(S_U - S_L)/6\sigma$

*Cf $=2\lambda/6\sigma$

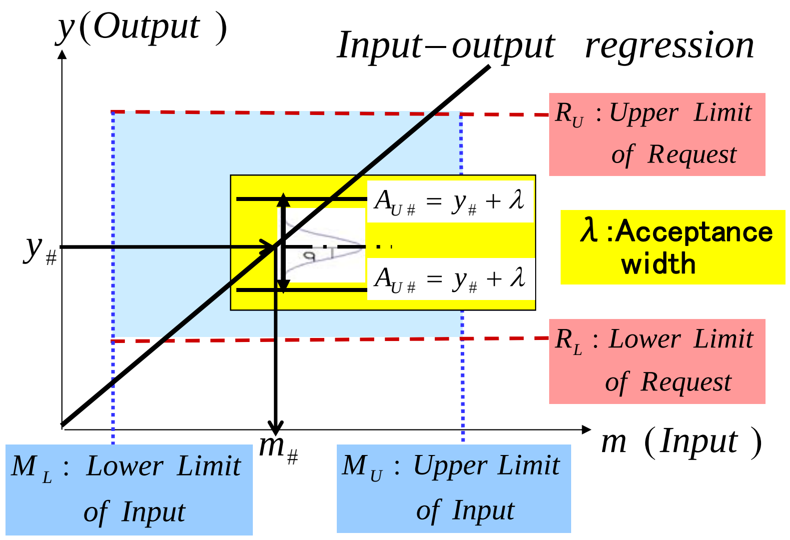


Figure 3: CRZ (Customer Request Zone)

3.5 Crossed Array Experiment and Summery Measures

We use the following CAE data in this paper.

* p quantitative design factors ($x_1, \cdots, x_p$)
* One qualitative nuisance factor with q levels
* One adjustment factor

The two-step method is an approach of searching for the best condition by the following two steps.

The first step:

Find a condition which the nuisance factors affect least.

The second step:

Adjust the total average to the target value or optimizing(maximizing or minimizing).

This paper focuses only on the first step and omits discussing the second step.

The strategy of robust design is to choose design factor settings that make a system less sensitive to the variation caused by nuisance factors. Average, variance and SNR are computed over the same run of inner array in the crossed array layout.

4. Mathematical Description of HOPE

In this chapter following simple case is focused to explain essence of HOPE easily.

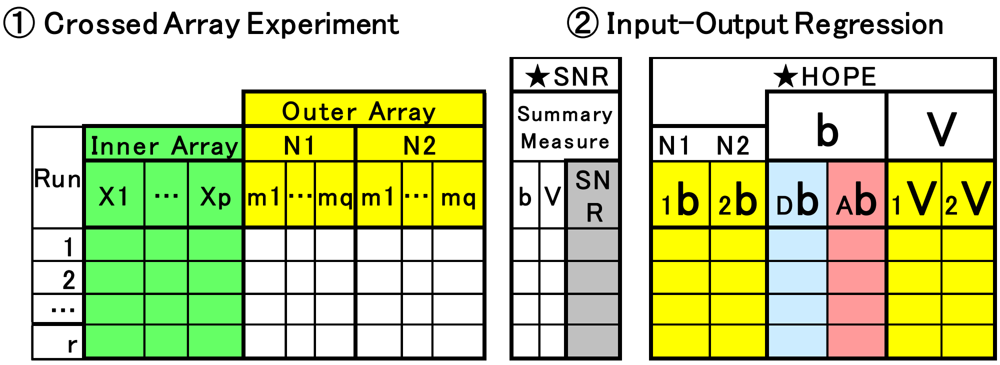
(1) Input-output regression without an intercept. .

(2) A Nuisance factor has only two levels.

4.1 Analysis (Preparation for Design)

The HOPE analysis is done by the following steps.

1. Data is gathered based on a Crossed Array Experiment.
2. Input-output regression parameters (I/O parameters) are estimated for each run..
3. Hyper-regressions for estimated I/O parameters are estimated.
4. Average of model parameters in two levels of a nuisance factor and difference between them are calculated for each run.



③ Hyper Regression *(Second order model decided by variable selection)*

[A] Quality (Average)

$$ {}_A b = \frac{{}_1 b + {}_2 b}{2} = {}_A^b g(\boldsymbol{x}) = {}_A^b c_0 + \sum_i^p {}_A^b c_i x_i + \sum_i^p \sum_{i<j}^p {}_A^b c_{ij} x_i x_j + \sum_i^p {}_A^b c_{ii} x_i^2 \quad (9) $$

[B]Robustness (Difference)

⇒Rage=Max−Min

$$ {}_D b = {}_1 b - {}_2 b = {}_D^b g(\boldsymbol{x}) = {}_D^b c_0 + \sum_i^p {}_D^b c_i x_i + \sum_i^p \sum_{i<j}^p {}_D^b c_{ij} x_i x_j + \sum_i^p {}_D^b c_{ii} x_i^2 \quad (10) $$

[C] Variance (Error)

$$ \ln_n V = {}_n^e g(\boldsymbol{x}) = {}_n^e c_0 + \sum_i^p {}_n^e c_i x_i + \sum_i^p \sum_{i<j}^p {}_n^e c_{ij} x_i x_j + \sum_i^p {}_n^e c_{ii} x_i^2 \quad (11) $$

Figure 4: Analysis based on Crossed Array Experiment

4.2 Design

Figure 5 shows the structure of design by HOPE. ”Quality”, “robustness” and “variance” are important. However, “cost”, “delivery”, “safety” and “environment” are also important. In Hope, one of the items is set to an objective function, and the other items are set as constraints in order to optimize these items mathematically. .

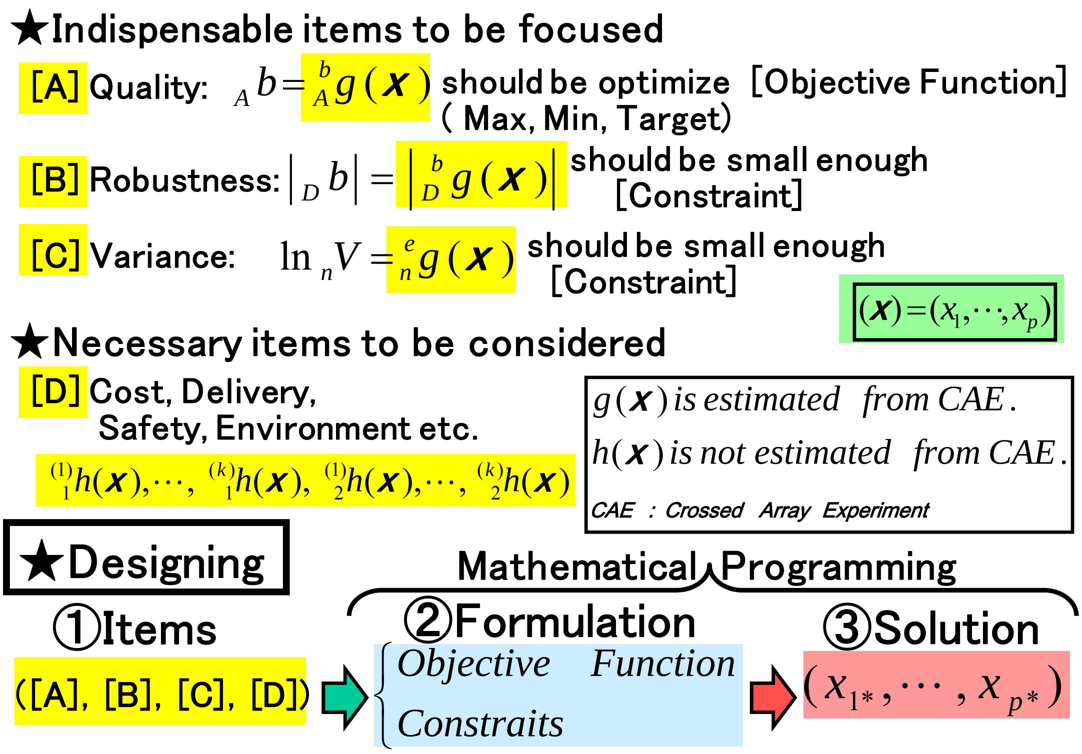


Figure 5: Design Based on Crossed Array Experiment

4.3 Robust Design Linked with Other Items

Items that should be considered here are C (Cost), D (Delivery), S (Safety), E (Environment) in addition to Q (Quality), R (Robustness) and V (Variance).
The expression of a general function of these items is shown below.

$$ {}^{(l)}u = {}^{(l)}h(x_1, \cdots, x_p), \quad l = 1, \cdots, k \tag{12} $$

5. PDCA (Plan-Do-Check-Act) Optimization

Stake-holders consists of customers, makers and society. We need to find the best solution under a lot of conditions requested by many people with various opinions.

A solution obtained by mathematical programming is a strictly mathematical result. In addition, the solution is temporary and transitional. So it should be examined by the stake-holders frequently.

PDCA Optimization is proposed in this paper. It is an approach to achieve all stake-holders’ agreement. It needs discussion based on PDCA concept. In case of robust design, PDCA cycle of optimization by using mathematical programming (MP) means as follows:

Plan: Formulate, Do: Analyze, Check: Examine, Act: Negotiate

The solution that achieves all stake-holders' agreement is called PDCA optimum solution. Mutual Agreement needs the insistence, the cooperation, and the concession through discussion.

6. Summary of HOPE Theory

6.1 Hierarchical-structure

The hierarchical-structure in HOPE is the same as a nested structure of matryoshka, which is a famous Russian doll with complicated structure.

(1) Products are "production facilities" for users.

* Products are means (facilities, equipments or devices) to generate the requested outputs.
* A user is a kind of manufacturing engineer in daily life, and he/she manufactures outputs he/she wants by products every day.
* Products must be evaluated same as manufacturing process based on process capability index Cp, therefore RMSE is important in RD.

(2) The function of a product is described by regression parameters such as intercept β_0, slope β_1 and variance σ^2 etc.

* The function of a product can be described by an input-output regression.
* The experiment is done in order to decide the level of its regression parameters.

(3)Hyper-regression: There are two kinds of regressions in RD. One is input-output regression (lower level regression) for customer's use of product and the other is hyper-regression for engineer's design of product (upper level regression). The latter is hyper- regression which means the regression of regression parameters.

(4)Hyper-index: There are two kinds of capability indices for evaluation of products and processes. One is function capability index (lower level index) Cf for customer's use of the product, and the other is process capability index (upper level index) Cp for engineer's manufacture of the product. The latter is hyper- capability index which means the capability index of capability index.

Cp: Capability index of the process. This is calculated based on the target and the specification limits.

Cf: Capability index of the function. This is calculated based on the target and the acceptance limits (specification limits determined by users).

6.2 Procedure of HOPE

HOPE consists of the following six procedures after a crossed array experiment. It is a democratic and mathematical design approach.

(1) Input-output regression parameters (I/O regression parameters) are estimated by least square method (LSM).
(2) Hyper-regressions for estimated I/O regression parameters are also estimated by LSM.
(3) The average and range (= maximum - minimum) of hyper-regression are calculated.
(4) Functions of relating items (C, D, S, and E etc.) and the design factors are provided.
(5) The PDCA optimization is done by using mathematical programming.
(6) The optimum solution (PDCA optimum solution) on which stake-holders can agree should be formed through discussion.

6.3 Summary of HOPE

This paper discusses HOPE which uses hierarchical-structure (hyper-regression and hyper-index) and mathematical programming to form reasonable solution. The core idea of

HOPE consists of following three points.

(1) It is necessary to think products to be production-facilities which generate outputs for users.

(2)We should consider hierarchical-structure (hyper-regression and hyper-index) in designing, manufacturing and using products.

(3)Reasonable solution which can be agreed by many stake-holders is formed by PDCA approach with mathematical programming based on hierarchical-structure.

7. Twin Rotor Paper Helicopter and Crossed Array Experiment

Paper helicopters are often used for teaching and design of experiments (Box 1991). The paper helicopter with twin rotors is shown in the Figure 6 and 7. This paper helicopter developed by Takahashi has the design factor of 20 and more. It is more complicated than the one in Box (1991). In this paper 9 factors of those are being used as the design factors from X_1 to X_9 as shown in the Figure 6 and 7. Eleven factors can be assigned in L_{12}. Two of eleven factors are used just for checking lack of fit. This experiment is very easy to do. Even beginners can prepare this experiment. However, it is enough to discuss the essential point of HOPE theory. The measurement unit for each design factors in helicopter design is mm (millimeter) .

A nuisance factor is paper material. Two kinds of paper, heavy/strong paper (copy-paper) and light/weak (poor-paper) are used. They are written respectively with H and L when short description is needed. An input factor is releasing height because it is clear for everybody that the releasing height has no interaction between every design factor and the nuisance factor. If the input factor is releasing height, zero intercept regression can be assumed. Every range of each design factor is very narrow in this experiment. So a liner model without interaction term and quadratic term is assumed for input-output relationship.

Y is the duration of flight and it distributes from 0.5 second to 2.0 second.1/100 second is used as the unit. Body width is fixed to 25mm which is reasonable size for beginners. Space= 35 and slit= 8 are determined for convenience to combine upper rotor and lower rotor by stapler.

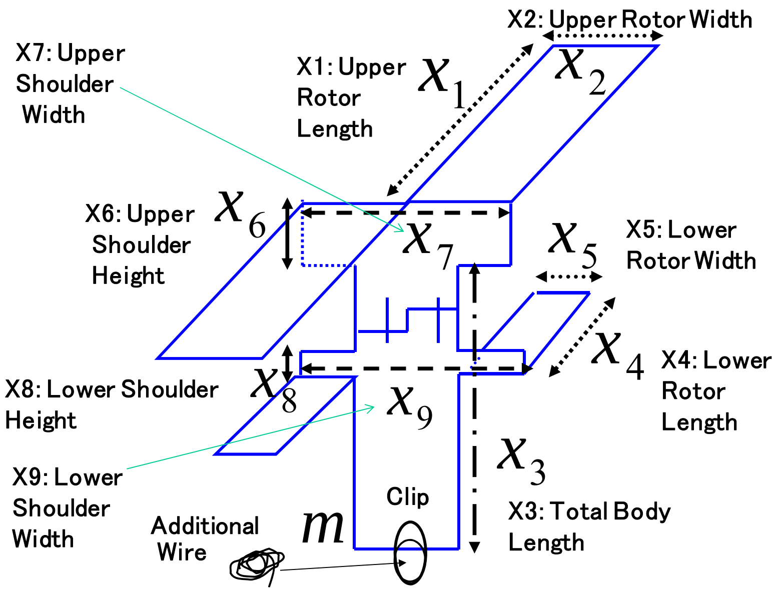


Figure 6: Paper Helicopter with 9 Factors after manufacturing

Figure 8 shows how to make twin rotor helicopters. It is easy for everybody to make the helicopters. Cutter knives are better than scissors for cutting papers with high quality and high productivity.

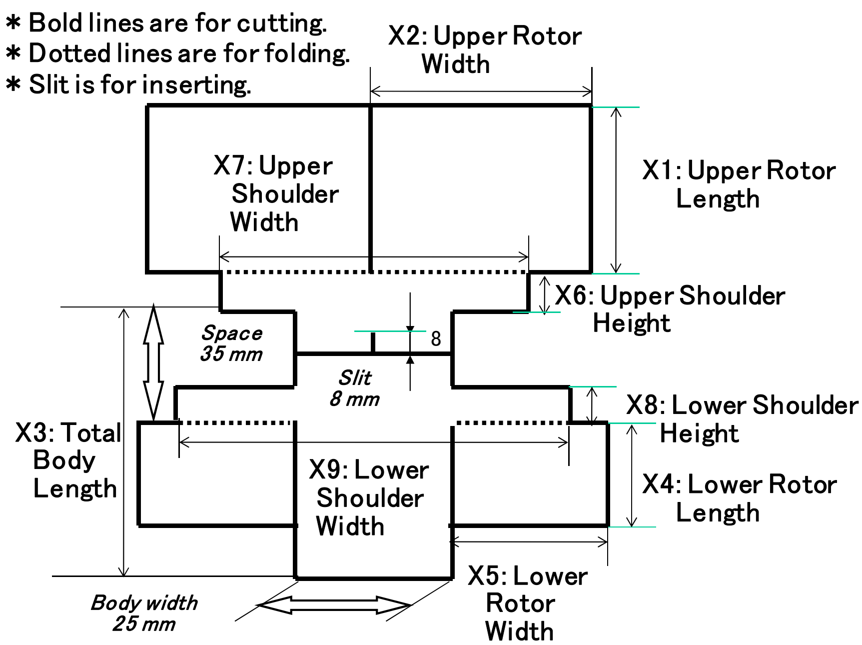


Figure 7: Paper Helicopter with 9 Factors before manufacturing

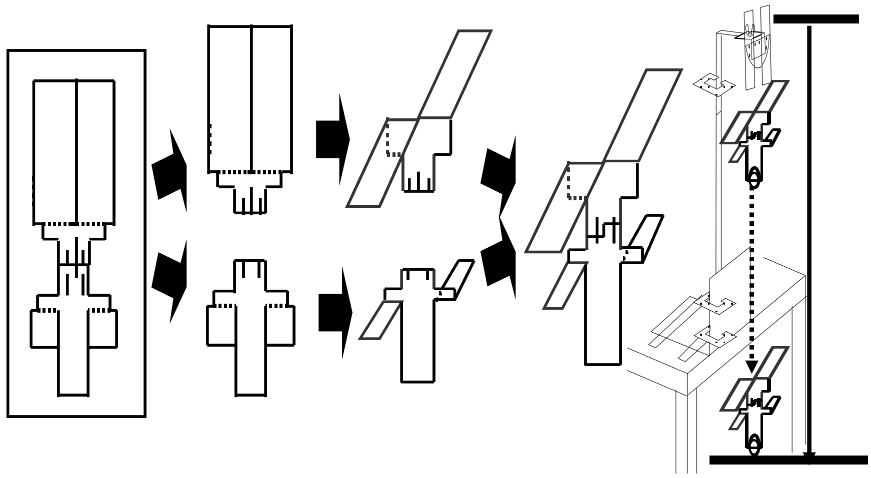


Figure 8: How to manufacture and fly Paper Helicopter with 9 Factors

Figure 9 shows the items to be considered in design. First group of items include three kinds of gaps and second one is area (total size) . The gaps are important in productivity and quality, and area is important in cost.

Figure 10 shows the examples with gap and without gap. To make gaps has many demerits as follows.

1. Hard work, 2. Nervous work, 3. Long operation time
4. Many workers, 5. Long manufacturing process
6. Many troubles, 7. Big dispersion, 8. Breakable product
Etc.

[Productivity], [Trouble], [Dispersion]

G1 (Gap1): Upper Gap

$X2 \times 2 - X7 = 0$

G2 (Gap2): Lower Gap

$X5 \times 2 + \text{Body Width}(25) - X9 = 0$

G3 (Gap3): Botom Gap

$X4 + 35 - X3 = 0$

[Cost]

A (Area): Total of Paper

$X1 \times X2 \times 2 + X3 \times \text{Body Width}(25)$

$+ X4 \times X5 \times 2 + X6 \times X7$

$+ X8 \times X9 + (8-X8) \times \text{Body Width}(25)$

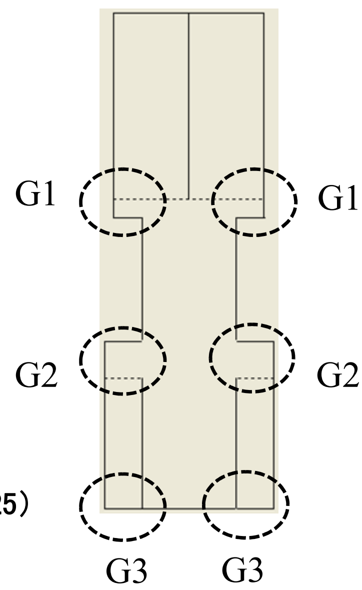


Figure 9: Items to be considered in design

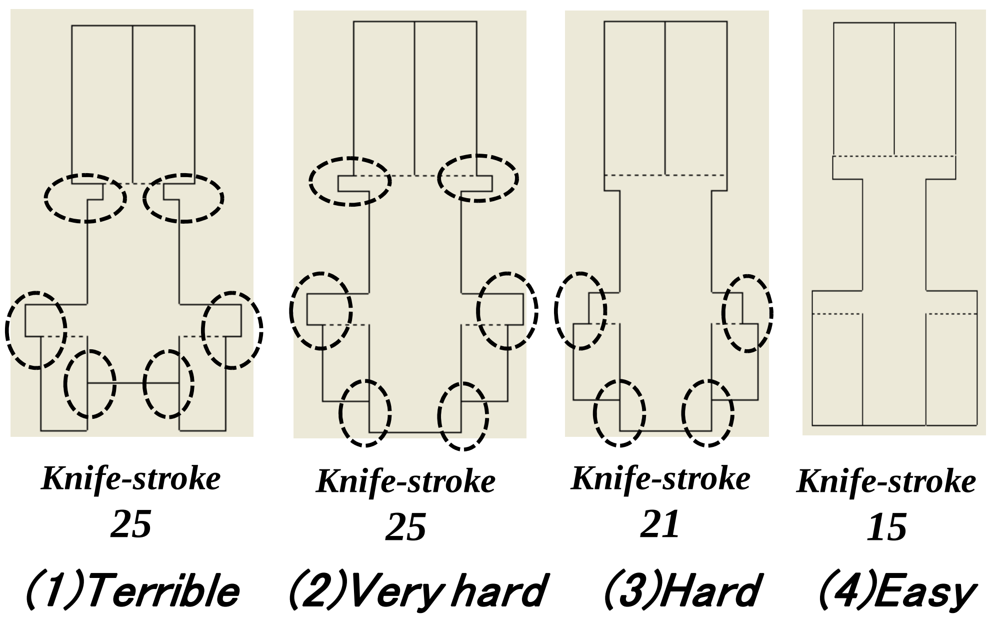


Figure 10: Examples with gaps & without gap
(It is easy to make the helicopter at the right-side.
But it is very terrible to make the left-side one)

8. Crossed Array Experiment

Figure.11 shows the data table of crossed array experiment and its' outline is as follows.

Output: Duration of flight (unit: 1/100 second)

Input factor: Releasing height with 4 levels (80cm, 120cm, 160cm, 200cm)

Nuisance factor: paper material with 2 levels (Heavy paper, Light paper)

Design factors: 9 factors (Figure 6 and 7) with 2 levels (Table 1)

Table 1: Design Information of CAE

Response

Response Name	Lower Limit	Upper Limit
Time	70	140

Model

Linear(No Intercept)

Input/Signal Factor

Factor Name	Type	L1	L2	L3	L4
Hight	C	80	120	160	200

Design/Control Factor

Factor Name	Type	L1	L2
X1(URL)	C	40	70
X2(URW)	C	20	30
X3(TBL)	C	70	80
X4(LRL)	C	20	40
X5(LRW)	C	15	30
X6	C	1	2
X7	C	1	2
X6(USH)	C	5	10
X7(USW)	C	30	50
X8(LSH)	C	5	10
X9(LSW)	C	55	65

Nuisance/Noise Factor

Factor Name	Type	L1	L2
Paper	N	Heavy	Light

Options

Show LnRMSE

□□Exampls

Columns (24/0)

X7(USW), X8(LSH), X9(LSW), Heavy[80], Heavy[120], Heavy[160], Heavy[200], Light[80], Light[120], Light[160], Light[200], G1, G2, G3, Area

Rows

All rows 12

Selected 0

Excluded 0

22/0 Cols	No	X1(URL)	X2(URW)	X3(TBL)	X4(LRL)	X5(LRW)	X6(USH)	X7(USW)	X8(LSH)	X9(LSW)	Heavy [80]	Heavy [120]	Heavy [160]	Heavy [200]	Light [80]	Light [120]	Light [160]	Light [200]	G1	G2	G3	Area
1	1	40	20	70	20	15	5	30	5	55	61	77	111	126	56	80	106	133	10	0	-15	4450
2	2	40	20	70	20	15	10	50	10	65	52	79	112	138	52	77	98	126	-10	-10	-15	5050
3	3	40	20	80	40	30	5	50	10	65	66	99	121	153	73	97	126	161	-10	20	-5	6850
4	4	40	30	70	40	30	10	30	5	65	60	89	123	164	65	100	129	159	30	20	5	7250
5	5	40	30	80	20	30	10	30	10	55	52	84	115	139	54	91	123	154	30	30	-25	6400
6	6	40	30	80	40	15	5	50	5	55	57	87	112	142	66	92	124	153	10	0	-5	6200
7	7	70	20	80	40	15	10	30	10	55	56	97	139	178	68	115	150	184	10	0	-5	6800
8	8	70	20	80	20	30	5	30	5	65	66	95	136	180	59	108	154	197	10	20	-25	6550
9	9	70	20	70	40	30	10	50	5	55	64	107	135	180	83	113	157	192	-10	30	5	7800
10	10	70	30	80	20	15	10	50	5	65	79	109	153	203	73	121	159	203	10	-10	-25	7700
11	11	70	30	70	40	15	5	30	10	65	66	99	150	174	70	116	154	201	30	-10	5	7900
12	12	70	30	70	20	30	5	50	10	55	70	110	151	175	73	122	163	217	10	30	-15	7900

Figure 11: Design Information and Data Table of Crossed Array Experiment

9. SNR Solution by JMP Software

Figure 12 shows two effect diagrams. The upper part shows SNR and the lower part shows sensitivity (S).

$$SNR = 10\log(b^2/V) \qquad (or\ \ SNR = \ln(b^2/V)\ )$$

$$S = 10\log b^2 \qquad (or\ \ S = \ln b^2)$$

Two-Step optimization is as follows.

Step 1 : Reduce variability.

Find the combination of design factors to maximize SNR.

Step 2 : Adjust sensitivity.

Adjust S (place it on the target, maximize it, minimize it) without changing the maximized SNR in the first step.

Figure 12 shows that X1 (URL) greatly influences both SNR and S. Especially, S is affected almost only by X1. So there are two problems.

Problem 1: If X1 is used to maximize SNR in step 1, then sensitivity can't be adjusted by using X1 in step2.

Problem 2: SNR is maximized at X1=40 in this example. But V, the denominator of SNR. is not maximized at X1=40. SNR is maximized at X1=40 just because b^2, the numerator of SNR, becomes larger at the level. A large SNR value doesn't always mean that the influence of nuisance factor is small.

These problems are caused from the following essential features of SNR.

[A] SNR and S share the square of slope b.

[B] SNR is summery measure of slope b and variance V.

$$SNR = 10\log(b^2/V) = 10\log b^2 - 10\log V$$

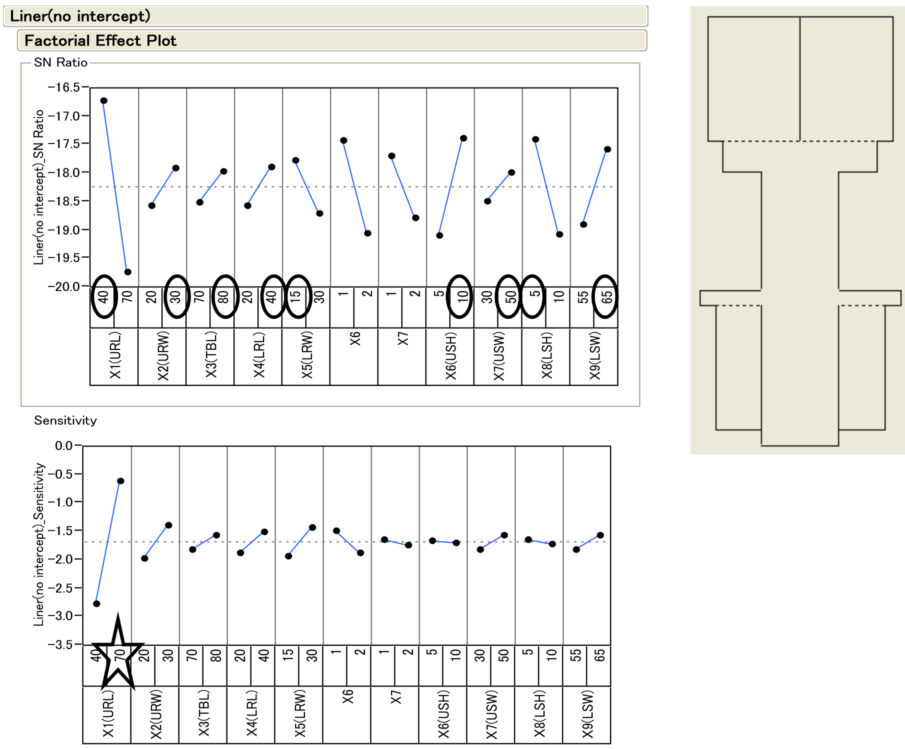


Figure 12: Two effect diagrams and helicopter design

SNR which is summery measure has the following limits.

＊It is difficult to understand what happens based on summery measure when the solution is obtained. Therefore, the optimum solution is not necessarily a solution that reflects the intention of stakeholders.

＊There are various, important items that are influenced by the design factor besides b and V such as quality, cost, delivery, safety and environment. The method of using this summery measure cannot consider these important items.

＊This method cannot consider CRZ (customer request zone) and Cf (function capability index) in designing.

10. HOPE Solution by JMP Software

10.1 Policies

I imposed the following three rules on participants in my education program for design of experiments.

(1) The quality assurance should be achieved in the first.

＊The influence of the nuisance factor can be evaluated by range of slopes. The range should be 0 or less than a small target value.

＊In this example, the acceptance width of customer is 15 (λ=15) then RMSE must be less than 3.75 to achieve Cf≧1.33 .

(2) The efficiency of the work should be improved.

＊To make work easier, decrease working mistakes, and shorten working hours.

＊The simplification of work often decreases the dispersion of products.

＊The number of cutting strokes should be reduced to make that simplify cutting and be made in a short time. This decreases cutting mistakes, cutting troubles and cutting dispersion as a result.

(3) The total amount of paper should be reduced after the above two conditions are achieved.

10.2 Analysis before Robust Design by Using Variable Selection for Hyper-regression

The variable selection of multiple regression analysis is carried out based on the p value. The criteria of selection is p >= 0.25, while the another criteria of removal is p < 0.25. Figure 13 shows the result of variable selection as follows.

1) The left-side of Figure 13 shows the results for slopes. The coefficient of determination is very large, and nuisance factor (paper type) is significant. However it can be reduced by robust design. Both of average and rage are necessary to be considered in designing.

2) The center of Figure 13 shows the result for RMSE. The coefficient of determination is not so large, however the effect cannot be ignored.

Nuisance factor is not significant, so that only average is necessary to be considered in designing.

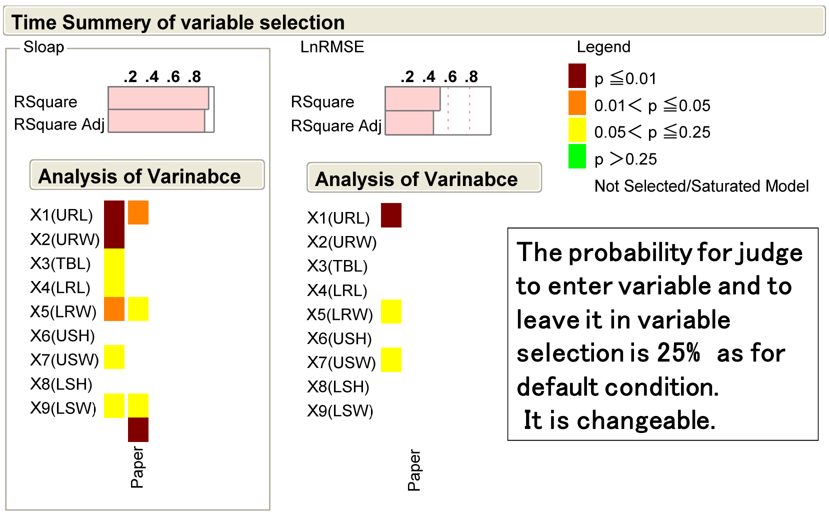


Figure 13: Results of variable selection for slope and LnRMSE

10.3 Design by JMP Software Based on HOPE Theory

RMSE is not considered here in order to simplify the explanation although it is easy to consider it with HOPE software. In this section cost, is evaluated with area of the airframe, and productivity is evaluated with number of cutting stroke.

10.3.1 Outline of Procedures and Display Layout of JMP Software Based on HOPE Theory

Outline of Procedures is as follows.

(1)Formulation: Show the requests of stakeholders mathematically.

［A］ Description: Set one objective function and many constraints.

(2)Optimization: Get solution by computer software.

［B］ Condition: Set mathematical conditions for optimization.

* Default condition is prepared in HOPE software.

(3)Inference: Consider the results statistically and professionally.

［C］ Solution: Check validity and acceptability of the solution.

［D］ Estimation: Recognize point estimations and interval estimations.

［E］ Regression: Compare the regression result with customer request zone.

Figure 14 shows the display layout of JMP Software

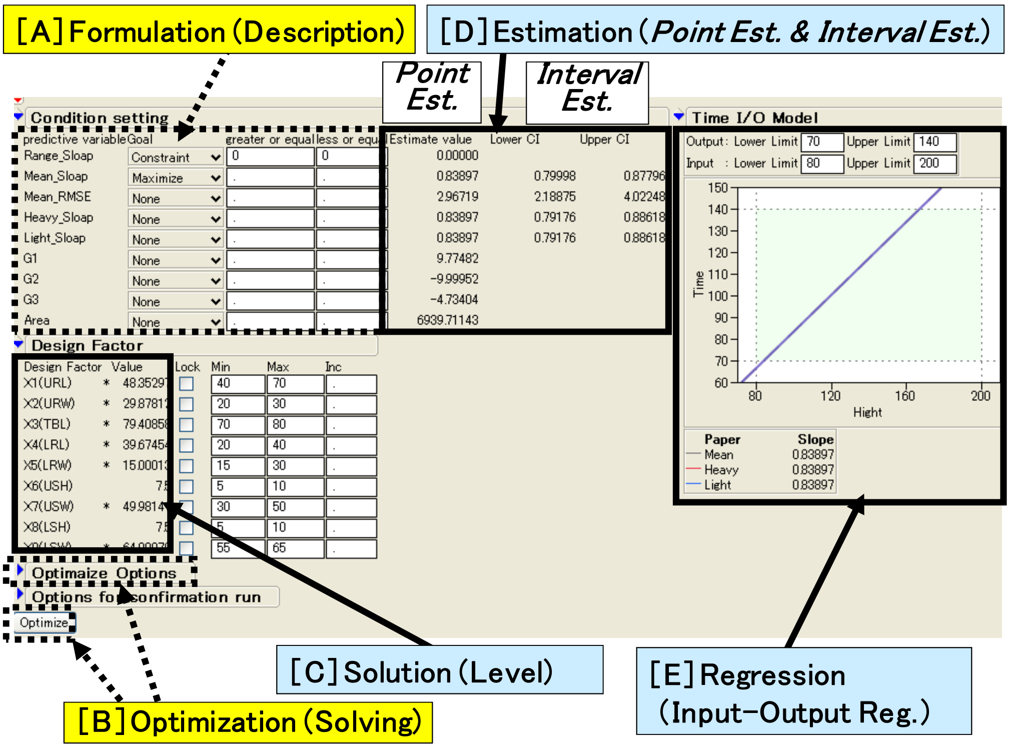


Figure 14: Display Structure of HOPE Software based on JMP

10.3.2 SNR Solution on JMP Software Based on HOPE Theory

Figure15 shows the SNR Solution on the display of JMP Software. It is easy to understand the condition of the SNR Solution. In this case, SNR Solution satisfies the CRZ fortunately. However, SNR Solution doesn't always satisfy CRZ. V (the denominator of SNR) isn’t always minimized when SNR is maximized.

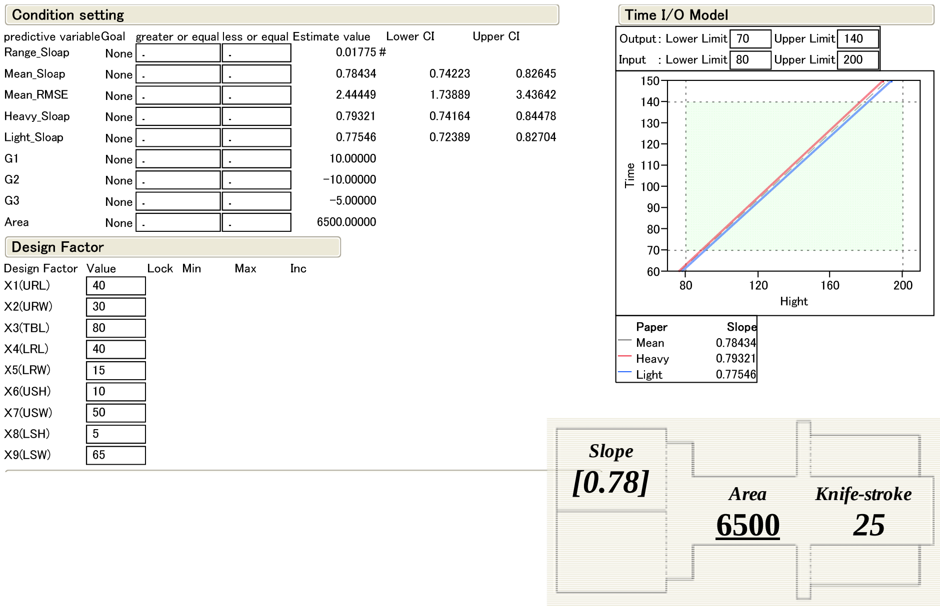


Figure 15: SNR Solution on Display of HOPE Software and Its Shape (**RMSE=2.44449≦3.75**)

10.3.3 Four Candidates Designed by JMP Software Based on HOPE Theory

(1) The First Candidate

This is the most basic solution in RD.

Objective function : Mean of slope ➔ Maximize

Constraints: Range of slope = 0 (or Range of slope≦c)

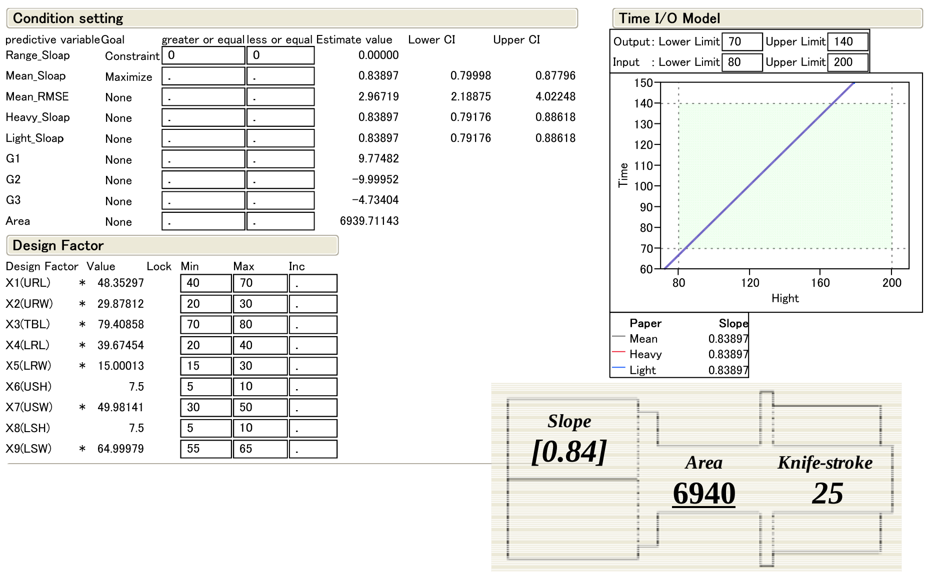


Figure 16: The First Candidate HOPE (1) and Its Shape (**RMSE=2.96719≦3.75**)

(2) The Second Candidate

By adding some conditions to a basic solution, another solution which has improved productivity can be created easily. Conditions (G1=0, G2=0, G3=0) of removing three kinds of gaps are added to constrains in this example.

Objective function : Mean of slope ➔ Maximize

Constraints: Range of slope = 0 (or Range of slope≦c)

G1=0, G2=0, G3=0

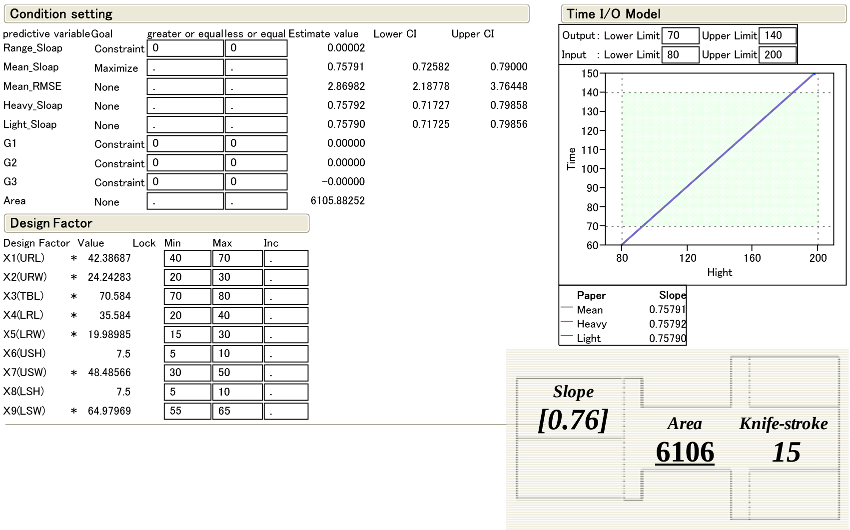


Figure 17: The Second Candidate HOPE (2) and Its Shape (**RMSE=2.86982≦3.75**)

(3) The Third Candidate

Because reducing the size of the airframe decreases the cost of materials, the warehousing expense, and the transportation rates and others, the total cost can be decreased much.

Objective function : Area (total size) ➔ Minimize

Constraints: Range of slope = 0 (or Range of slope≦c)

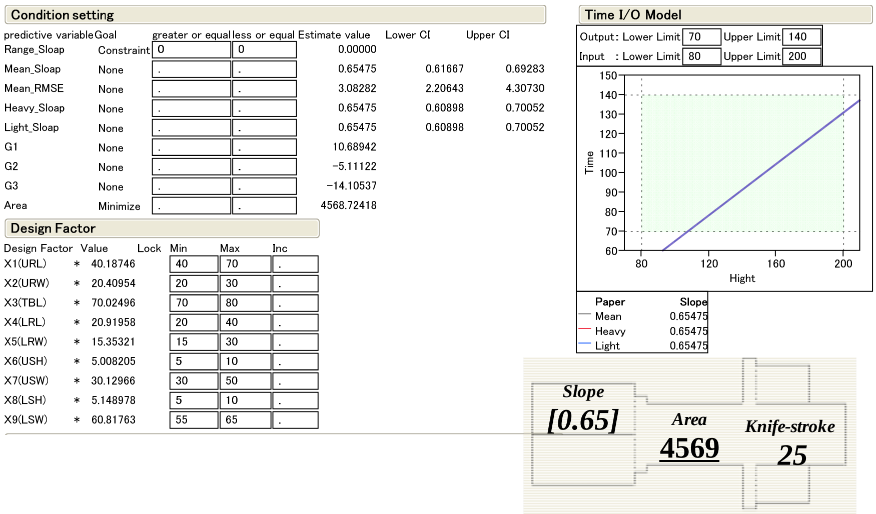


Figure 18: Initial Approach to Create Third Candidate HOPE (3) and Its Shape

Unfortunately, this solution does not satisfy CRZ, so that it cannot be accepted. Then, to satisfy CRZ, this solution should be improved. Because it is necessary that y exceed 140 when m is 200, the condition of slope≧0.70 is added to constraints.

Objective function : Area (total size) ➔ Minimize

Constraints: Range of slope = 0 (or Range of slope≦c)

slope≧0.70

[caution] It is necessary to make the condition which considers RMSE in case of strict formulation.

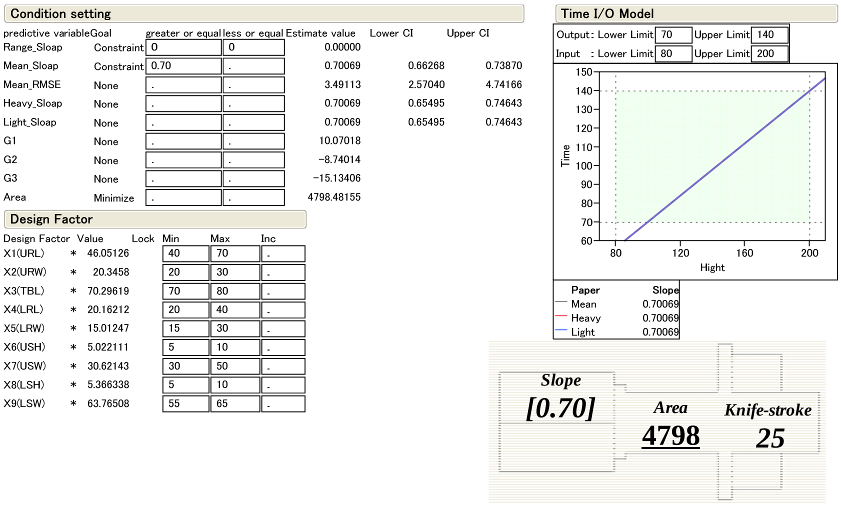


Figure 19: The Third Candidate HOPE (3) and Its Shape (**RMSE=3.49113≦3.75**)

(4) The Fourth Candidate

Previous solution has the smallest size, but it also has many gaps, so that its productivity is low. To improve the productivity it is needed to add conditions (G1=0, G2=0, G3=0) of removing three kinds of gaps to constrains.

Objective function : Area (total size) ➔ Minimize

Constraints: Range of slope = 0 (or Range of slope≦c)

slope≧0.70

G1=0, G2=0, G3=0

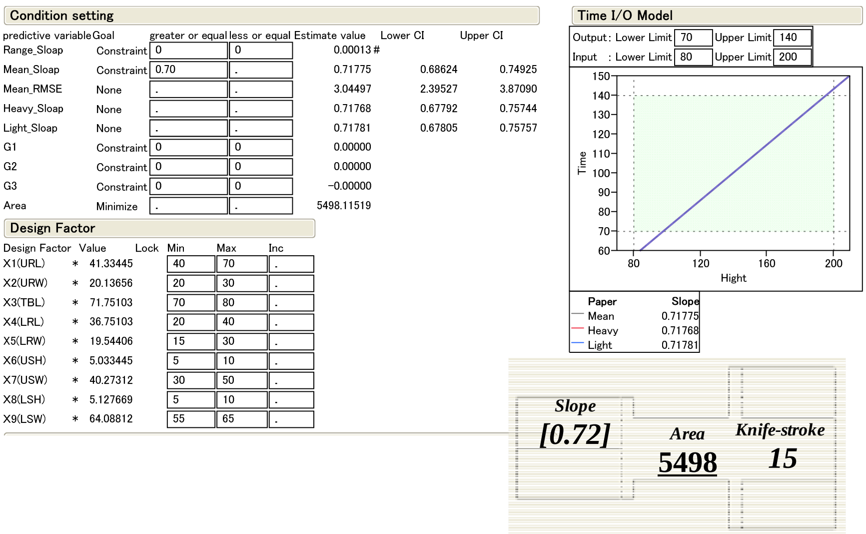


Figure 20:The Fourth Candidate HOPE (4) and Its Shape (**RMSE=3.04497≦3.75**)

When HOPE theory is used as stated above, various candidates are created. It is very easy and quick to create many candidates based on HOPE theory by using HOPE Software.

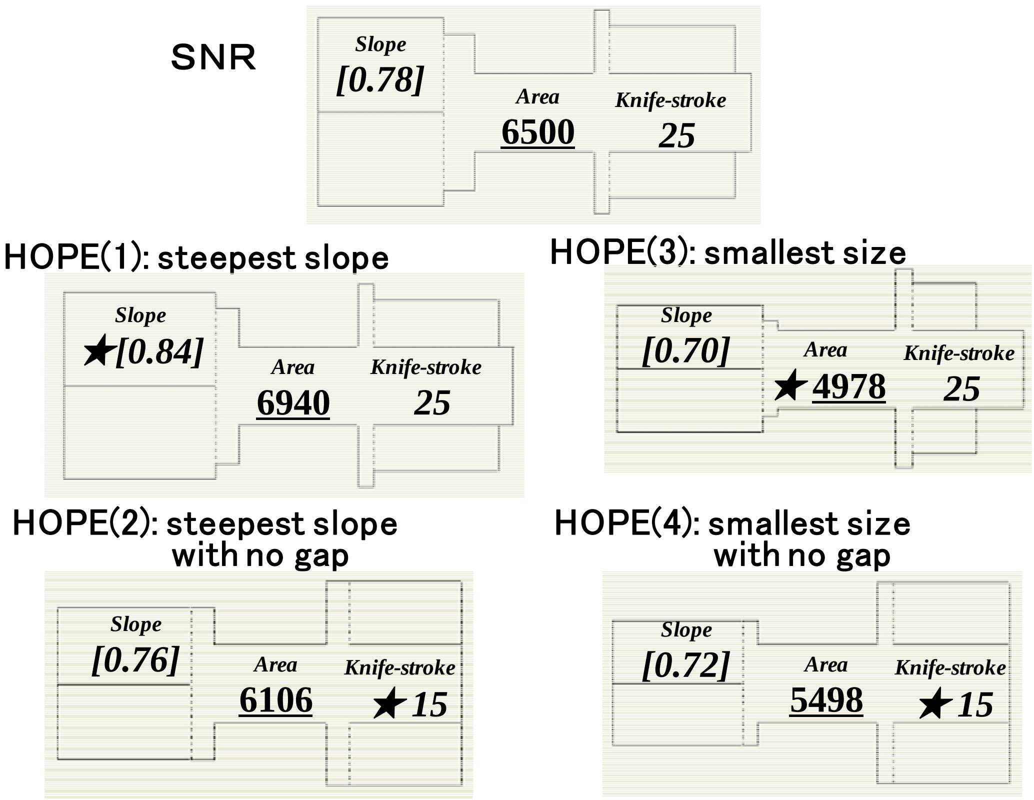


Figure 21: SNR Solution and Four Candidates by HOPE and Each Shape

10.3.4 How to Narrow Down the Candidates

Confirmation run is indispensable for RD, however time and money have not enough capacity, so that candidates to be examined are limited. Therefore it is needed to narrow many candidates created down to a few candidates. Priority is very important to do it and four examples are prepared as follows. Before 4 priorities are mentioned, it must be confirmed that all candidates achieve RMSE$\leqq$ 3.75 which means Cf$\geqq$1.33.

[A] Slope is very important, Cost or Productivity is not so important.
HOPE (1) is the best candidate .
[B] Cost is very important, Productivity or Slope is not so important.
HOPE (3) is the best candidate .
[C]Productivity is the first, Slope is the second and Cost is the third.
HOPE (2) is the best candidate .
[D] Productivity is the first, Cost is the second and Slope is the third.
HOPE (4) is the best candidate .

SNR solution is not flexible because it cannot consider the priorities like the above. Needless to say, it is necessary to confirm more candidates if you have moretime and money.

10.3.5 Estimation and Confirmation Run

HOPE software displays two estimations for mean of slopes: point estimation and interval estimation.

For this purpose, an option for conformation run is prepared in HOPE Software.

11. Conclusion

The SNR proposed by Taguchi is an indicator to consider variation and sensitivity. Although this indicator is useful, it should be handled carefully since the optimal solution based on the SNR is not always reasonable in the real cases.

HOPE proposed by Takahashi enables us to get useful and practical solutions easily especially when there are various inner arrays in crossed array experiment. In this paper, L_{12} proposed by Placket-Burman (1946) is used for inner array of crossed array experiment in robust design and uses first order model in analysis because it is easy and useful for many cases.

HOPE method provides reasonable solutions based on estimated regressions. The example of paper helicopters shows the usefulness and effectiveness of HOPE method for crossed array experiments on many control factors.

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