Applications of Bayesian Methods Using JMP

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Overview

- Introduction to Bayesian statistical methods in JMP
- Reliability examples
  - Bearing cage field-failure data analysis
  - Rocket motor field data analysis
  - An accelerated test to estimate telecommunications laser lifetime
  - An accelerated test to estimate the life time of a new-technology IC processor
- Concluding remarks

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Reliability

- ***Probability*** that a system, vehicle, machine, device, and so on, will perform its intended function under ***encountered*** operating conditions, for a specified period of time (Meeker and Escobar 1998)
- Quality over time (Condra 1993)
- Failure avoidance
- A highly quantitative engineering discipline, often requiring complicated **statistical and probabilistic** analyses

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Bayesian Statistical Methods

- Over the past 30 years, there has been a **revolution** in the world of statistical modeling and data analysis. Bayesian methods are now widely used in many areas of application.
- Reasons for the revolution
  - Rediscovery and further developments of Markov-chain Monte Carlo (MCMC) methods
  - Spectacular improvements in computing power
  - Development of relatively easy-to-use software (BUGS, Stan, SAS proc MCMC, and now JMP)
- Motivation for using Bayesian methods
  - Provides a means combine prior information with limited data leading to improved inferences
  - Modeling and analysis using Bayesian methods for some complicated models is easier than using classical methods like maximum likelihood
- Downside? “You have to think.”

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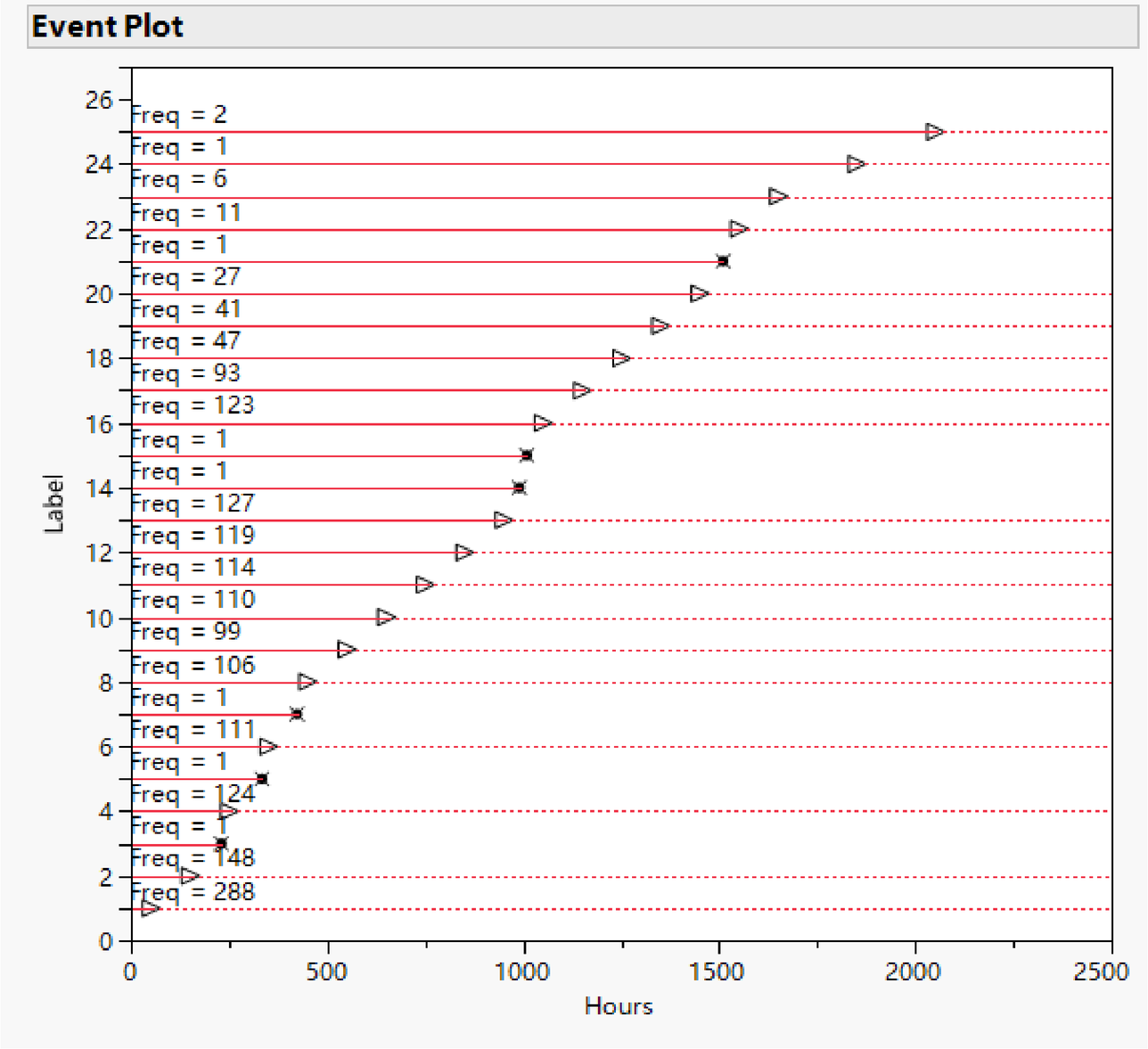
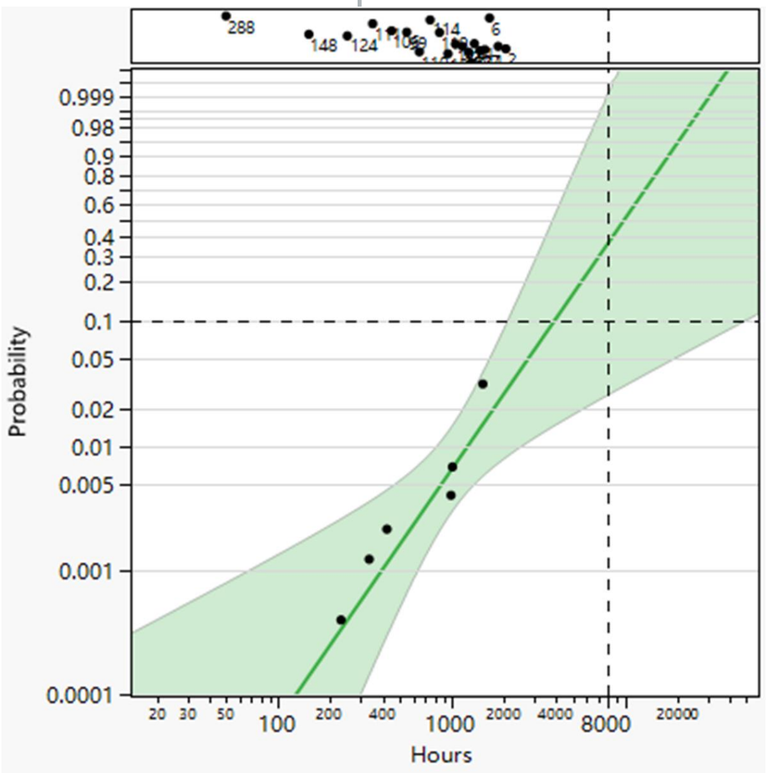
Aircraft Engine Bearing Cage Field-Failure Data

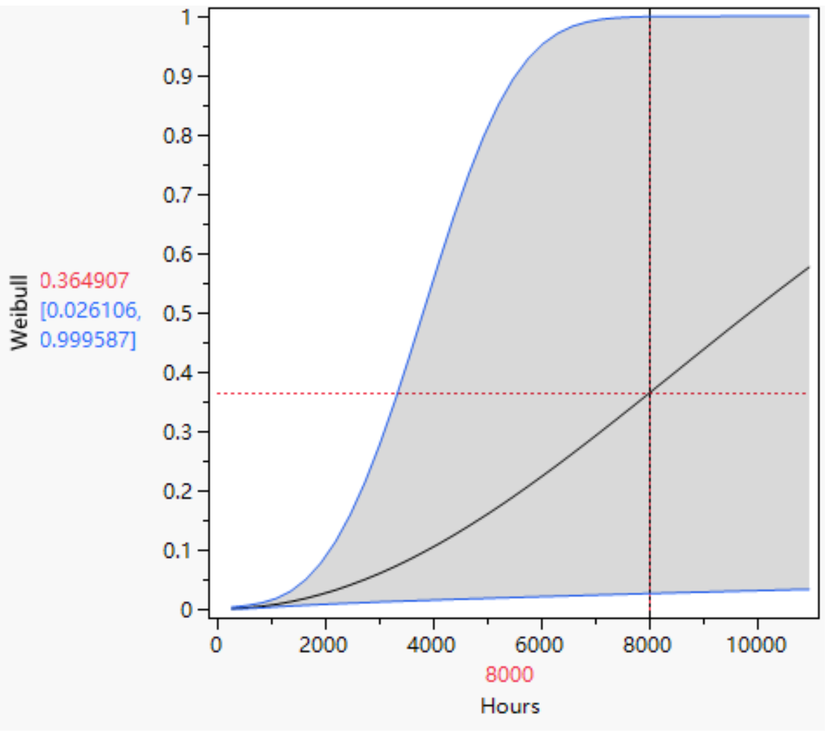
- Data from the *Weibull Handbook (1984)*
- 1703 units had been introduced into the field over time; oldest unit at 2220 hours of operation.
- Design life specification was B10 = 8000 hours of operation
- 6 units had failed
- Do we have a serious problem? Re-design needed?

Bearing Cage Field-Failure Data Event Plot



Bearing Cage Weibull Maximum Likelihood Fitting



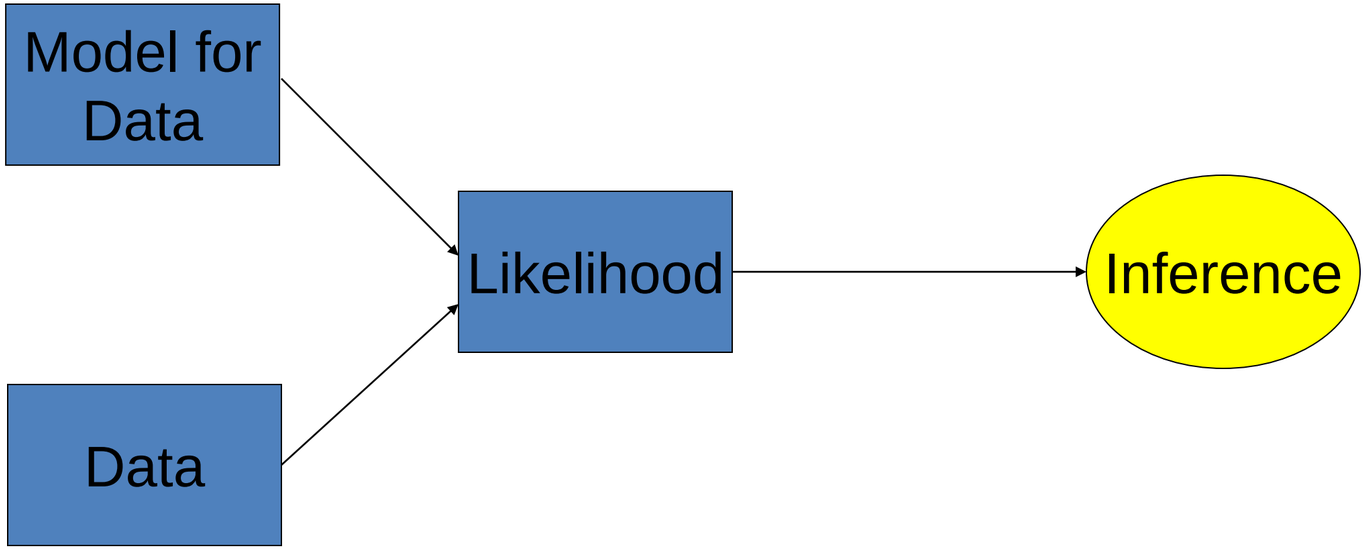


Parametric Estimate - Weibull

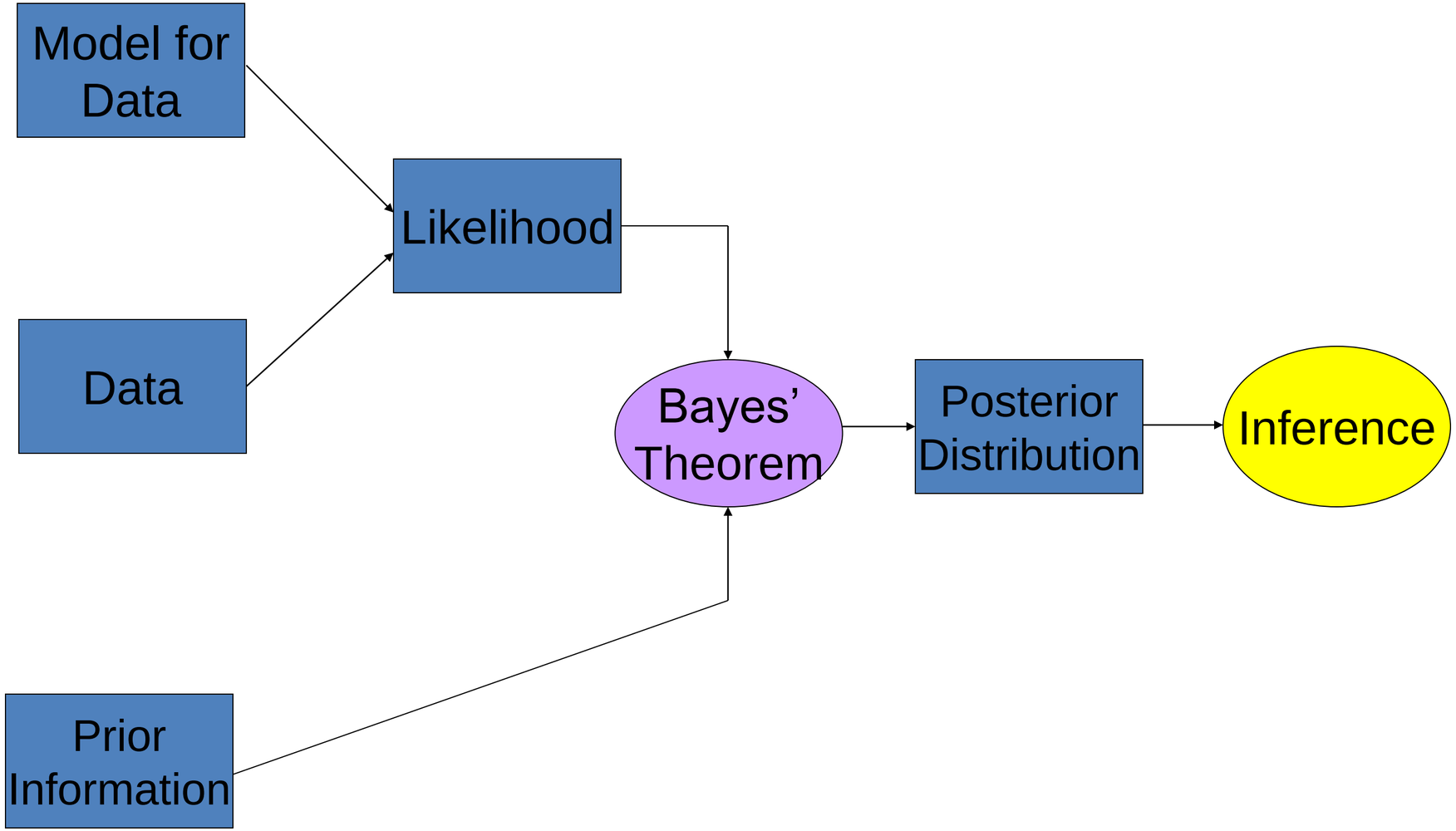
Parameter	Estimate	Std Error	Lower 95%	Upper 95%
location	9.375	0.8351	7.7383	11.012
scale	0.491	0.1607	0.1764	0.806
Weibull α	11792.178	9848.1267	2294.6744	60599.215
Weibull β	2.035	0.6657	1.2403	5.670
Mean	10447.606	8771.4265	2015.4834	54156.970

Criterion	
-2*LogLikelihood	152.87379
AICc	156.88085
BIC	167.75409

Likelihood Inference



Bayesian Inference



Bearing Cage Field Data Joint Prior Distribution

Prior for B10: Weakly informative B10 ~ Lognormal<1000, 50,000>

Prior for Weibull shape β: Informative: β ~ Truncated Normal<1.5, 3>

Bayesian Estimates - Result 1

Method: Simple Rejection

Priors

Name	Prior Distribution		Value		Value		Value
Quantile	Lognormal	Probability	0.1000	Lower 99% Limit	1000.000	Upper 99% Limit	50000.00
Weibull β	Normal			Lower 99% Limit	1.5000	Upper 99% Limit	3.0000

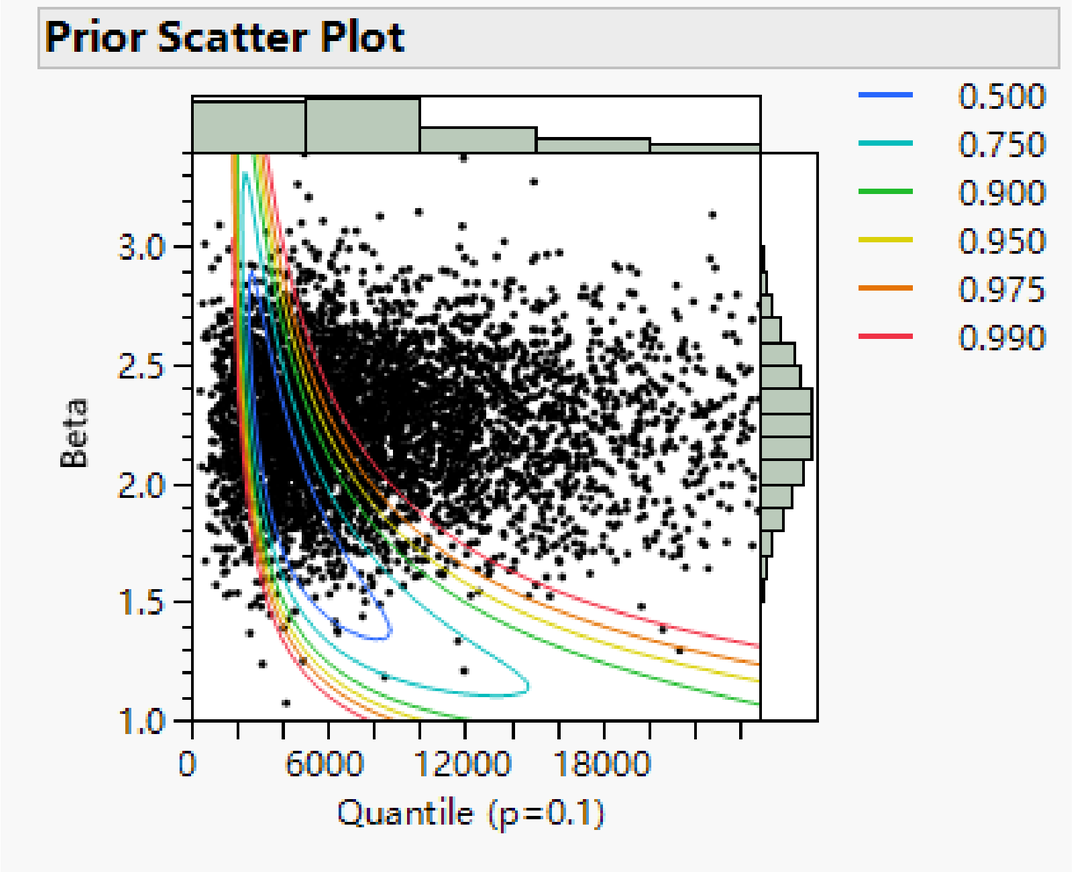
Random Seed 267411951

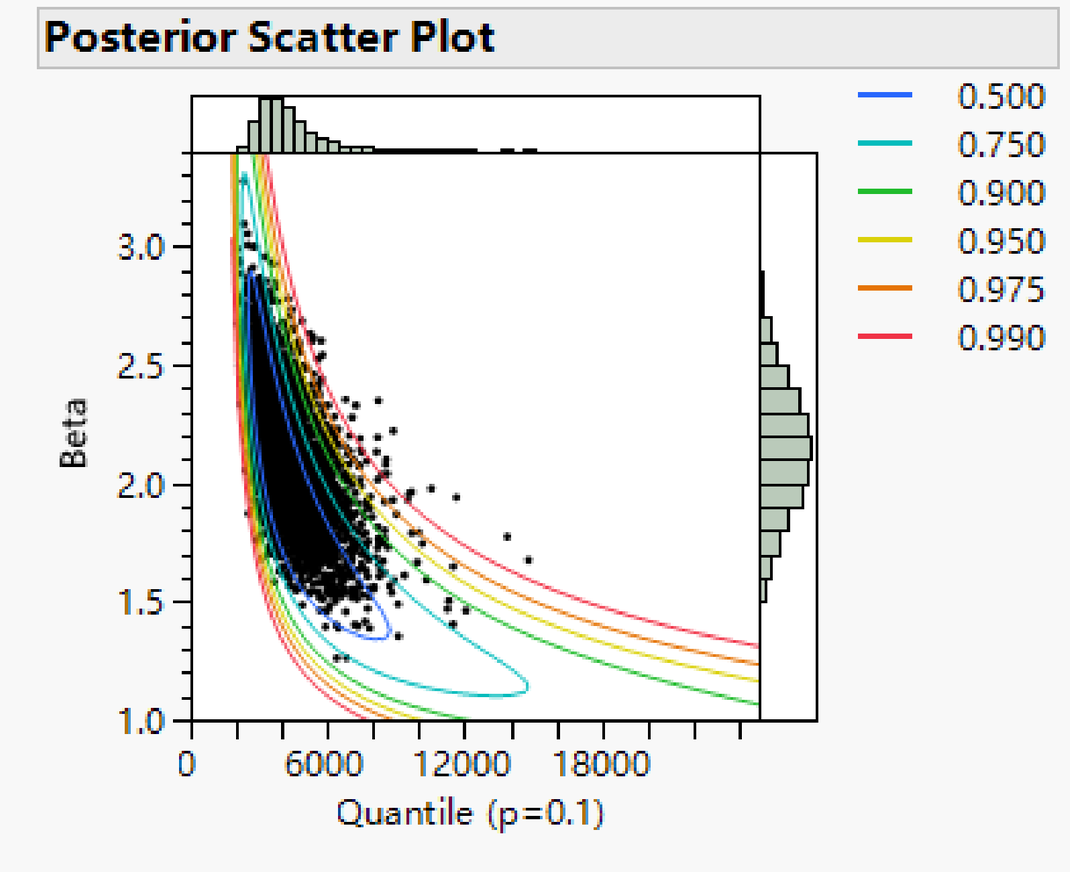
Posterior Estimates

Parameter	Median	Lower 95%	Upper 95%	Mean	StdDev
Location	9.329585	8.754473	10.17909	9.362648	0.36504
Scale	0.465643	0.373434	0.610345	0.473213	0.061353
Quantile (p=0.1)	3924.917	2582.887	7081.7	4161.087	1193.62
Weibull β	2.147493	1.635727	2.676513	2.147535	0.268934

Note: JMP does not have TNORM, Normal<1.5, 3> is equivalent to TNORM<1.5, 3> here, with negligible differences.

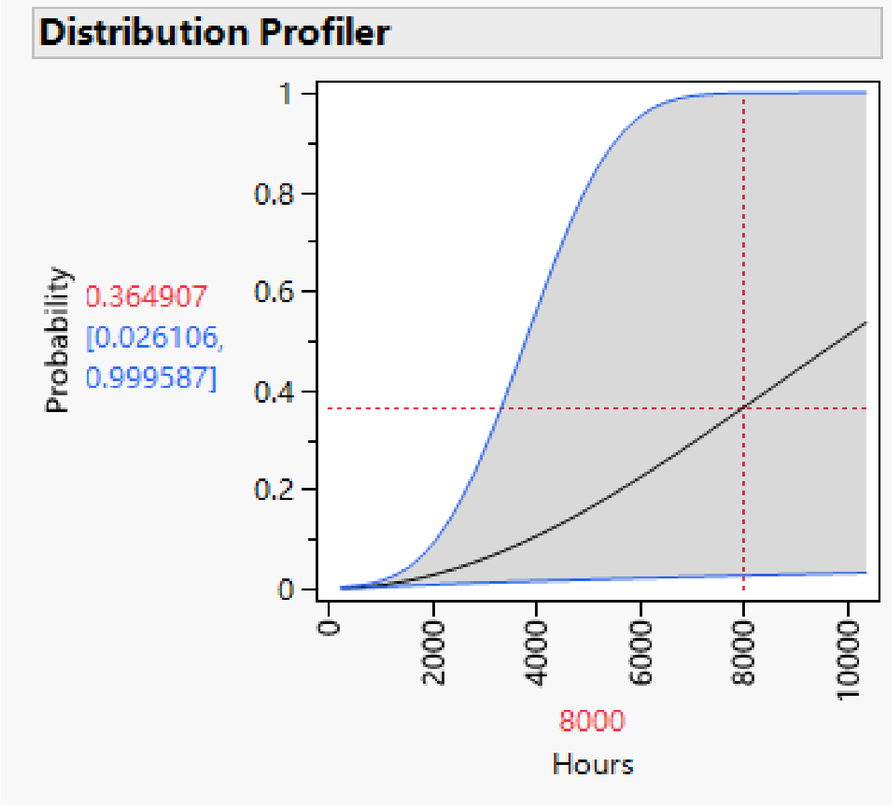
Bearing Cage Field Data
Comparison of Joint Prior and Posterior Distributions



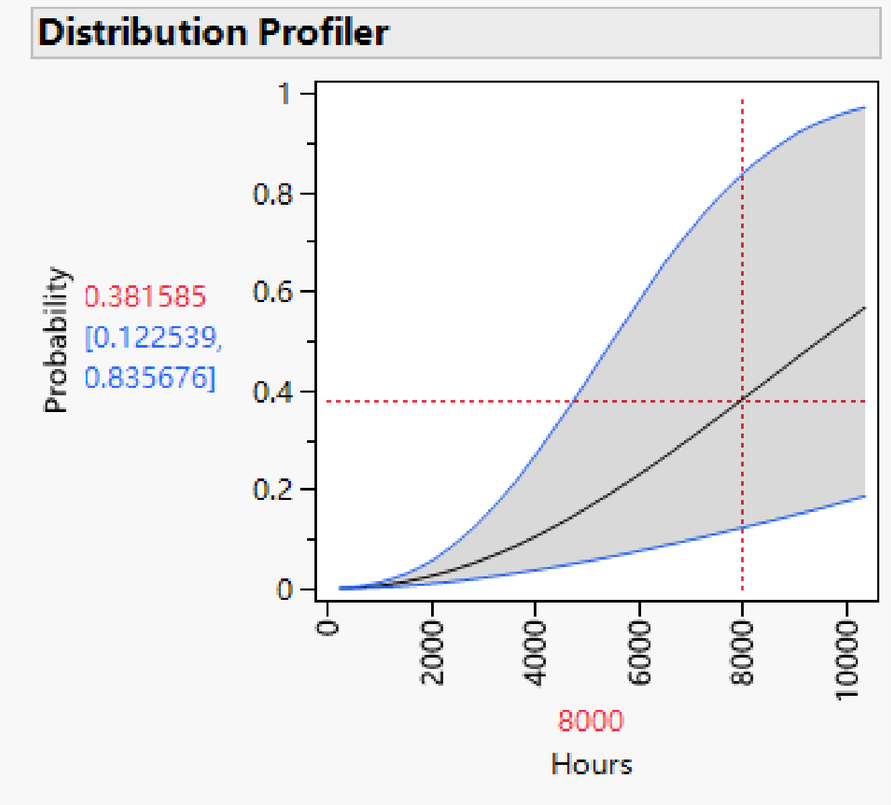


Bearing Cage Field Data
Comparison of ML and Bayesian Inferences

Maximum Likelihood



Bayesian with Informative Prior for β



Lessons Learned

- With a small number of failures, not much can be said about reliability
- Engineers often have information about the Weibull shape parameter, based on knowledge of the failure mechanism
- Using the prior information will often lead to improved, more useful inferences
- Bayesian methods provide a formal statistical method to combine information from different sources

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Rocket Motor Field Data Analysis

- Rocket motor is one of five critical missile components
- Approximately 20,000 missiles in inventory
- 1,940 firings over the life of the missile; catastrophic motor failures for three ***older*** missiles
- Failures thought to be due to thermal cycling, but only age information is available
- Failure times not directly observed (1,937 right censored, 3 left censored observations)
- Concern about a possible wearout failure mode and the distribution of remaining life of the stockpile

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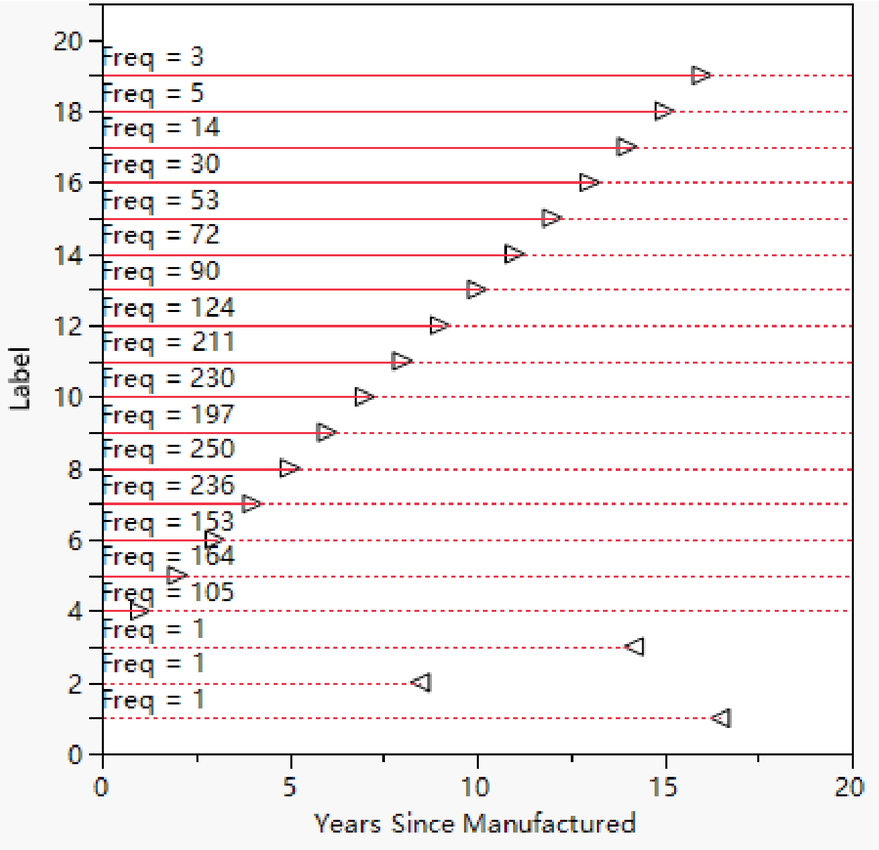
Rocket Motor Lifetime Data

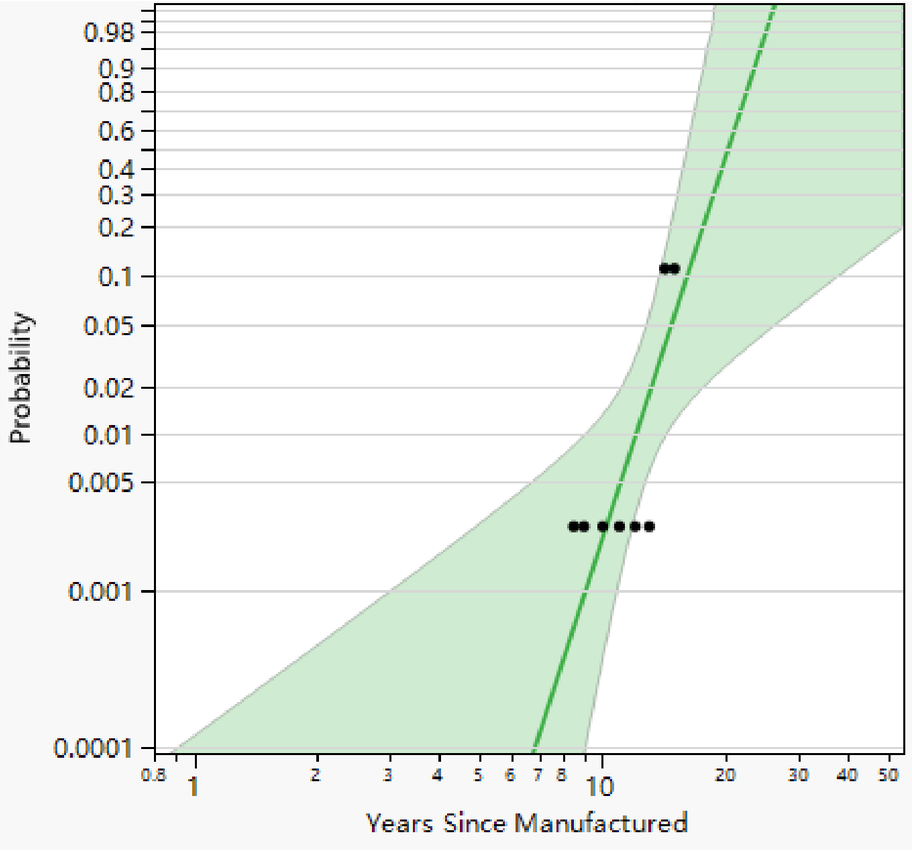
Years	Number of Motors	Years	Number of Motors	Years	Number of Motors
> 1	105	> 8	211	> 14	14
> 2	164	> 9	124	> 15	5
> 3	153	> 10	90	> 16	3
> 4	236	> 11	72	< 8.5	1
> 5	250	> 12	53	< 14.2	1
> 6	197	> 13	30	< 16.5	1
> 7	230				

Rocket Motor Life Data Analysis

Event Plot



Weibull Probability Plot

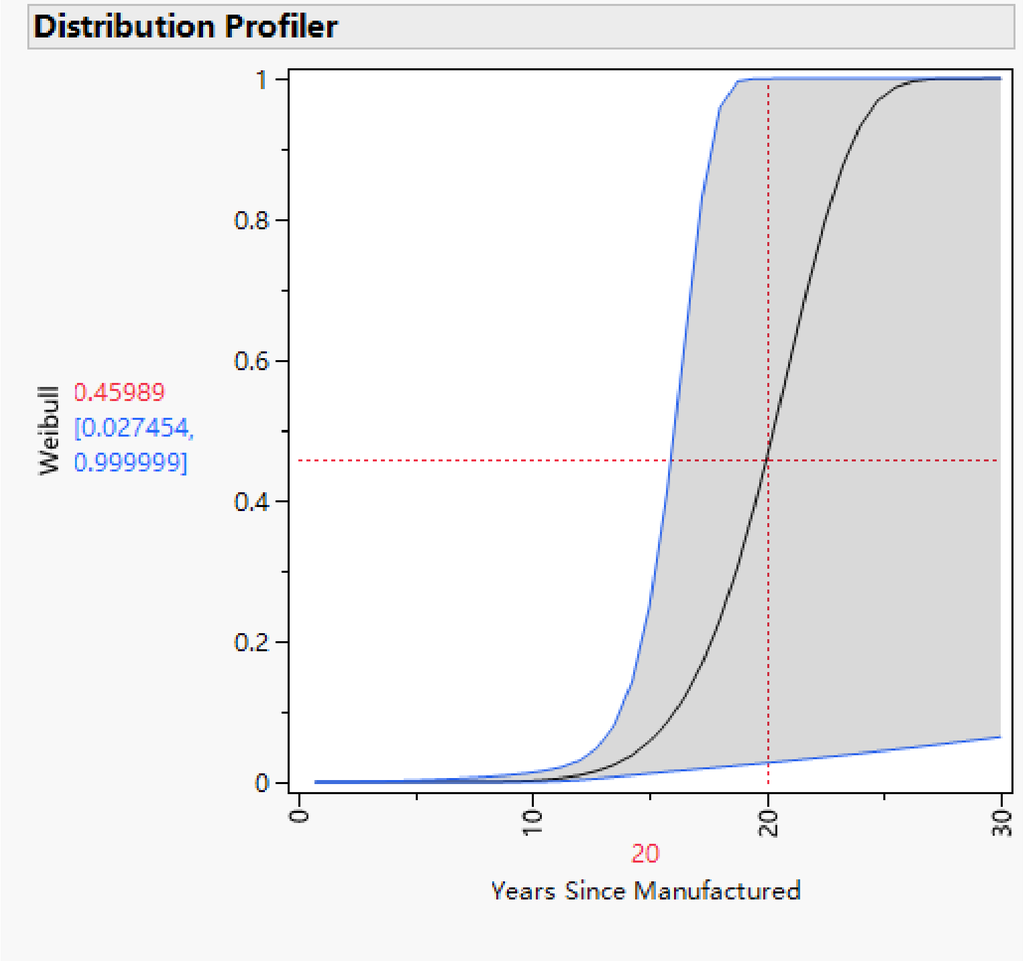


Rocket Motor Weibull ML Analysis

Parametric Estimate - Weibull

Parameter	Estimate	Std Error	Lower 95%	Upper 95%
location	3.055358	0.2162706	2.631476	3.479241
scale	0.123058	0.0480368	0.028908	0.217209
Weibull α	21.228791	4.5911623	13.894260	32.435089
Weibull β	8.126236	3.1721454	4.603869	34.592770
Mean	20.007199	3.9737988	13.555700	29.529127

Weibull shape parameter estimate was 8.1 (very large)



Rocket Motor Bayesian Weibull Analysis

- The ML estimate of beta=8.1 was much larger than expected.
- Estimate of fraction failing at 20 years was 0.46
- LR Confidence Interval=(0.023, 0.9999); not useful
- Prior distributions:
  - Informative: β ~ Truncated Normal<1, 5>
  - Weakly informative: B10 ~ Truncated Normal<5, 400>
- 10,000 MCMC draws obtained from the joint posterior distribution

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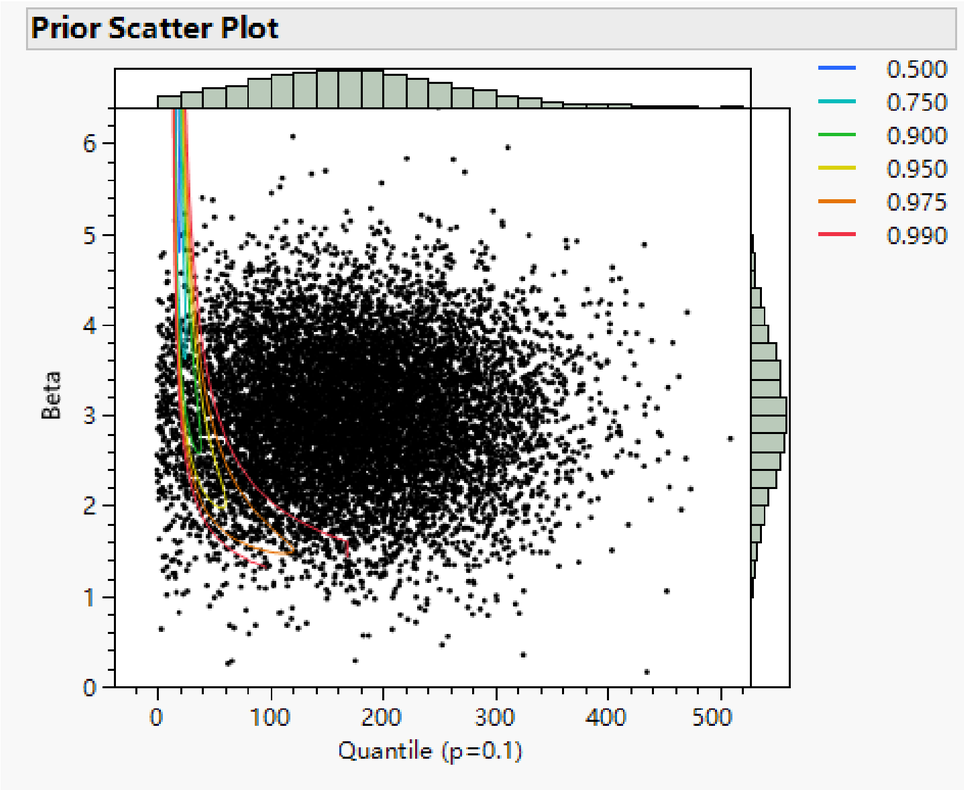
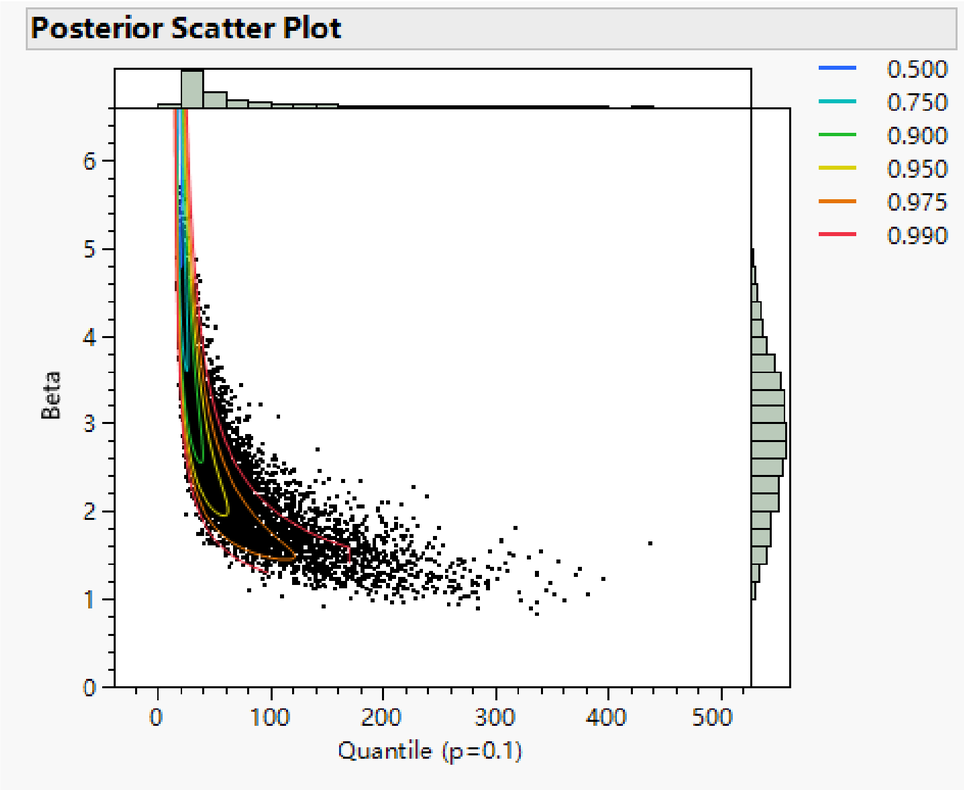
Rocket Motor Comparison of the Likelihood Contours and Samples from the Prior and Posterior Distributions Using Prior Information on β



Prior Sample



Posterior Sample

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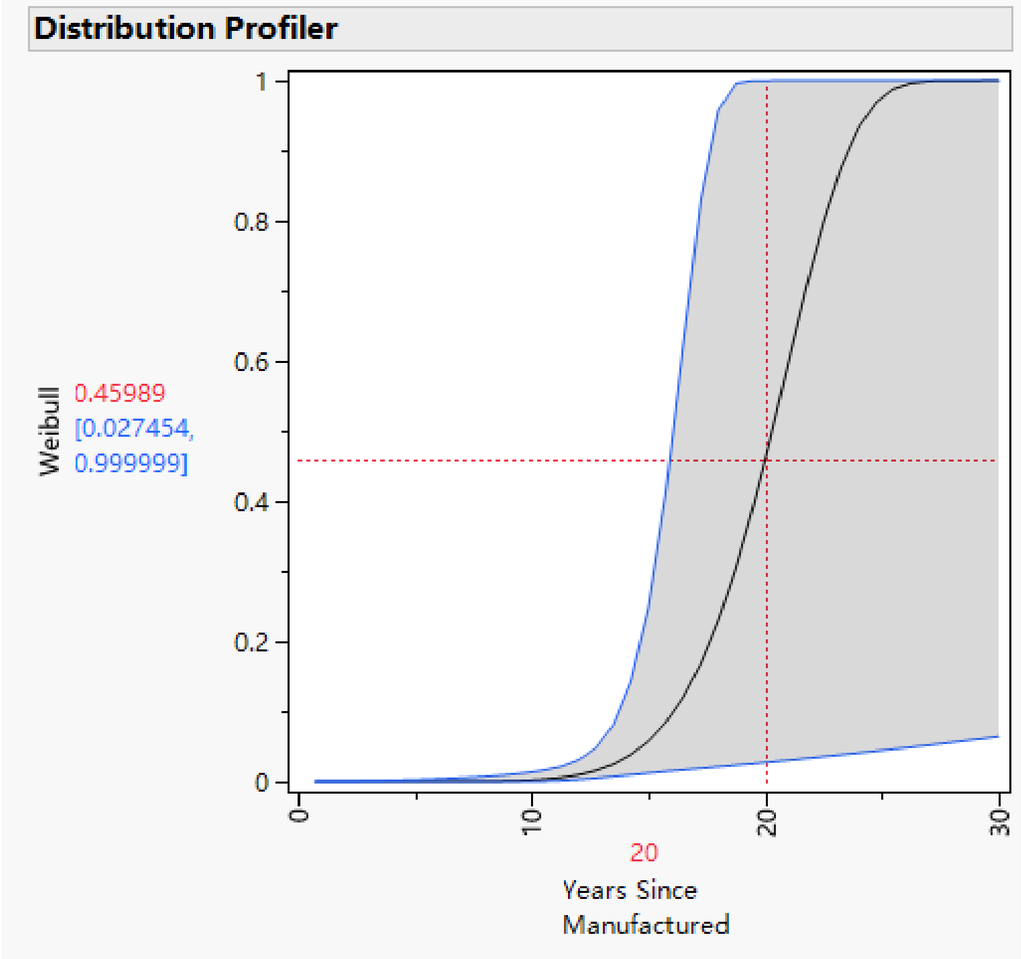
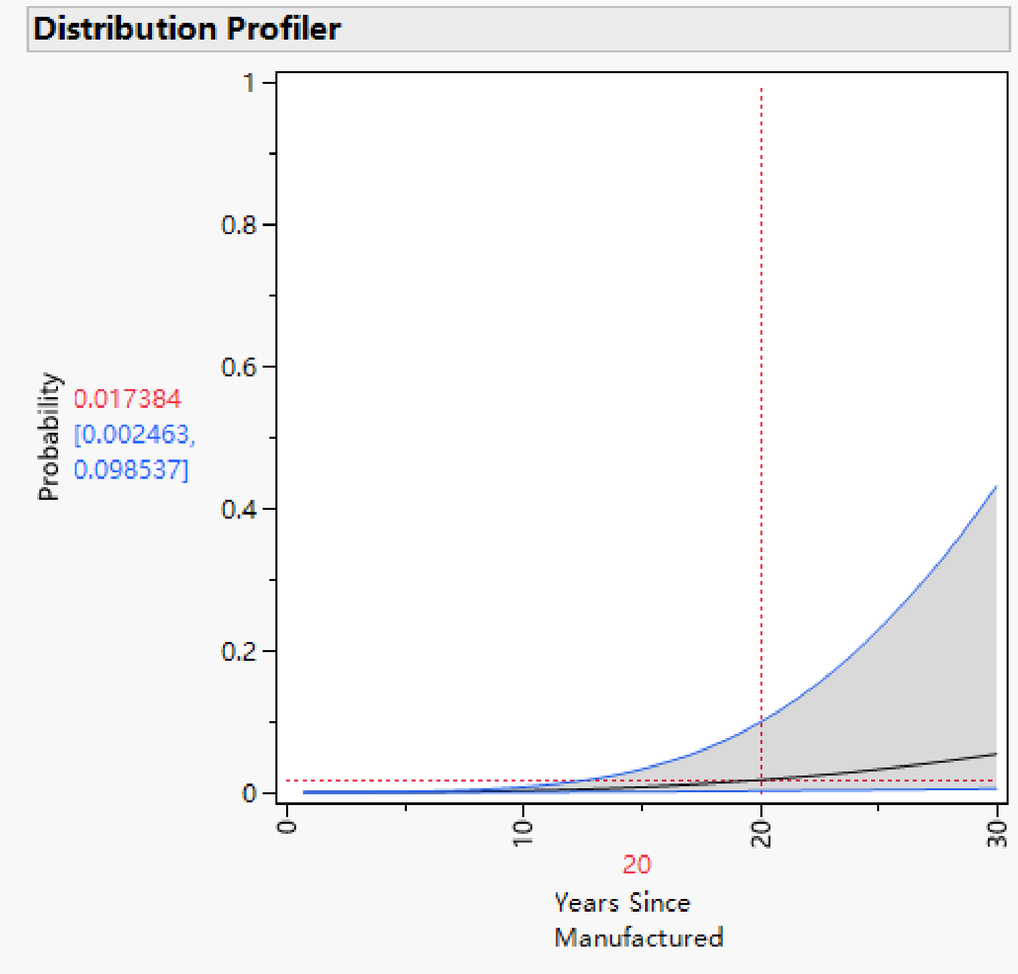
Rocket Motor Comparison of ML Estimates and Bayesian Estimates Using Prior Information on β

Maximum Likelihood



Bayesian



Lessons Learned

- Even when no actual failure times are observed, there is still reliability information in the data.
- With very few failures, there is little information in the data
- The limited information can be supplemented by using knowledge about the failure mode and other engineering information
- JMP's Bayesian analysis tools make Bayesian analyses easy.

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Accelerated Testing

- In product design, engineers need to obtain reliability information quickly
- Test units at high levels of temperature, voltage, stress, or other “accelerating variable” to get reliability information quickly
- Use a physically-motivated model to extrapolate to use conditions
- Extrapolation is dangerous; assumed model may not hold outside the range of the data

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Accelerated Life Test of a Laser

- Data from Hooper and Amster (1990)
- Accelerated Life Test with temperature acceleration at 40˚C, 60˚C, and 80˚C
- Units tested at use conditions 10˚C, but none failed
- Test lasted 5,000 hours
- Interest is in estimating fraction failing at 30,000 hours (~3.5 years) at 10˚C.

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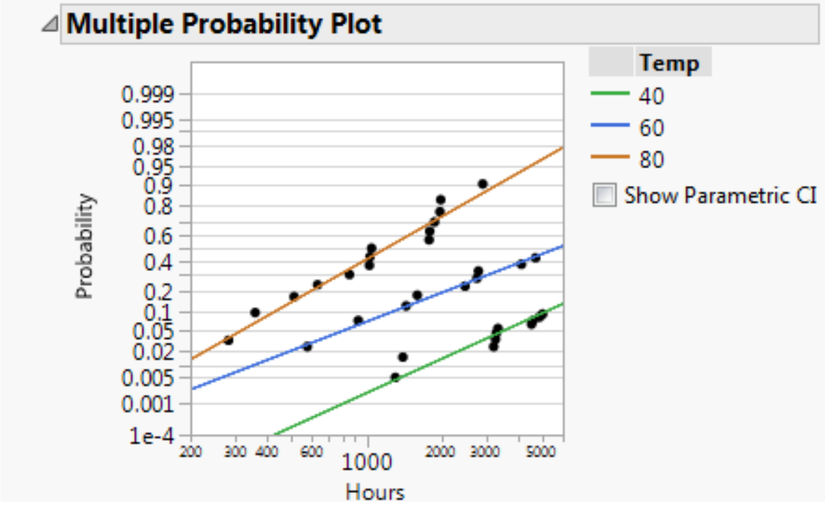
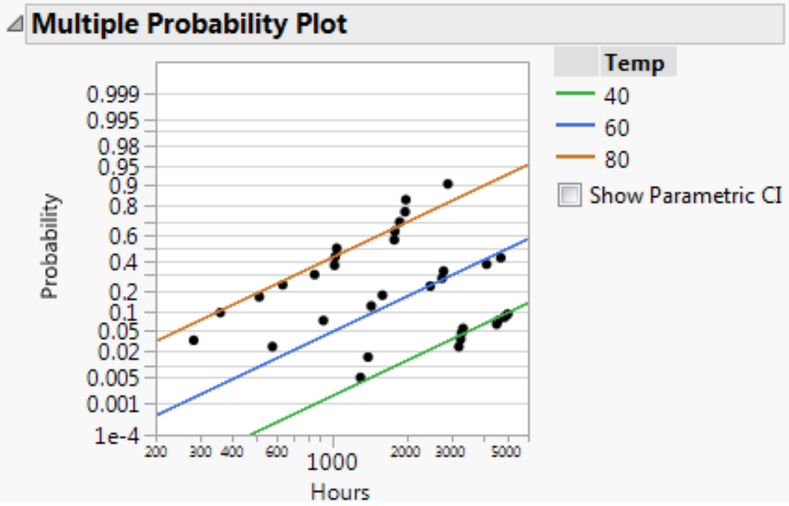
Fit Life by X Multiple Probability Plots to Assess Model Adequacy

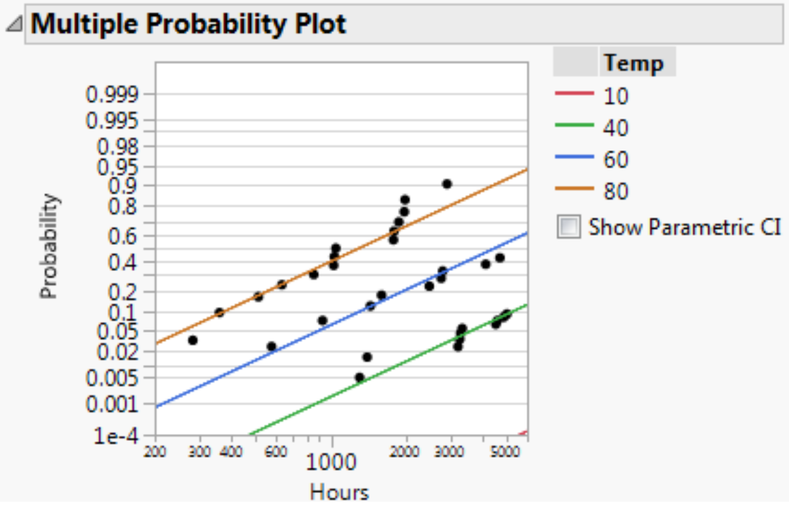
Separate distributions



Separate distributions; common slope



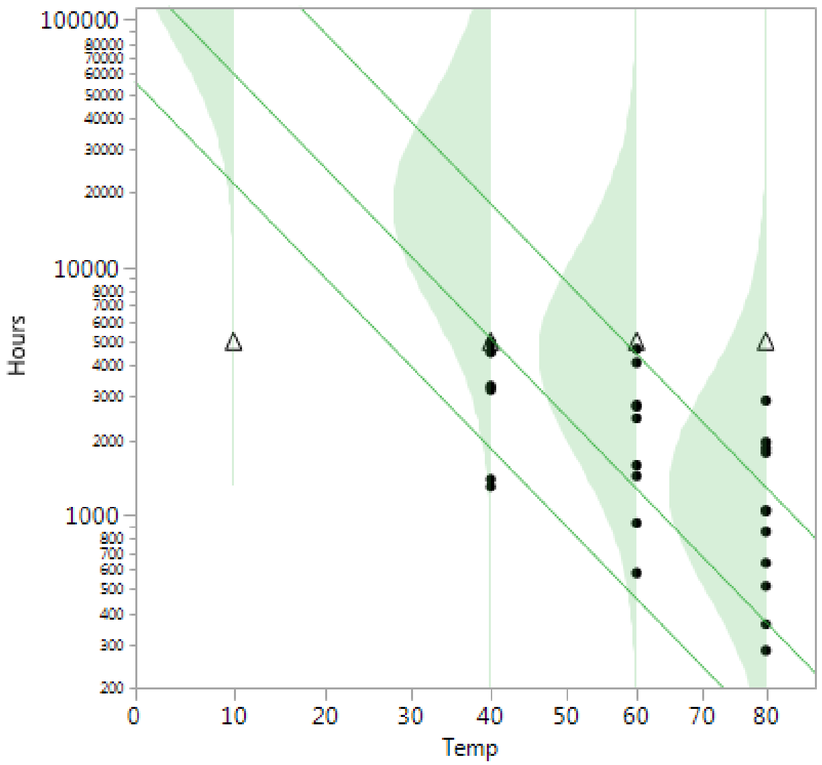
Arrhenius model



Test Results

Description	L-R ChiSquare	DF	Prob>ChiSq
No Effect vs. Regression	80.71407	1	<.0001*
Regression vs. Separate Location	0.387349	2	0.8239
Separate Location vs. Separate Location and Scale	1.503157	2	0.4716

Arrhenius Model Fit to the Laser Data



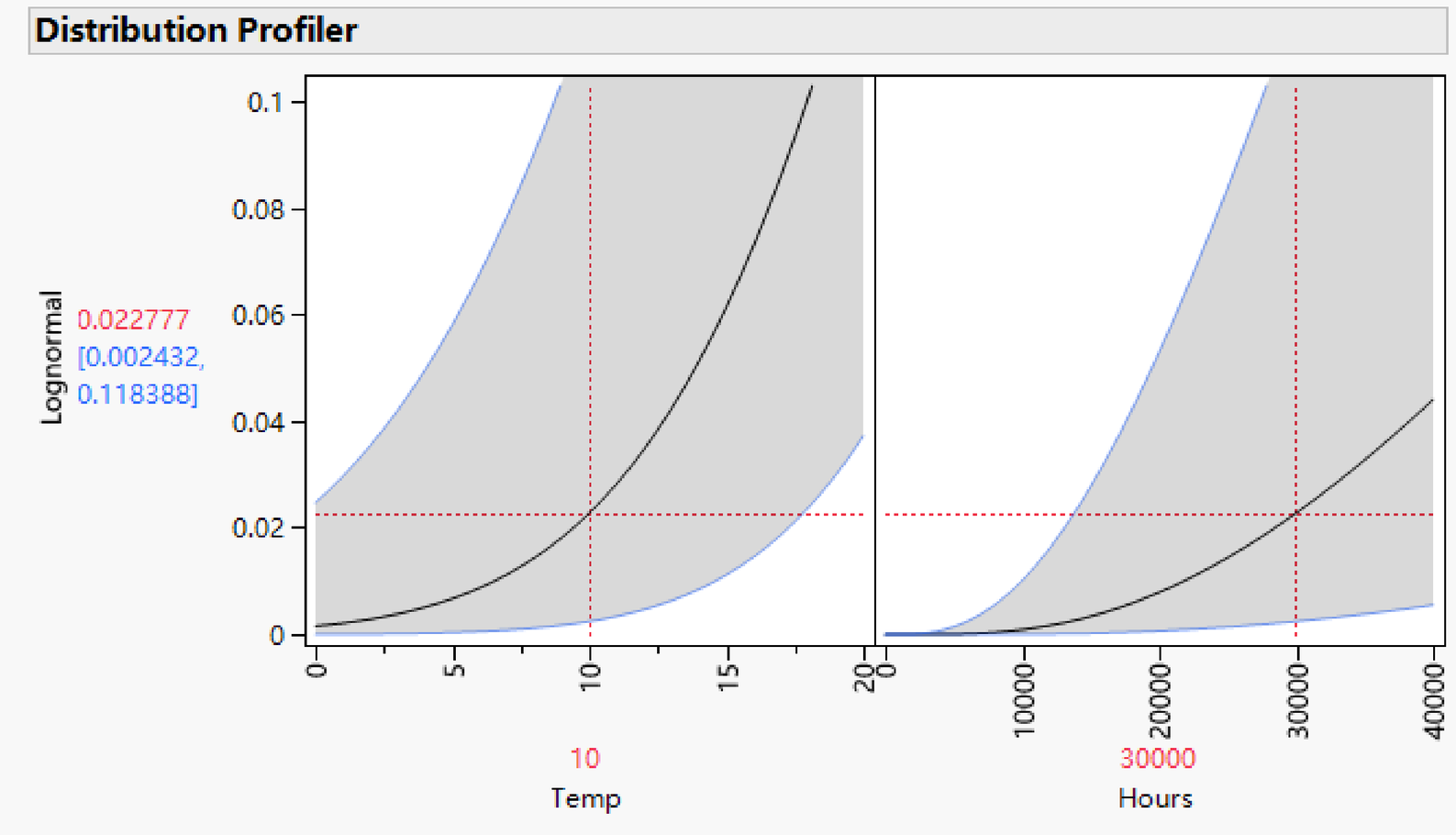
Estimates

Parameter	Estimate	Std Error	Lower 95%	Upper 95%
β0	-13.46865	2.8871953	-19.12745	-7.809851
β1	0.62788	0.0828422	0.46551	0.790247
σ	0.97782	0.1326468	0.71784	1.237806

$$\mu = -13.46865 + \frac{(0.627879 \cdot 11604.518122)}{(\text{Temp} + 273.15)}$$

Note: Arrhenius constant has been updated with more significant digits in JMP16.
This would not change existing conclusions that used 110605.

Laser Lifetime Distribution Profilers to Estimate Fraction Failing at 10˚C 30,000 Hours



Laser Lifetime Bayesian Analysis

Bayesian Estimates - Result 1

Method: Simple Rejection

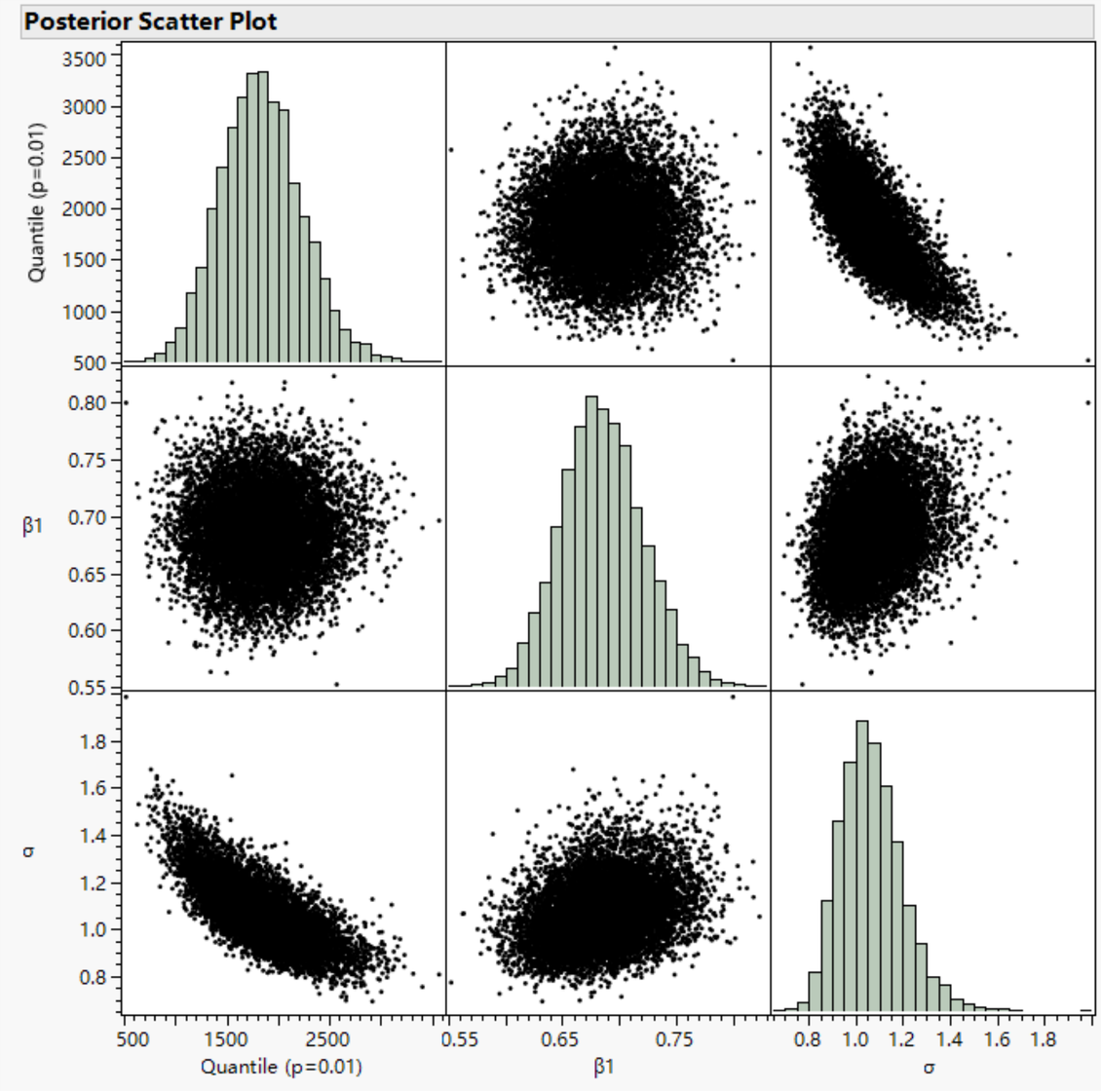
Priors

Name	Prior Distribution	Temp	Probability		Value		Value
Quantile	Lognormal	40.0000	0.0100	Lower 99% Limit	100.0000	Upper 99% Limit	32000.00
β1	Lognormal			Lower 99% Limit	0.6000	Upper 99% Limit	0.8000
σ	Lognormal			Lower 99% Limit	0.0500	Upper 99% Limit	20.0000

Random Seed 663475382

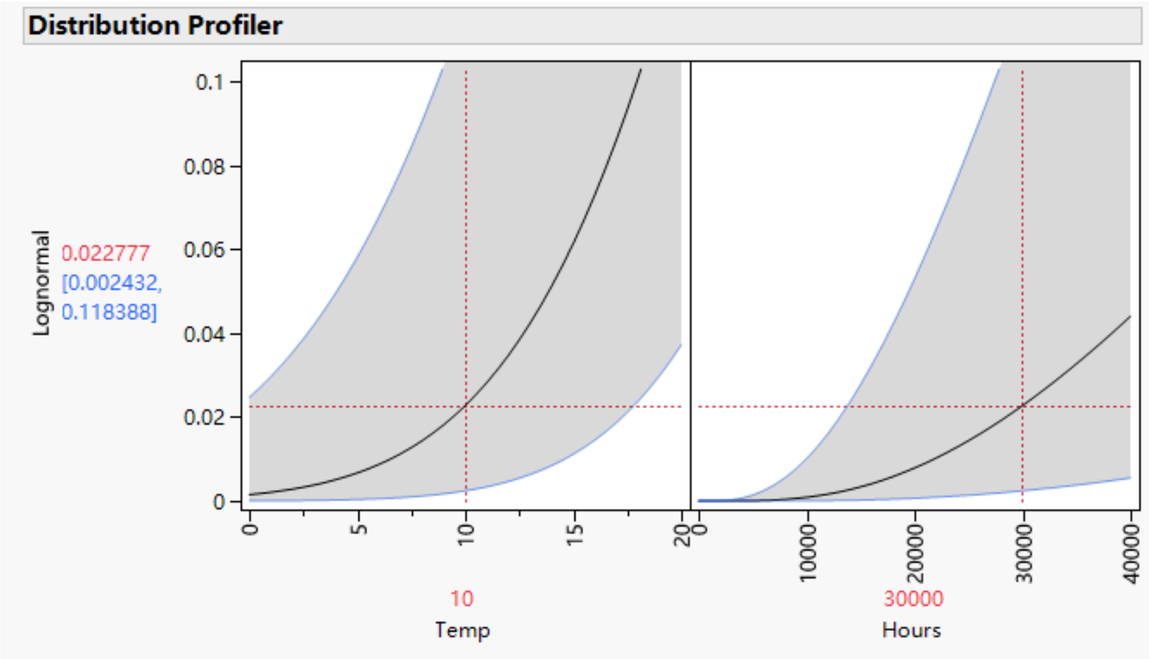
Posterior Estimates

Parameter	Median	Lower Bound	Upper Bound	Mean	StdDev
β0	-15.3686	-17.9516	-13.102	-15.4146	1.246922
β1	0.683537	0.618232	0.757823	0.684884	0.035652
σ	1.05233	0.847889	1.358022	1.064764	0.130064
Quantile (p=0.01)	1815.188	1111.535	2633.421	1829.903	390.0859

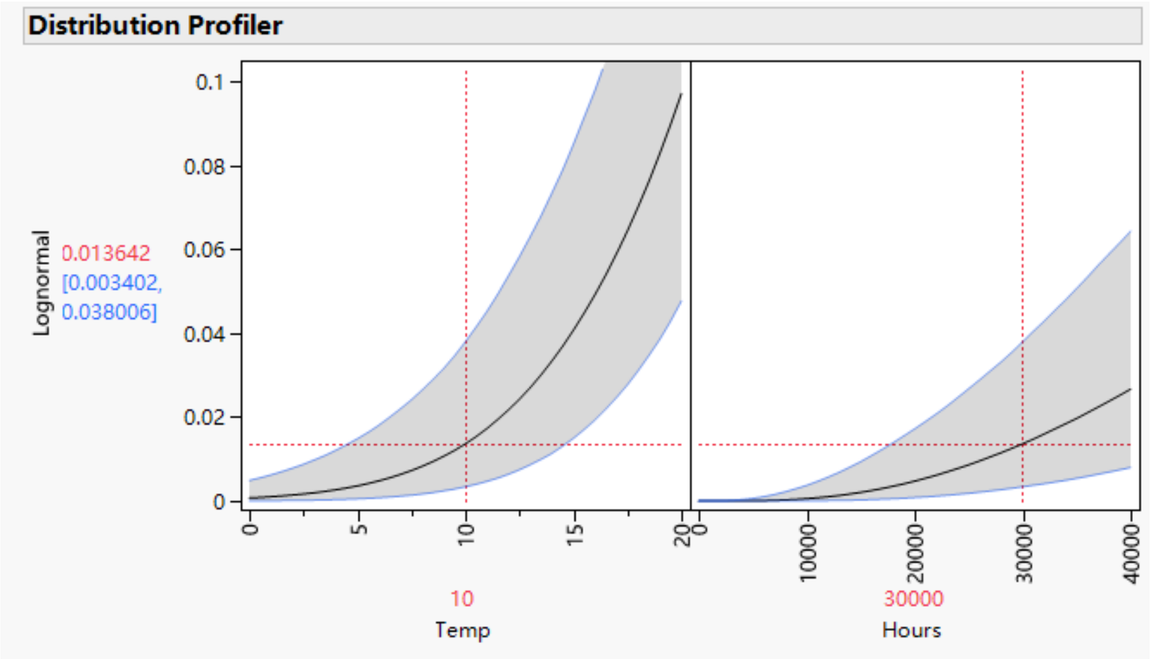


Laser Lifetime Comparison of Fraction Failing at 10˚C 30,000 Hours ML Estimates and Bayesian Estimates Using Prior Information on β_1

Maximum Likelihood



Bayesian



Lessons Learned

- Accelerated life tests provide reliability information quickly
- Engineers often have information about the effective activation energy that can be used to improve precision (or reduce cost through the use of smaller sample sizes).
- Bayesian methods provide an appropriate method to combine the engineer's information with the ALT data.

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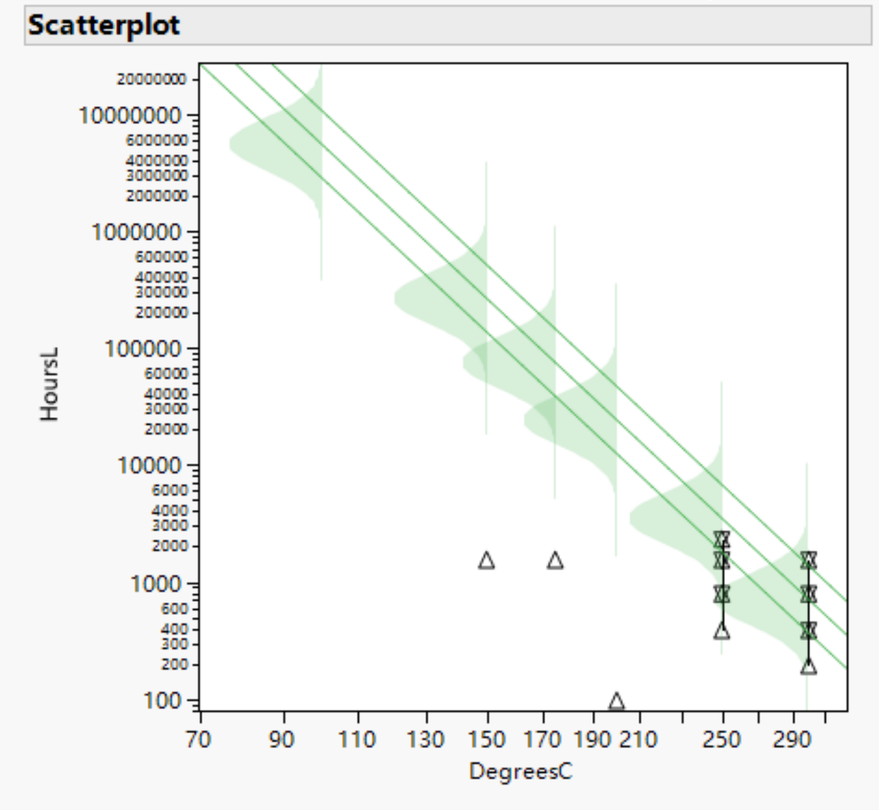
Analysis of Interval-Censored ALT data for a New-Technology IC Device

- An accelerated life test was run to evaluate the life time of a new **processor IC device**
- Tests run at 150, 175, 200, 250, and 300 degrees C
- Interval-censored data
- Failure only at 250 and 300 degrees C
- Developers interested in estimating the 0.01 Quantile of the life distribution at 100 degrees C

New-Technology IC Device ALT Data

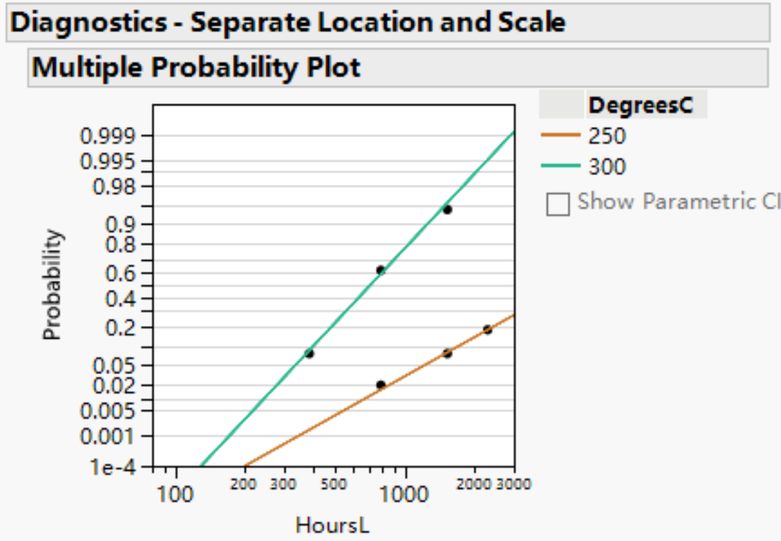
Hours Lower	Hours Upper	Status	Number of Devices	Temperature Degrees C
	1536	Right Censored	50	150
	1536	Right Censored	50	175
	96	Right Censored	50	200
384	788	Failed	1	250
788	1536	Failed	3	250
1536	2304	Failed	5	250
	2304	Right Censored	41	250
192	384	Failed	4	300
384	788	Failed	27	300
788	1536	Failed	16	300
	1536	Right Censored	3	300

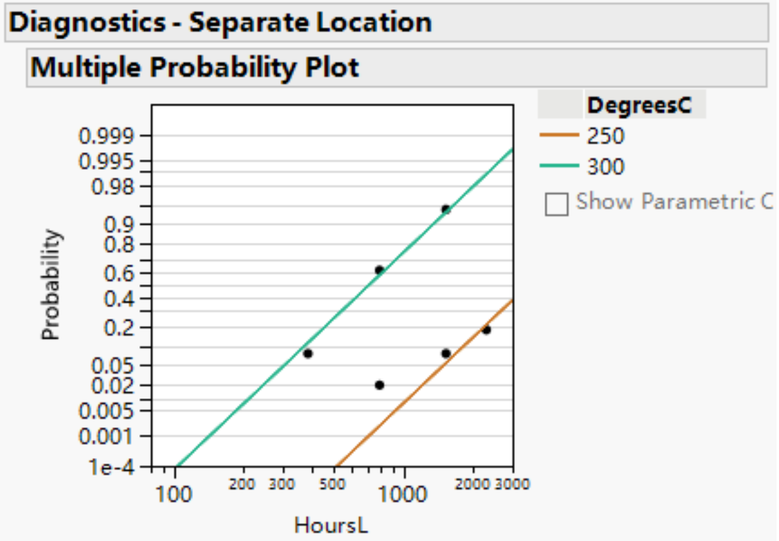
ML Estimation for the New-Technology IC Device



Test Results

Description	L-R ChiSquare	DF	Prob>ChiSq	Note
No Effect vs. Regression	190.0685	1	<.0001*	
Regression vs. Separate Location	2.39e-12	3	1.0000	Count extra 3 degrees of freedom for excluded zero-failure groups.
Separate Location vs. Separate Location and Scale	4.719007	1	0.0298*	





Failures at 300C were caused by a different failure mechanism that would never be seen at use conditions. Need to drop those data.

ML Estimation for the New-Technology IC Device Using only the 250 C Data

Lognormal Results

Statistics | Distribution | Quantile | Hazard | Density | Acceleration Factor | Custom Estimation | Bayesian Estimates

Statistics

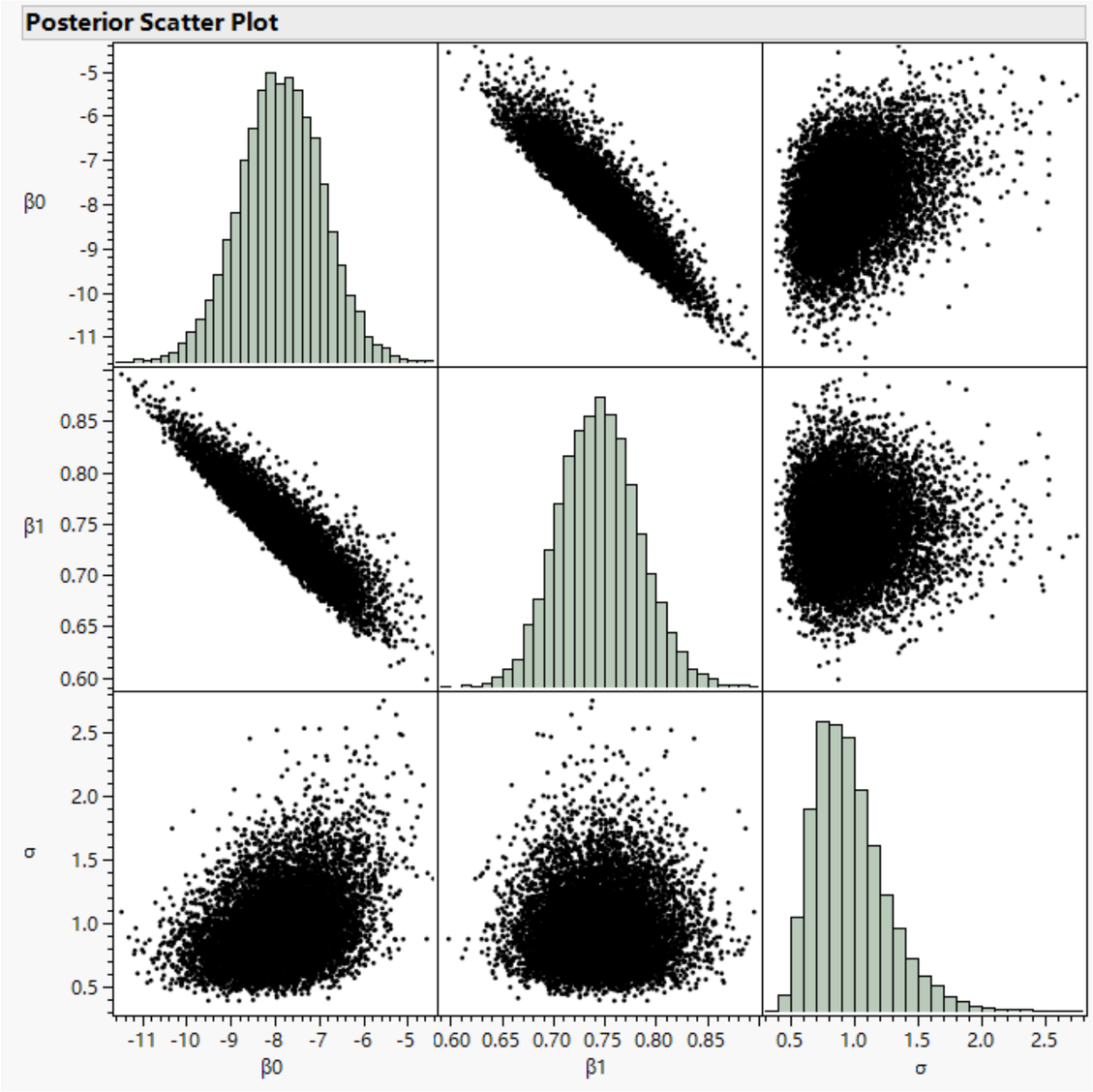
Estimates

Parameter	Estimate	Std Error	Lower 95%	Upper 95%
β0	-21.08816	118853.71	-232970.1	232927.91
β1	1.33568	5358.11	-10500.4	10503.04
σ	0.87098	0.26	0.51290026	1.70

$$\mu = -21.08816 + \frac{(1.335684 \cdot 11604.518122)}{(\text{DegreesC} + 273.15)}$$

With only one temperature level, there is not enough information to fit the ALT regression model

Bayesian Estimation Joint Posterior Distributions for the New-Technology IC Device Using only the 250 C Data with Prior Information for the Activation Energy



Bayesian Estimation for the New-Technology IC Device Using Only the 250 C Data with Prior Information for the Activation Energy

Bayesian Estimates - Result 1

Method: Simple Rejection

Priors

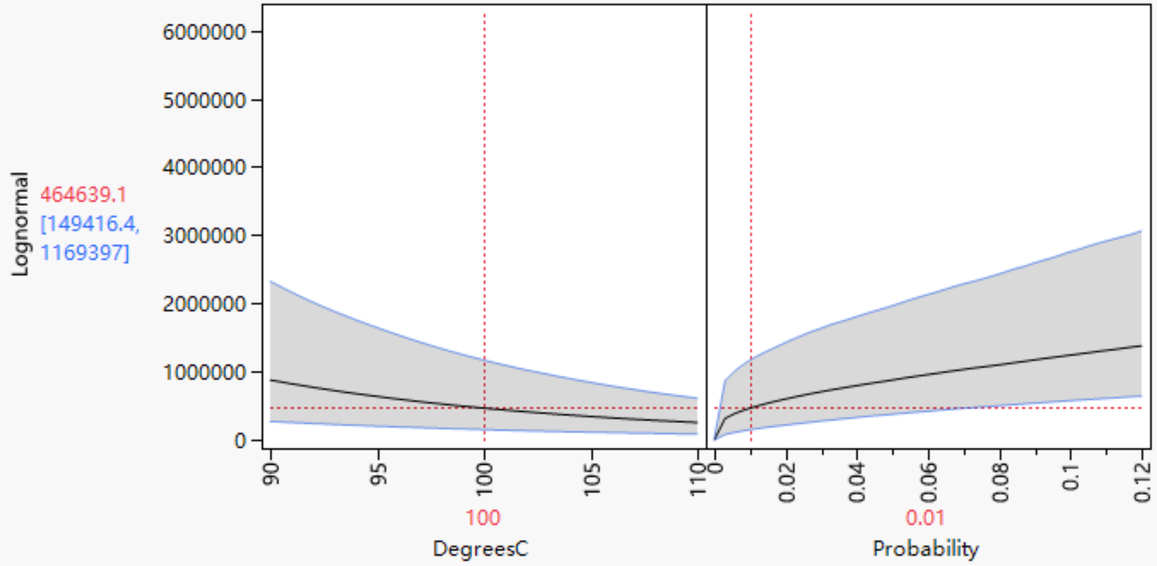
Name	Prior Distribution	DegreesC	Probability		Value		Value
Quantile	Lognormal	250.0000	0.1000	Lower 99% Limit	100.0000	Upper 99% Limit	10000.00
β1	Lognormal			Lower 99% Limit	0.6500	Upper 99% Limit	0.8500
σ	Lognormal			Lower 99% Limit	0.0500	Upper 99% Limit	5.0000

Random Seed 663475382

Posterior Estimates

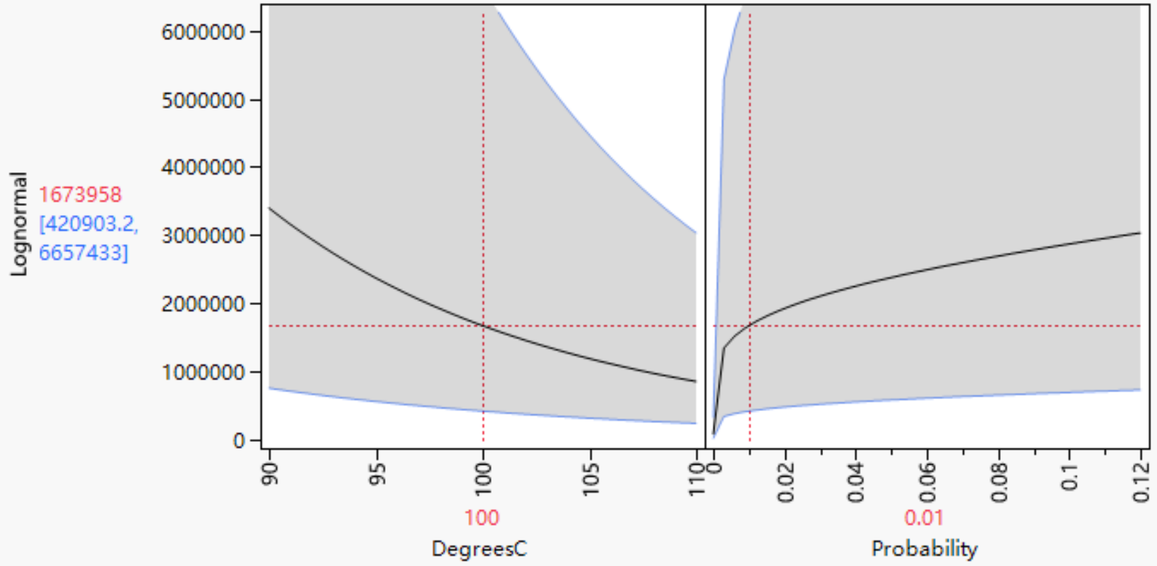
Parameter	Median	Lower Bound	Upper Bound	Mean	StdDev
β0	-7.87011	-9.73772	-6.07678	-7.87481	0.935606
β1	0.74384	0.672005	0.822789	0.744284	0.038769
σ	0.921229	0.547209	1.654914	0.967207	0.289131
Quantile (p=0.1)	1646.892	1060.24	2375.213	1661.293	331.649

Quantile Profiler



Maximum likelihood estimates with bad data

Quantile Profiler



Lessons Learned

- In some applications, interval censoring arises. Appropriate statistical methods exist for handling such data.
- Using excessive levels of an accelerating variable is likely to cause failures from mechanisms that will never be active in actual use
- Even with failures at only one level of temperature, we can estimate life at the use conditions (and quantify statistical uncertainty) if we have prior information about the effective activation energy (slope of the regression line) and use Bayesian methods.

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Concluding Remarks

- Improvements in computing hardware and software have greatly advanced our ability to analyze reliability data.
- The use of Bayesian methods will continue to increase, allowing the effective use of available engineering information to improve the precision of reliability inferences and to reduce the costs in reliability testing.
- JMP already has powerful tools for applying Bayesian methods in life data analysis (Life Distribution) and accelerated testing (Fit Life by X).
- Although these Bayesian capabilities are in the reliability part of JMP, they can certainly be used for non-reliability applications.

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References

Li, M. and W.Q. Meeker (2014) Application of Bayesian Methods in Reliability Data Analyses. *The Journal of Quality Technology*, 46, 1–23.

Meeker, W. Q., L. A. Escobar, and F. Pascual (2021), *Statistical Methods for Reliability Data*, Second Edition, John Wiley and Sons, New York.

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JMP Implementation

A two-stage algorithm:

1. Simple Rejection Algorithm
   Tried-and-true; impractical if rejection rate is high.
2. Random Walk Metropolis–Hastings Algorithm
   Efficient; fail undetectably if the likelihood is “irregular”, e.g. flat.

- When there are few or no failures, the likelihood is relatively flat. Rejection rate is not bad, and simple rejection suffices.
- When there are more failures, the likelihood becomes more “regular”. Simple rejection becomes impractical, and random walk MH is more promising to succeed.

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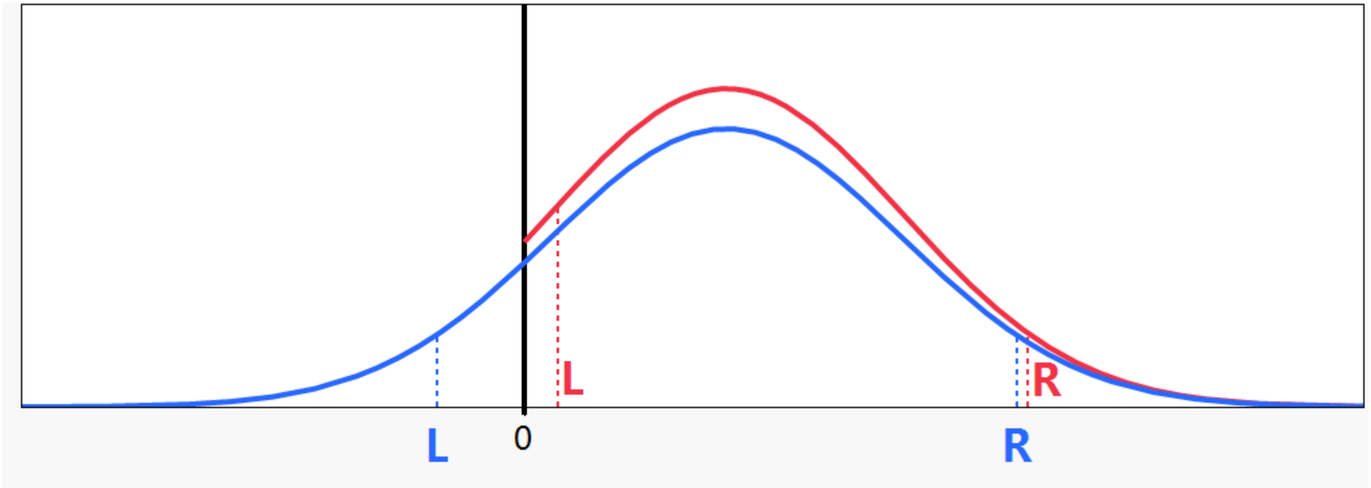
JMP Implementation

Specify truncated normal prior in JMP.

1. JMP does not have built-in truncated normal prior now.
2. An equivalent normal prior can be used for
   - Quantile, if the distribution belongs to log-location-scale family.
   - σ or Weibull β

   Because negative values will be thrown out automatically.



E.g.

TNorm< **5**, **400** >

Norm< **-75**, **399** >

Supplement script:
tnorm_config_to_norm_config.jsl

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