State-space/ARIMA Modeling of Glacier Melting Rate

Patrick Giuliano, Mason Chen and Charles Chen

STEM Approach

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- Science – Geoscience (the study of interconnected processes that form and shape the surface of the Earth)
- Technology – Gravity Recovery and Climate Experiment Follow-On (GRACE-FO) satellites
- Engineering – Investigate the impact of COVID-19 on the Antarctic glacier mass trend
- Mathematics – Application of non-seasonal and seasonal ARIMA models and other Time Series techniques

Thwaites Glacier Crisis

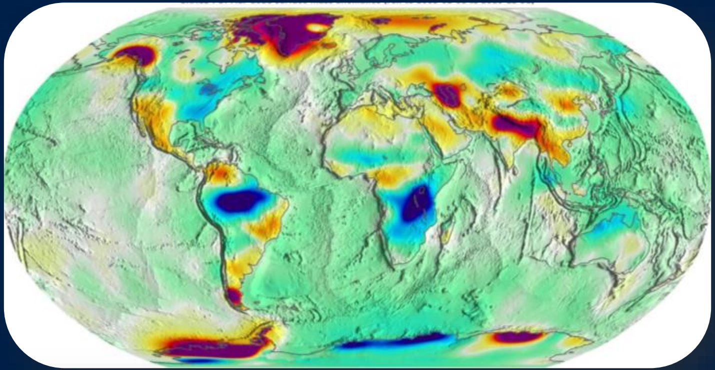
Scientists recently found that more warm water was flowing underneath the Antarctic glacier, the widest on the planet, than previously thought

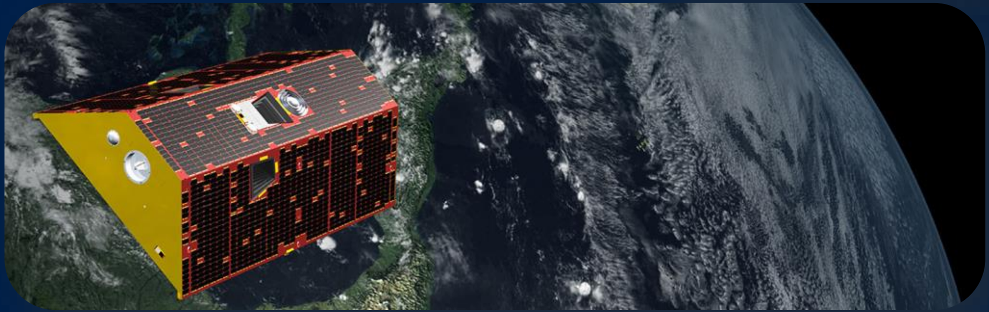
Greenland and Ice Loss

The estimated rate of ice loss from the Greenland ice sheet in September 2019 to August 2020 was roughly half of that from the preceding year

GRACE-FO

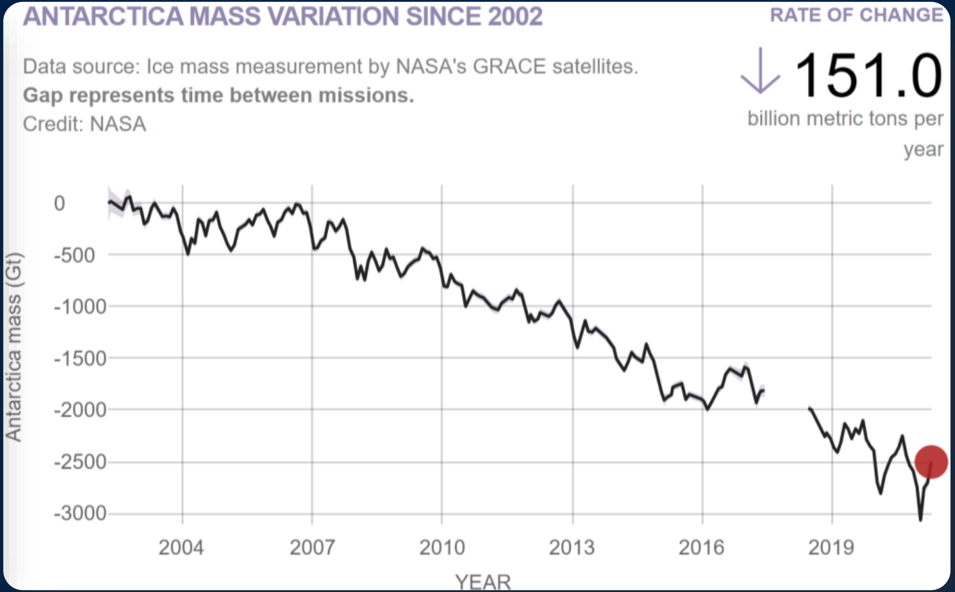
The Gravity Recovery and Climate Experiment Follow-On mission is a successor to the original GRACE mission which orbited Earth from 2002 to 2017.





Data

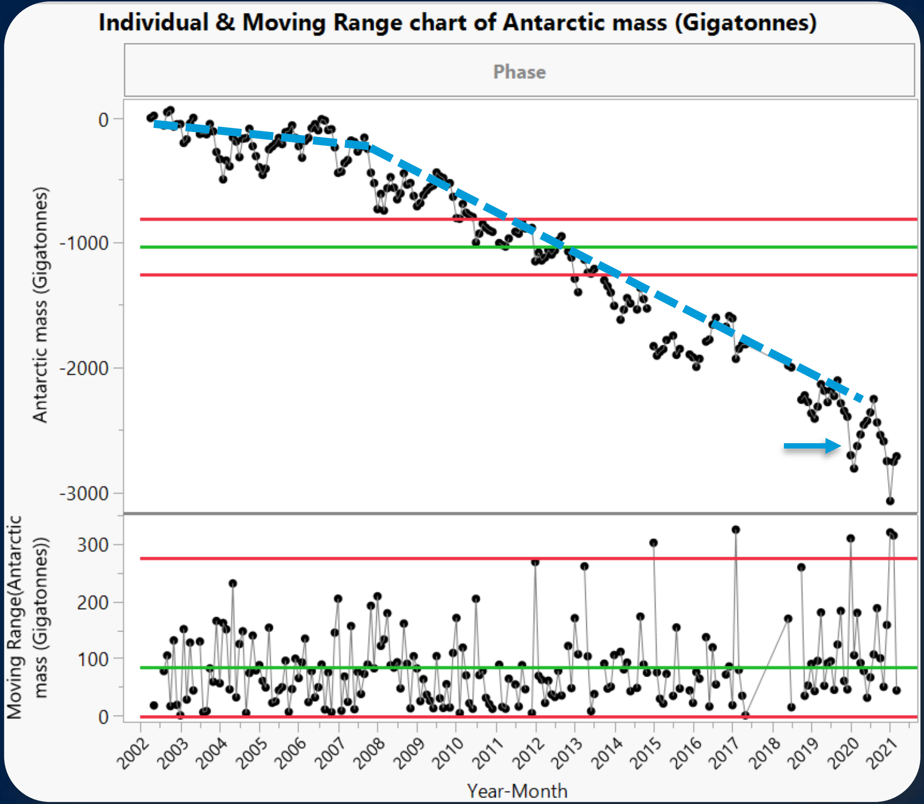
	Antarctic mass (Gigatonnes)	Year	Month	Year-Month
1	•	2002	1	01/2002
2	•	2002	2	02/2002
3	•	2002	3	03/2002
4	0	2002	4	04/2002
5	18.36	2002	5	05/2002
6	•	2002	6	06/2002
7	•	2002	7	07/2002
8	-59.82	2002	8	08/2002
9	45.54	2002	9	09/2002
10	62.69	2002	10	10/2002
11	-69.03	2002	11	11/2002
12	-49.78	2002	12	12/2002
13	-48.71	2003	1	01/2003
14	-200.03	2003	2	02/2003
15	-171.49	2003	3	03/2003
16	-43.66	2003	4	04/2003
17	0.79	2003	5	05/2003
18	•	2003	6	06/2003
19	-128.94	2003	7	07/2003
20	-122.41	2003	8	08/2003
21	-130.92	2003	9	09/2003
22	-48.06	2003	10	10/2003
23	-107.58	2003	11	11/2003
24	-273.11	2003	12	12/2003



Control Chart

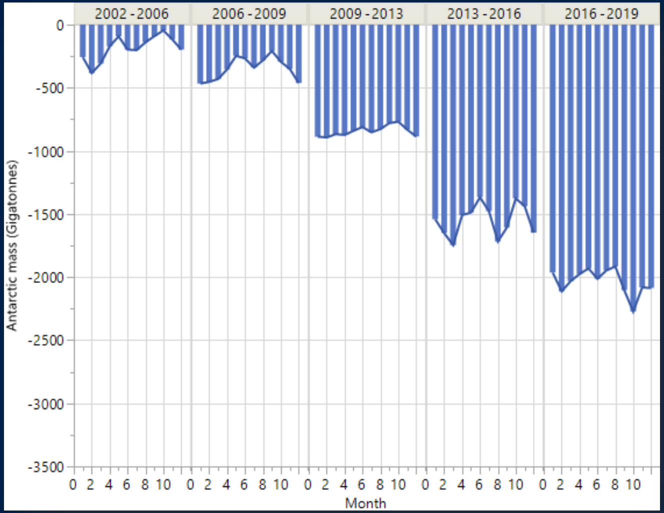
The seasonal pattern is most obvious at around the COVID-19 pandemic

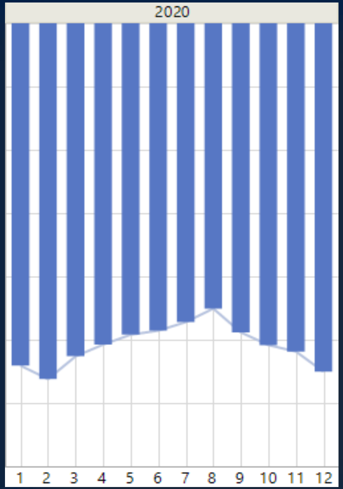
Can the Time Series Analysis detect a difference in the trend and seasonal patterns when including/excluding the period?



Historical Glacier Mass Patterns

Unlike the 2020 season, the years from 2002 to 2019 displayed multiple peaks

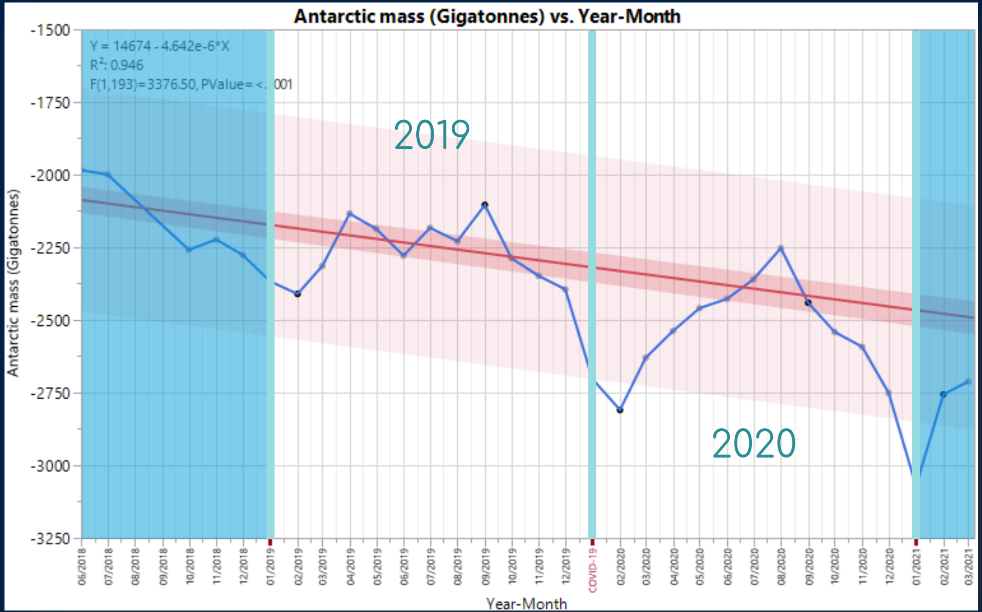




	Antarctic mass (Gigatonnes)	
Month	Mean	Std Dev
1	-1242	907
2	-1252	880
3	-1212	872
4	-956	781
5	-954	806
6	-1059	822
7	-1012	775
8	-889	789
9	-790	807
10	-880	894
11	-982	874
12	-1060	852

2019 vs 2020 Glacier Pattern

A smoother seasonal pattern was observed in 2020 as compared to the 2019 season



ARIMA

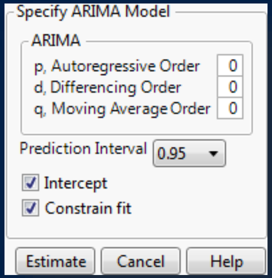
Non-seasonal and seasonal

Non-Seasonal ARIMA Models

ARIMA – autoregressive integrated moving average (p, d, q)

- Autoregression (p) – a variable that depends on prior values
- Integrated (d) – values are replaced by differences between that value and previous values
- Moving average (q) – residual errors depend on a moving average model based on prior values

$$y'_t = c + \phi_1 y'_{t-1} + \cdots + \phi_p y'_{t-p} + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t,$$



Nonseasonal Model Comparison

Both datasets have the same top two non-seasonal ARIMA models ((1, 1, 1) and (0, 1, 0))

Including the COVID-19 period

Model Comparison

Report	Graph	Model	DF	Variance	AIC ^	SBC	RSquare	-2LogLH	Weights	.2 .4 .6 .8	MAPE	MAE
☑	☑	ARIMA(1, 1, 1)	175	10197.384	2151.3981	2160.9435	0.986	2145.3981	0.963334		74.959965	72.448281
☑	☑	I(1)	177	10785.478	2159.0394	2162.2212	0.986	2157.0394	0.021110		62.700425	74.804409
☐	☐	IMA(1, 1)	176	10846.548	2161.0360	2167.3996	0.986	2157.036	0.007779		63.129137	74.803350
☐	☐	ARI(1, 1)	176	10846.587	2161.0366	2167.4002	0.986	2157.0366	0.007777		63.049414	74.803733
☐	☐	AR(1)	193	11713.654	2387.0054	2393.5514	0.961	2383.0054	0.000000		.	92.407556
☐	☐	ARMA(1, 1)	192	11774.366	2389.0023	2398.8213	0.961	2383.0023	0.000000		.	92.408142
☐	☐	MA(1)	193	239036.1	2971.5227	2978.0687	0.647	2967.5227	0.000000		.	402.59757
☐	☐	ARIMA(0, 0, 0)	194	685348.46	3174.7316	3178.0046	0.000	3172.7316	0.000000		.	710.03999

Excluding the COVID-19 period

Model Comparison

Report	Graph	Model	DF	Variance	AIC ^	SBC	RSquare	-2LogLH	Weights	.2 .4 .6 .8	MAPE	MAE
☑	☑	ARIMA(1, 1, 1)	160	8582.4782	1942.3764	1951.6577	0.985	1936.3764	0.927749		78.005993	67.226825
☑	☑	I(1)	162	9053.4594	1948.6479	1951.7417	0.984	1946.6479	0.040326		66.638406	68.995913
☐	☐	IMA(1, 1)	161	9100.7843	1950.4897	1956.6772	0.984	1946.4897	0.016057		69.978249	68.864405
☐	☐	ARI(1, 1)	161	9102.1258	1950.5134	1956.7009	0.984	1946.5134	0.015868		69.422994	68.876985
☐	☐	AR(1)	178	10184.665	2178.7192	2185.1051	0.953	2174.7192	0.000000		.	87.578156
☐	☐	ARMA(1, 1)	177	10233.054	2180.5683	2190.1472	0.954	2174.5683	0.000000		.	87.374822
☐	☐	MA(1)	178	196423.18	2707.6696	2714.0555	0.614	2703.6696	0.000000		.	357.76248
☐	☐	ARIMA(0, 0, 0)	179	514316.35	2878.9220	2882.1149	-0.00	2876.922	0.000000		.	612.93623

(0, 1, 0) Models

p-values for the parameter estimates are similar for both datasets

Including the COVID-19 period

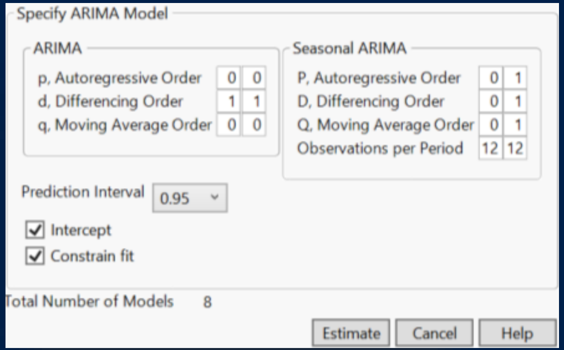
Parameter Estimates							
Term	Lag	Estimate	Std Error	t Ratio	Prob>|t|	Constant Estimate	Mu
Intercept	0	-10.42635	7.758467	-1.34	0.1807	-10.426349	-10.426349

Excluding the COVID-19 period

Parameter Estimates							
Term	Lag	Estimate	Std Error	t Ratio	Prob>|t|	Constant Estimate	Mu
Intercept	0	-9.439448	7.430668	-1.27	0.2058	-9.4394479	-9.4394479

Seasonal ARIMA Models

- Seasonal ARIMA model is denoted by (p, d, q)(P, D, Q)m
- Based on the AIC, the best model is (0, 1, 0)(0, 1, 1)12 for both datasets



Including the COVID-19 period

Model Comparison

Report	Graph	Model	DF	Variance	AIC ^	SBC	RSquare	-2LogLH	Weights	.2 .4 .6 .8	MAPE	MAE
✓	✓	Seasonal ARIMA(0, 1, 0)(0, 1, 1)12	142	10764.574	1752.4447	1758.3843	0.987	1748.4447	0.719820		40.440597	61.979200
		Seasonal ARIMA(0, 1, 0)(1, 1, 1)12	141	10849.195	1754.4228	1763.3322	0.987	1748.4228	0.267725		40.240381	62.018713
		Seasonal ARIMA(0, 1, 0)(1, 1, 0)12	142	11610.319	1760.5585	1766.4981	0.986	1756.5585	0.012455		39.800649	66.297904
		Seasonal ARIMA(0, 1, 0)(0, 1, 0)12	143	13807.891	1782.4022	1785.3720	0.984	1780.4022	0.000000		48.302923	71.137889
		Seasonal ARIMA(0, 1, 0)(1, 0, 1)12	175	8310.8597	2124.4716	2134.0170	0.988	2118.4716	0.000000		54.600886	64.560322
		Seasonal ARIMA(0, 1, 0)(1, 0, 0)12	176	9575.9072	2140.2722	2146.6358	0.987	2136.2722	0.000000		56.562634	69.144073
		Seasonal ARIMA(0, 1, 0)(0, 0, 1)12	176	9992.2064	2147.1390	2153.5025	0.987	2143.139	0.000000		59.312703	70.874107
✓	✓	I(1)	177	10785.478	2159.0394	2162.2212	0.986	2157.0394	0.000000		62.700425	74.804409

Excluding the COVID-19 period

Model Comparison

Report	Graph	Model	DF	Variance	AIC ^	SBC	RSquare	-2LogLH	Weights	.2 .4 .6 .8	MAPE	MAE
✓	✓	Seasonal ARIMA(0, 1, 0)(0, 1, 1)12	127	7930.7758	1534.8583	1540.5780	0.987	1530.8583	0.706305		43.783544	54.857455
		Seasonal ARIMA(0, 1, 0)(1, 1, 1)12	126	8031.1673	1536.6141	1545.1936	0.987	1530.6141	0.293578		43.174966	54.752947
		Seasonal ARIMA(0, 1, 0)(1, 1, 0)12	127	9502.447	1552.2700	1557.9896	0.985	1548.27	0.000117		43.495682	60.180079
		Seasonal ARIMA(0, 1, 0)(0, 1, 0)12	128	12024.307	1578.9967	1581.8565	0.982	1576.9967	0.000000		52.640753	65.795781
		Seasonal ARIMA(0, 1, 0)(1, 0, 1)12	160	6922.6274	1913.2733	1922.5546	0.987	1907.2733	0.000000		57.132352	60.452697
		Seasonal ARIMA(0, 1, 0)(1, 0, 0)12	161	8128.3022	1933.2993	1939.4868	0.986	1929.2993	0.000000		60.874553	64.513225
		Seasonal ARIMA(0, 1, 0)(0, 0, 1)12	161	8440.7022	1938.8789	1945.0664	0.985	1934.8789	0.000000		63.374081	65.903559
✓	✓	I(1)	162	9053.4594	1948.6479	1951.7417	0.984	1946.6479	0.000000		66.638406	68.995913

(0, 1, 0)(0, 1, 1)12 Models

The slope for the period excluding the pandemic is now steeper

Including the COVID-19 period

Parameter Estimates

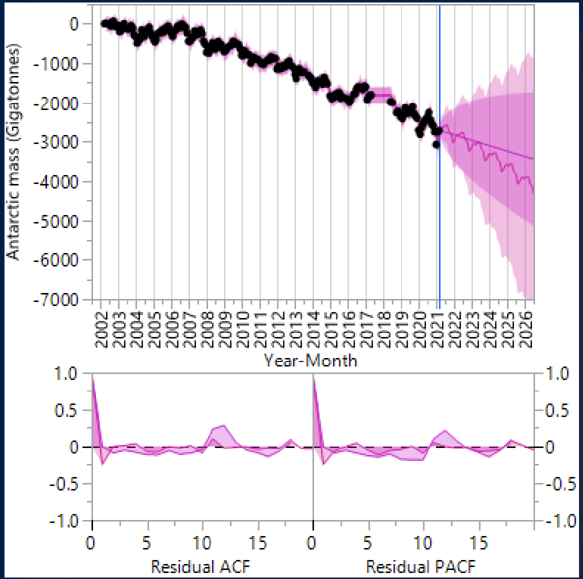
Term	Factor	Lag	Estimate	Std Error	t Ratio	Prob>|t|	Constant Estimate	Mu
MA2,12	2	12	0.632670	0.100669	6.28	<.0001*	-2.3815202	-2.3815202
Intercept	1	0	-2.381520	4.317669	-0.55	0.5821		

Excluding the COVID-19 period

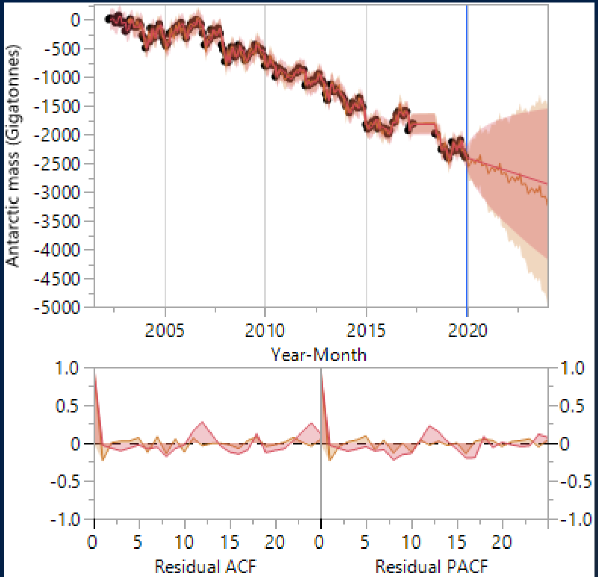
Parameter Estimates

Term	Factor	Lag	Estimate	Std Error	t Ratio	Prob>|t|	Constant Estimate	Mu
MA2,12	2	12	0.760786	0.101940	7.46	<.0001*	-2.9080765	-2.9080765
Intercept	1	0	-2.908076	3.234437	-0.90	0.3703		

ACF and PACF Plots



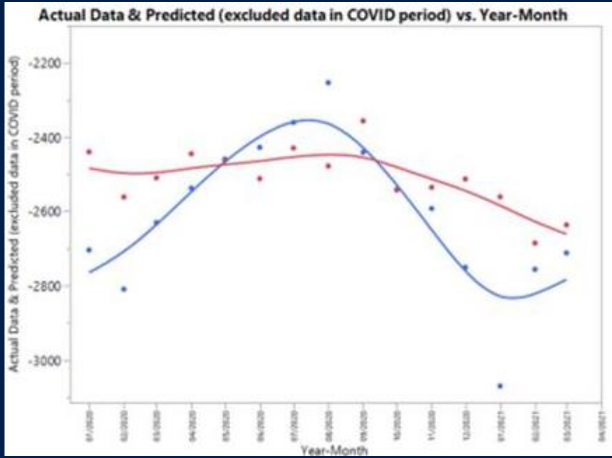
Including the COVID-19 period



Excluding the COVID-19 period

Seasonal ARIMA Forecasts

- The (0, 1, 0)(0, 1, 1)12 model cannot accurately forecast the glacier mass variation during the COVID-19 period
- The predicted curve has a much weaker seasonal pattern than the actual data



Conclusion

- Control charts and histogram analyses suggested that the pandemic may have been responsible for a smoother seasonal pattern in glacier mass variation
- The non-seasonal and seasonal ARIMA models did not reveal any notable differences between the two datasets
- Forecasting the glacier mass data during the COVID-19 period revealed that the past data has a weaker seasonal component
- Future work can consider other climate factors

Thanks!

Any questions?