STEAMS Methodology of Designing a Modern Partial Deck AKQJ Poker Game

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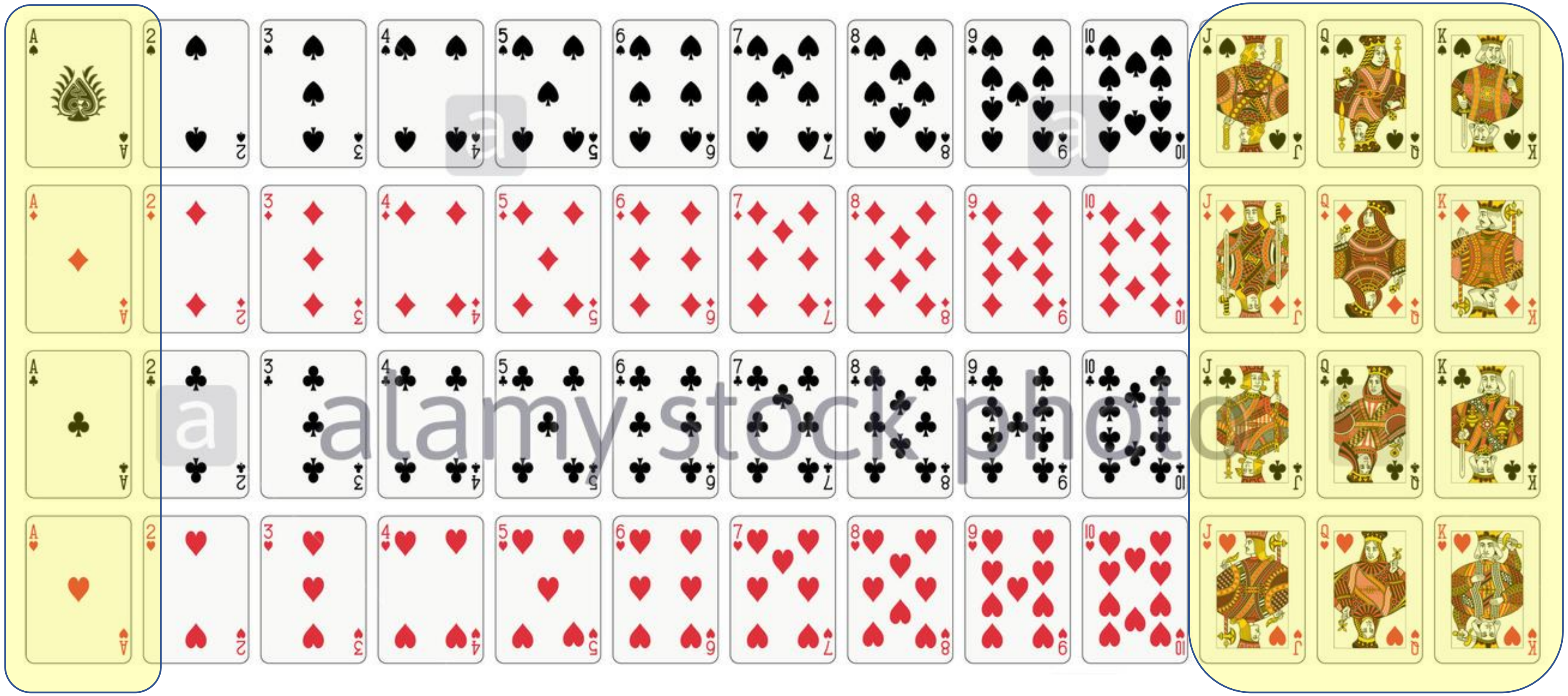


Science: Gambling Disorder Psychology

Gambling disorder, also known as compulsive gambling, pathological gambling, or gambling addiction, is the irresistible impulse to continue gambling. 3-4% of Americans have a gambling disorder.

Causes

The causes of compulsive gambling are not established. It may be caused by a variety of reasons.

Symptoms

Gambling addiction could lead to personal problems and problems with finances.

How to determine the expected winning probability and help prevent the gambling disorder behavior

Technology: General Poker Terminology

Flush — The flush contains any five of the thirteen ranks, all of which belong to one of the four suits, minus the 40 straight flushes.

Two pair — The pairs can have any two of the thirteen ranks, and each pair can have two of the four suits. The final card can have any one of the eleven remaining ranks, and any suit.

One Pair — The pair can have any one of the thirteen ranks, and any two of the four suits. The remaining three cards can have any three of the remaining twelve ranks, and each can have any of the four suits.

No pair — A no-pair hand contains five of the thirteen ranks, discounting the ten possible straights, and each card can have any of the four suits, discounting the four possible flushes.

Mathematics: Compare Full Deck and Partial Deck

Full Deck

Total Permutations

$$P\binom{52}{5} = \frac{52!}{(52-5)!}$$

$= 311{,}875{,}200$

60X Higher Trials

Partial Deck

Total Permutations

$$P\binom{24}{5} = \frac{24!}{(24-5)!}$$

$= 5{,}100{,}480$

$$C\binom{13}{1} * C\binom{48}{1}$$

$= 624$

5X Higher Events

$$C\binom{6}{1} * C\binom{20}{1}$$

$= 120$

Probability= 624/311,875,200 **< 0.001%**

12X

Probability= 120/5,100,480= **0.002%**

Reduce total card numbers (partial deck) can increase the chance of getting big patterns such as (Four of a Kind or Full House)

Mathematics: Derive General Partial Deck Formula

	4*m Partial Track		
	Trial	Event	Probability
Royal Straight	C(4m,5)	C(4,1)	C(4,1)/C(4m,5)
Straight Flush		C(4,1)*C(m-5,1)	C(4,1)*C(m-5,1)/C(4m,5)
Four of a Kind		C(m,1)*C(m-1,1)*C(4,1)	C(m,1)*C(m-1,1)*C(4,1)/C(4m,5)
Full House		C(m,1)*C(m-1,1)*C(4,3)* C(4,2)	C(m,1)*C(m-1,1)*C(4,3)* C(4,2)/C(4m,5)
Flush		C(4,1)*C(m,5)-C(4,1)*C(m-4,1)	[C(4,1)*C(m,5)-C(4,1)*C(m-4,1)]/C(4m,5)
Straight		C(m-4,1)* [C(4,1)^5-C(4,1)]	C(m-4,1)* [C(4,1)^5-C(4,1)]/C(4m,5)
Three of a Kind		C(m,1)* C(m-1,2)*C(4,3)*C(4,1)*C(4,1)	C(m,1)* C(m-1,2)*C(4,3)* C(4,1)*C(4,1)/C(4m,5)
Two Pair		C(m,2)*C(m-2,1)*C(4,2)*C(4,2)*C(4,1)	C(m,2)*C(m-2,1)* C(4,2)*C(4,2)*C(4,1)/C(4m,5)
One Pair		C(m,1)*C(m-1,3)*C(4,2)*C(4,1)*C(4,1)*C(4,1)	C(m,1)*C(m-1,3)*C(4,2)* C(4,1)*C(4,1)*C(4,1)/C(4m,5)
Nothing		[C(m,5)-(m-4)]*[C(4,1)^5-C(4,1)]	[C(m,5)-(m-4)]*[C(4,1)^5-C(4,1)]/C(4m,5)

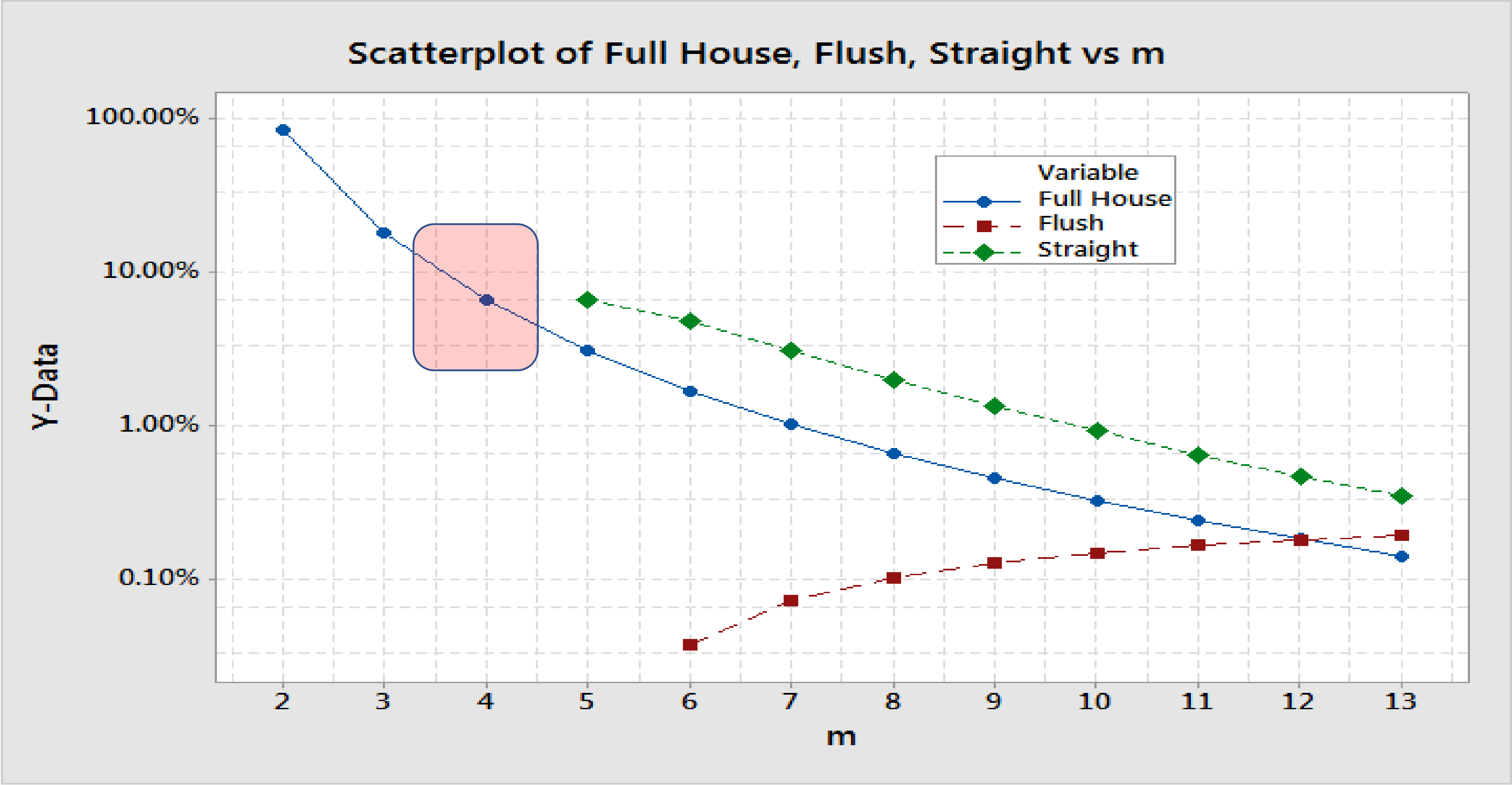
Use JAVA to simulate these Poker Probability on any partial deck

Mathematics: Odds Ratio of Full vs. Partial Deck

	Full Deck			24 Partial Deck			
	Trial	Event	Probability	Trial	Event	Probability	Ratio
Royal Straight	C(52, 5) 2,598,960	C(4,1)	0.000%	C(48,5) 42,504	C(4,1)	0.009%	61.1
Straight Flush		C(4,1)*C(9,1)	0.001%		C(4,1)*C(1,1)	0.009%	6.5
Four of a Kind		C(13,1)*C(12,1)*C(4,1)	0.024%		C(6,1)*C(5,1)*C(4,1)	0.282%	11.7
Full House		C(13,1)*C(12,1)* C(4,3)*C(4,2)	0.144%		C(6,1)*C(5,1)* C(4,3)*C(4,2)	1.694%	11.8
Flush		C(4,1)*C(13,5)- C(4,1)*C(10,1)	0.197%		C(4,1)*C(6,5)- C(4,1)*C(2,1)	0.038%	0.2
Straight		C(10,1)*[C(4,1)^5- C(4.1)]	0.392%		C(2,1)*[C(4,1)^5- C(4,1)]	4.800%	12.2
Three of a Kind		C(13,1)*C(12,2)* C(4,3)*C(4,1)*C(4,1)	2.113%		C(6,1)*C(5,2)* C(4,3)*C(4,1)*C(4,1)	9.034%	4.3
Two Pair		C(13,2)*C(11,1)* C(4,2)*C(4,2)*C(4,1)	4.754%		C(6,2)*C(4,1)* C(4,2)*C(4,2)*C(4,1)	20.327%	4.3
One Pair		C(13,1)*C(12,3)*C(4,2)* C(4,1)*C(4,1)*C(4,1)	42.257%		C(6,1)*C(5,3)*C(4,2)* C(4,1)*C(4,1)*C(4,1)	54.207%	1.3
Nothing		[C(13,5)-10]* [C(4,1)^5-4]	50.118%		[C(6,5)-2]* [C(4,1)^5-4]	9.599%	0.2

- Partial Deck has significantly increased the matching probability except for Flush and Nothing Cases

Mathematics: Partial Deck Poker Probability



Poker AKQJ Game:

- m=4, total 16 cards available
- Simplify situations: no Flush and no Straight
- **Winning Patterns: Four of a Kind, Full House, Three of a Kind, and Two-Pairs**

- By adjusting the partial deck card number, the winning probability and ranking have been changed. Poker Game is more excited when playing less cards.

AI: Use JAVA to Simulate Probability (2 Players)

- To simplify the simulation model, we only consider Full House as the only winning pattern for this case study
- We will JAVA Random Generator to pick two random cards (one for Player A and one for Player B) from the remaining 18 cards

Player A

Player B

2 out of 18 to get "A Full House"

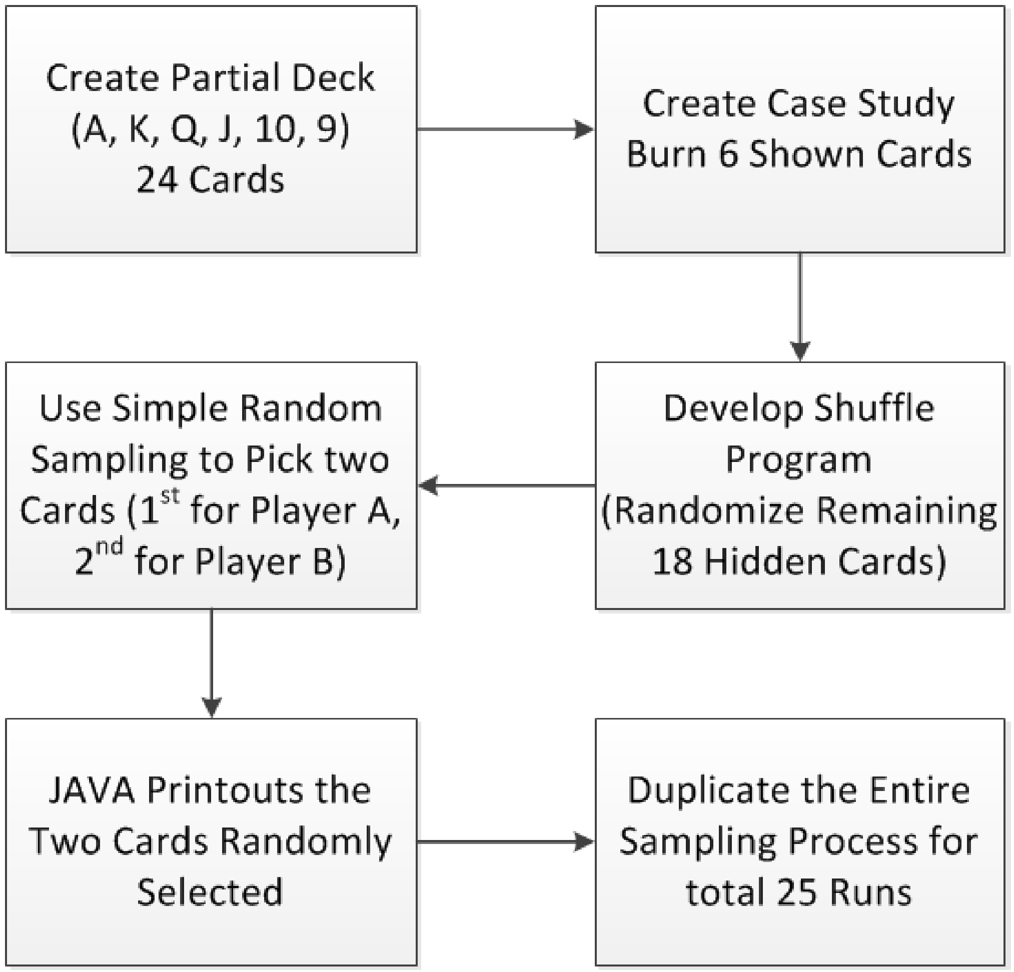
2 out of 18 to get "K Full House"

2 out of 18 to get "A Full House"

2 out of 18 to get "J Full House"

AI: JAVA Algorithm and Output (2 Players)

JAVA Flow Chart



Output

JAVA	JAVA Random Card		Full House?		Who Won
	Player A	Player B	Player A	Player B	
1	9 of Heart	10 of Spade	Not	Not	Tie
2	Queen of Heart	9 of Club	Not	Not	Tie
3	9 of Heart	9 of Spade	Not	Not	Tie
4	Queen of Spade	9 of Heart	Not	Not	Tie
5	10 of Spade	9 of Diamond	Not	Not	Tie
6	9 of Club	Jack of Heart	Not	J	B
7	9 of Club	King of Club	Not	Not	Tie
8	Jack of Club	9 of Heart	Not	Not	Tie
9	9 of Diamond	9 of Spade	Not	Not	Tie
10	King of Heart	10 of Heart	K	Not	A
11	Ace of Club	Jack of Diamond	A	J	A
12	King of Club	Jack of Heart	K	J	A
13	Queen of Spade	Ace of Diamond	Not	A	B
14	Jack of Club	King of Heart	Not	Not	Tie
15	King of Heart	Jack of Heart	K	J	A
16	Jack of Diamond	Queen of Spade	Not	Not	Tie
17	Jack of Club	10 of Spade	Not	Not	Tie
18	Jack of Club	Queen of Club	Not	Not	Tie
19	9 of Club	Queen of Heart	Not	Not	Tie
20	9 of Heart	Queen of Club	Not	Not	Tie
21	Ace of Diamond	10 of Spade	A	Not	A
22	9 of Heart	Ace of Club	Not	A	B
23	1o of Club	9 of Diamond	Not	Not	Tie
24	King of Heart	Queen of Club	K	Not	A
25	Jack of Diamond	Ace of Club	Not	A	B

Statistics: Verify JAVA Simulation (2 Players)

Tally for Discrete Variables: Who Won

Who Won	Count	Percent
A	6	24.00
B	4	16.00
Tie	15	60.00
N=	25	

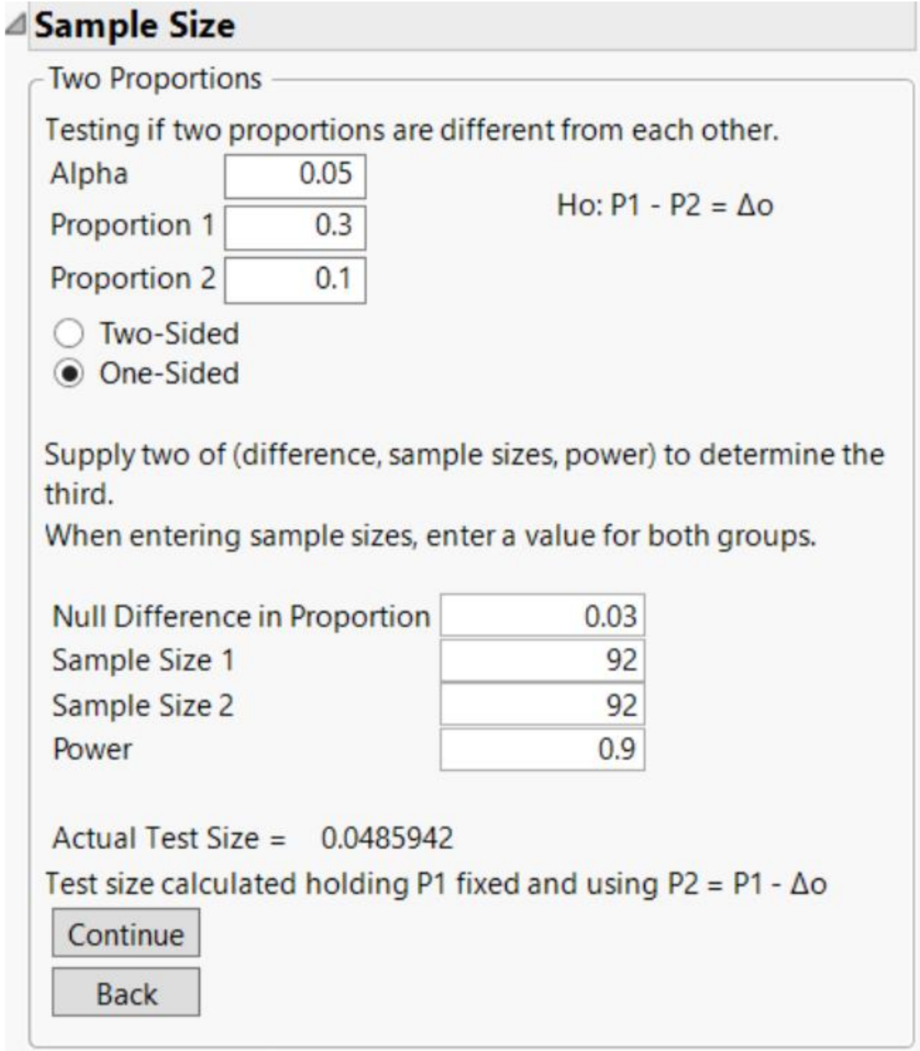
	1-Proportion
A= 20.3%	P= 0.804
B= 19.7%	P= 0.804
Tie = 60.2%	P= 1.000

- JAVA Random Simulation method can match the expected probability reliably
- Player A has a slightly higher chance to win over Player B (Because Player A K Full House > Player B J Full House)

Statistics: Power and Sample Size

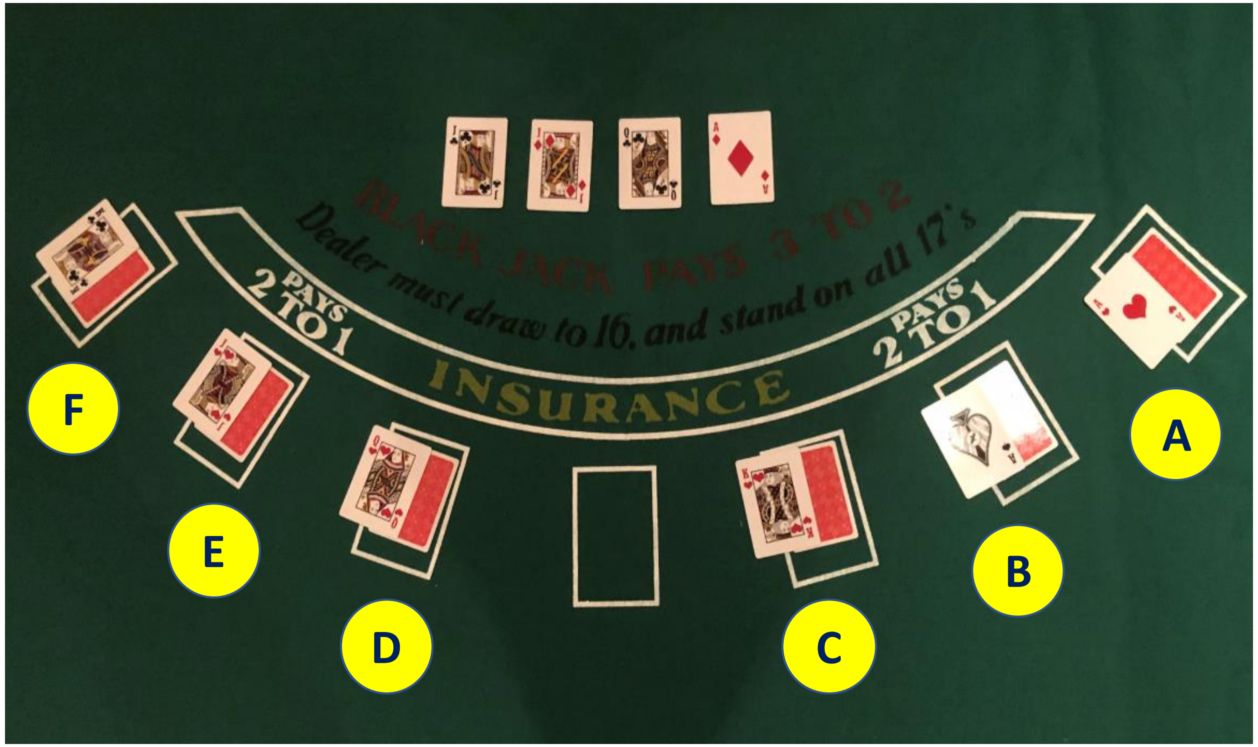
JMP >> DoE >> Design Dignostics >> Sample Size and Power

- Determine the minimum sample size (how many AKQJ datasets).
- Conduct JMP 2-proportions Power Test using Normal Approximation
  - Estimating the best player winning @ 30% and the worst player @ 10%
  - Set 5% Alpha (95% Confidence) and 10% Beta (90% Power)
  - Consider minimum 3% Null Difference to differentiate the players
  - **Sample size is 92 data sets needed**
  - Check normal approximation (skewness)= 92x0.167 (overall mean) >10 (pass)
  - Real game is judged by how many chips left, therefore sample size needed should be less (more continuous)



Technology: Modern Poker AKQJ Game (6 Players)

- Full deck is too complicated to calculate winning probability during poker game
- Partial deck increases the winning probability and simplify the winning situation
- By calculating the winning probability, players can prevent irrational gambling



Entry : 1 Chip

Betting Round: 2 Chips

Statistics: Make Card Sets based on Random Generation

Spades A,K,Q,J
Hearts A,K,Q,J
Diamonds A,K,Q,J
Clubs A,K,Q,J

Card Type	Card Order	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10
S-A	1	S-K	D-K	D-Q	C-Q	H-K	C-J	D-J	C-Q	C-J	D-A
S-K	2	C-J	S-Q	C-J	S-K	D-K	H-Q	D-Q	S-Q	C-A	D-K
S-Q	3	S-A	C-A	D-K	H-Q	H-J	S-J	S-Q	D-Q	D-Q	H-J
S-J	4	C-K	D-A	S-J	S-J	S-Q	D-K	D-A	C-J	C-Q	C-J
H-A	5	S-Q	D-J	S-K	S-Q	H-Q	C-Q	H-J	H-K	H-A	S-J
H-K	6	D-A	H-A	H-K	C-K	D-J	H-J	H-A	S-J	S-K	S-K
H-Q	7	S-J	H-Q	D-J	H-J	H-A	D-A	C-Q	S-A	H-J	S-A
H-J	8	D-K	S-A	C-K	D-J	D-A	D-Q	C-K	H-Q	D-J	C-K
D-A	9	D-Q	H-J	H-A	S-A	D-Q	S-Q	D-K	D-A	C-K	D-Q
D-K	10	H-Q	C-Q	S-A	H-A	S-J	D-J	C-A	C-K	D-A	D-J
D-Q	11	C-Q	C-K	S-Q	C-J	C-K	S-K	S-A	H-J	H-K	H-K
D-J	12	H-J	S-K	C-A	C-A	C-Q	C-A	H-K	D-J	S-Q	C-Q
C-A	13	H-K	S-J	C-Q	D-A	C-A	C-K	H-Q	H-A	S-J	C-A
C-K	14	H-A	D-Q	H-Q	D-K	S-A	S-A	C-J	C-A	H-Q	H-Q
C-Q	15	D-J	H-K	D-A	H-K	S-K	H-A	S-K	D-K	S-A	H-A
C-J	16	C-A	C-J	H-J	D-Q	C-J	H-K	S-J	S-K	D-K	S-Q

- The cards for each run are also created by random generation to prevent any card shuffling bias

Mathematics: Simplify Probability Algorithm (6 Players)

In the real time Gambling Situation, it's very difficult to do comprehensive probability calculation in time to determine the betting decision. Therefore, find another simpler and alternative calculation method is necessary.

- We will use the Worst Scenario Case to simplify the winning probability algorithm
- The worst case of Player A when against Player B is Player B has the hidden card= "A"
- Player A would look at the table and count how many "A" cards still not shown
- P(A vs.B)=1 if Player B has no chance to get "A" as hidden card, otherwise P(A vs. B)=1.
- Overall P(A) would be calculated based on how many players that player A can win at the worst case scenario
- The left table has demonstrated the calculation algorithm

Player A	Worst Scenario	Individual Winning%
B	Win	100%
C	Lose	0%
D	Win	100%
E	Win	100%
F	Lose	0%
Overall Winning %		**60%**

Original Method: 1st Run Overall Winning Probability

- The overall winning probability of Player B is when Player B can win over all the other players.
- Therefore the overall winning probability P(B)= P(B vs. A)*P(B vs. C)*…*P(B vs. F)
- Same calculation would be applicable to the other Players
- For the 1st Run, Player B has the hidden card Heart Q, other five players have their hidden cards: Dimond Q, Club Q, Heart J, Heart K and Heart A.
- The left table has listed the win, tie or lose situation for Player B against the other players based on five hidden card scenarios.

Player B	A	C	D	E	F
D-Q	Win	Win	Win	Win	Win
C-Q	Win	Win	Win	Win	Win
H-J	Tie	Lose	Lose	Tie	Lose
H-K	Lose	Win	Lose	Win	Win
H-A	Lose	Lose	Win	Win	Lose
	50%	**60%**	**60%**	**90%**	**60%**
Overall			**10%**		

Original Method: 1ST Round Overall Winning Probability

- The overall winning probability of Player A is when Player A can win over all the other players.
- P(B)= P(B vs. A)*P(B vs. C)*…*P(B vs. F)
- Same calculation would be applicable to the other Players C-F

Player B	A	C	D	E	F
D-Q	Win	Win	Win	Win	Win
C-Q	Win	Win	Win	Win	Win
H-J	Tie	Lose	Lose	Tie	Lose
H-K	Lose	Win	Lose	Win	Win
H-A	Lose	Lose	Win	Win	Lose
	50%	**60%**	**60%**	**90%**	**60%**
Overall	**10%**				

Player C	A	B	D	E	F
D-Q	Win	Lose	Win	Win	Tie
C-Q	Win	Lose	Win	Win	Tie
H-J	Lose	Lose	Lose	Lose	Lose
H-K	Lose	Lose	Lose	Win	Lose
H-A	Lose	Lose	Lose	Tie	Lose
	40%	**0%**	**40%**	**70%**	**20%**
Overall	**0%**				

Player D	A	B	C	E	F
D-Q	Win	Win	Win	Win	Win
C-Q	Win	Win	Win	Win	Win
H-J	Win	Win	Win	Win	Win
H-K	Lose	Tie	Win	Win	Win
H-A	Win	Win	Lose	Win	Lose
	80%	**90%**	**80%**	**100%**	**80%**
Overall	**46%**				

Player E	A	B	C	D	F
D-Q	Tie	Lose	Lose	Tie	Lose
C-Q	Tie	Lose	Lose	Tie	Lose
H-J	Tie	Lose	Lose	Tie	Lose
H-K	Lose	Lose	Lose	Lose	Lose
H-A	Lose	Lose	Lose	Lose	Lose
	30%	**0%**	**0%**	**30%**	**0%**
Overall	**0%**				

Player F	A	B	C	D	E
D-Q	Win	Win	Win	Win	Win
C-Q	Win	Win	Win	Win	Win
H-J	Win	Win	Win	Win	Win
H-K	Win	Lose	Win	Win	Win
H-A	Win	Win	Win	Win	Win
	100%	**80%**	**100%**	**100%**	**100%**
Overall	**80%**				

Simplified Method: Simulate Psychology Behavior

Simply the probability calculation by against the other players' best card scenario (Worst Case) and make the folding decision

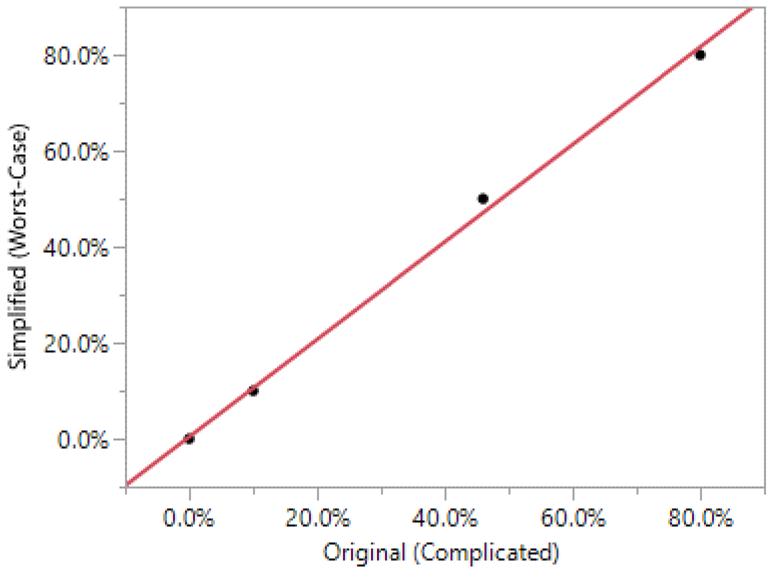
1st Run	Shared Cards				Player A (0%)		Player B (15%)	
	Card 1	Card 2	Card 3	Card 4	Open	Hidden	Open	Hidden
1st Game Cards' Distrbution	S-J	D-K	D-J	C-A	S-K	D-Q	C-J	H-Q
Actrual Matching					2-Pairs		J-Three	
Worst Case Overall Winning Probability					No need to Calculate		10% Chance	
Stay or Fold in the Betting Round					Always Stay		Fold	
Results (win or lose chips)					-3		-1	
1st Run	Player C (30%)		Player D (45%)		Player E (60%)		Player (75%)	
	Open	Hidden	Open	Hidden	Open	Hidden	Open	Hidden
1st Game Cards' Distrbution	S-A	C-Q	C-K	H-J	S-Q	H-K	D-A	H-A
Actual Matching	2-Pairs		J-Full House		2-Pairs		A-Full House	
Worst Case Overall Winning Probability	0% Chance		50% Chance		0% Chance		80% Chance	
Stay or Fold in the Betting Round	Fold		Stay		Fold		Stay	
Results (win or lose chips)	-1		-3		-1		9	

Players' Gambling Psychology Characters:

- Player A will bet blindly no matter what situation: conditional winning probability threshold @ 0%
- Player B will bet very aggressive with little probability calculation sense: conditional winning probability threshold @ 15%
- Players C, D & E will bet more cautiously with stronger probability calculation sense: conditional winning probability threshold @ 30%,45%, 60%
- Player F will bet very conservatively with professional probability calculation capability: conditional winning probability threshold @ 75%
- Based on the Character setting and simulation, three players will stay in the game and Player F won this round with best cards

Statistics: 1st Trial Correlation between two Methods

Players	Original (Complicated)	Simplified (Worst-Case)
B	10%	10%
C	0%	0%
D	46%	50%
E	0%	0%
F	80%	80%



JMP >> Analyze >> Fit Y by X

Linear Fit

Simplified (Worst-Case) = 0.0038399 + 1.0152945*Original (Complicated)

Summary of Fit

RSquare	0.997707
RSquare Adj	0.996942
Root Mean Square Error	0.019706
Mean of Response	0.28
Observations (or Sum Wgts)	5

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	0.50683501	0.506835	1305.171
Error	3	0.00116499	0.000388	Prob > F
C. Total	4	0.50800000		<.0001*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0038399	0.011666	0.33	0.7637
Original (Complicated)	1.0152945	0.028103	36.13	<.0001*

Two different methods of calculating the overall winning probability have shown extremely high correlations

- The simplified method could provide equivalent winning prediction capability
- The simplified method could **save calculation time by 3X-5X** and make it feasible < 1 minute for each player to make the betting decision on time

Statistics: 5th Trial, Compare two Methods.

Original Method

Player B	A	C	D	E	F
Q	T	W	W	W	W
K	L	T	T	T	T
Q	T	W	W	W	W
A	L	L	L	L	L
A	L	L	L	L	L
	20%	50%	50%	50%	50%
Overall	1.3%				

Player C	A	B	D	E	F
Q	T	T	W	W	W
J	T	T	W	W	L
Q	T	T	W	W	W
A	L	L	L	L	L
A	L	L	L	L	L
	30%	30%	60%	60%	40%
Overall	1.3%				

Player D	A	B	C	E	F
Q	L	L	W	T	W
J	L	L	L	W	L
K	L	L	L	L	L
A	L	L	L	L	L
A	L	L	L	L	L
	0%	0%	20%	30%	20%
Overall	0%				

Player E	A	B	C	D	F
Q	W	W	W	W	W
J	W	W	L	W	L
K	L	L	W	W	W
Q	W	W	W	W	W
A	L	L	L	T	L
	60%	60%	60%	90%	60%
Overall	12%				

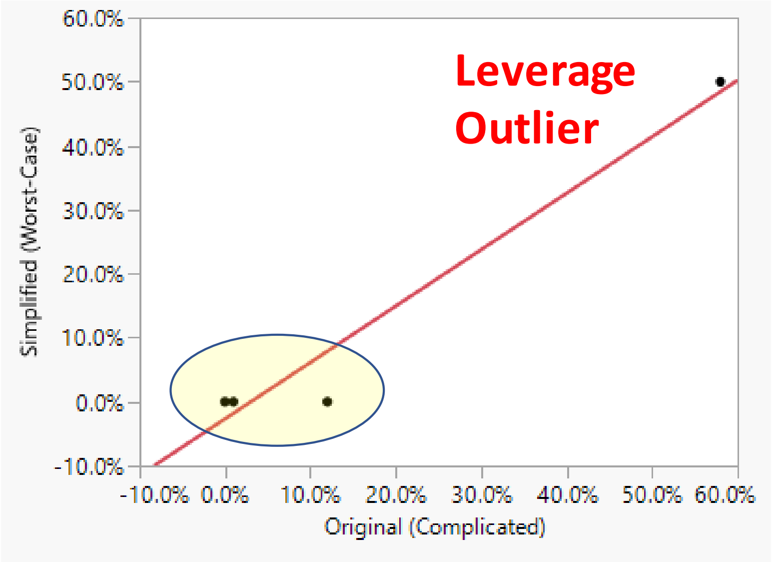
Player F	A	B	C	D	E
Q	W	W	W	W	W
J	W	W	W	W	W
K	W	W	W	W	W
Q	W	W	W	W	W
A	L	L	T	W	W
	80%	80%	90%	100%	100%
Overall	58%				

Simplified (Worst-Case) Method

5th Run	Shared Cards				Player A (0%)		Player B (15%)	
	Card 1	Card 2	Card 3	Card 4	Open	Hidden	Open	Hidden
1st Game Cards' Distrbution	H-A	D-A	S-K	C-J	H-K	D-Q	D-K	S-J
Actrual Matching					2-Pairs		2-Pairs	
Worst Case Overall Winning Probability					No need to Calculate		0% Chance	
Stay or Fold in the Betting Round					Always Stay		Fold	
Results (win or lose chips)					5		-1	
5th Run	Player C (30%)		Player D (45%)		Player E (60%)		Player (75%)	
	Open	Hidden	Open	Hidden	Open	Hidden	Open	Hidden
1st Game Cards' Distrbution	H-J	C-K	S-Q	C-Q	H-Q	C-A	D-J	S-A
Actual Matching	2-Pairs		2-Pairs		A-3 Kind		A- Full House	
Worst Case Overall Winning Probability	0% Chance		0% Chance		0% Chance		50% Chance	
Stay or Fold in the Betting Round	Fold		Fold		Fold		Fold	
Results (win or lose chips)	-1		-1		-1		-1	

Statistics: 5th Trial Correlation between two Methods

Players	Original (Complicated)	Simplified (Worst-Case)
B	1%	0%
C	1%	0%
D	0%	0%
E	12%	0%
F	58%	50%



JMP >> Analyze >> Fit Y by X

Linear Fit

Simplified (Worst-Case) = -0.026929 + 0.8814491*Original (Complicated)

Summary of Fit

RSquare	0.96078
RSquare Adj	0.947706
Root Mean Square Error	0.051134
Mean of Response	0.1
Observations (or Sum Wgts)	5

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	0.19215591	0.192156	73.4907
Error	3	0.00784409	0.002615	Prob > F
C. Total	4	0.20000000		0.0033*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-0.026929	0.027243	-0.99	0.3958
Original (Complicated)	0.8814491	0.102821	8.57	0.0033*

Even the 5th Trial's Worst-Case consistency is below 50%, two different methods of calculating the overall winning probability have still shown high correlations

- Though, there is one leverage outlier observed. If excluding this leverage outlier, the correlation will be very poor near the lower range.

Statistics: AKQJ Card Distribution and Matching Probability

		Actual Card Distribution					
Runs	Worst Case Consistency	1-Pair	2-Pairs	3-Kinds	Full House	4-Kinds	Winner (Should be)
1	70%	0	3	1	2	0	Full House
2	77%	0	4	2	0	0	3-Kinds
3	83%	0	4	0	2	0	Full House
4	57%	0	3	1	2	0	Full House
5	48%	0	4	1	1	0	Full House
	67%	0%	60%	17%	23%	0%	

Card Distribution based on 5 trials: most actual winners are having Full House.

	A	B	C	D	E	F
1	-3	-1	-1	-3	-1	9
2	-3	9	-1	-1	-3	-1
3	-3	3	-1	3	-1	-1
4	-3	-1	-3	-1	-1	9
5	5	-1	-1	-1	-1	-1
Total	-7	9	-7	-3	-7	15

Simplified Worst Case model can predict the actual winners 80% based on 5 trials

	Worst-Case Results			
Runs	Best Card Winning%	Players Stay	W-C Winner	WC Matching Actual Winner
1	80%	3	Full House	Yes
2	80%	3	3-Kinds	Yes
3	90%	3	Full House	Yes
4	80%	3	Full House	Yes
5	50%	1	2-Pairs	No
	76%	2.6		80%

Player F conservative character has the best returning case: win big and lose small

Results and Conclusions

- Apply both Poker Probability and JAVA programming on simulating Poker Winning Probability
  - ✓ Combination and Conditional Probability
  - ✓ Developed the Worst-Case Scenario to Shorten Betting Time <1 mins
  - ✓ Expected Probability vs. JAVA Simulated Probability
  - ✓ JAVA Simple Random Sampling and Shuffle Algorithm
- Knowing Poker probability may take huge advantage when the Partial Deck is getting smaller
- When sample size is too small, most cards will be known and uncertainty is reduced