Determining the Right Sample Size for an MSA Study

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Outline

1. Measurement Systems Analysis
2. Previous Study
3. Current Study Design
4. Results
   - Two Factors Crossed
   - Two Factors Nested
5. Conclusions

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- **Typical Solution:** Set negative variance components to zero.
- **Some practitioners were not happy with zeroed variance components either!!**

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- We generalized Portnoy and Sahai's modified Jeffrey's Prior and implemented it in JMP's Variability platform.
- JMP's default behavior is to use REML estimates if no variance components have been zeroed and use the Bayesian estimates otherwise. We will refer to this as a **Hybrid method**.

Previous Study

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   - Automotive Industry Action Group's (AIAG) *Gauge R&R* method

EMP Classification System

- **Intraclass Correlation Coefficient (ICC)** - ρ - ratio of product variance to total variance

$$\rho = \frac{\sigma_p^2}{\sigma_p^2 + \sigma_e^2} = \frac{\sigma_p^2}{\sigma_x^2}$$

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- EMP Classifications:

Classification	$\hat{\rho}$	**Probability of Warning***
First Class	0.80 – 1.00	0.99 – 1.00
Second Class	0.50 – 0.80	0.88 – 0.99
Third Class	0.20 – 0.50	0.40 – 0.88
Fourth Class	0.00 – 0.20	0.03 – 0.40

* Probability of a warning for a $3\sigma_p$ shift within 10 subgroups using Test 1.

AIAG's Classification System

- AIAG uses Percent Gauge R&R to classify the health of a measurement system.

$$\%GRR = 100\sqrt{\frac{\hat{\sigma}_e^2}{\hat{\sigma}_p^2 + \hat{\sigma}_e^2}} = 100\frac{\hat{\sigma}_e}{\hat{\sigma}_x} = 100\sqrt{1 - \hat{\rho}}$$

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- AIAG Classifications:

Classification	$\%GRR$	$\hat{\rho}$
Acceptable	$0\% - 10\%$	$0.99 - 1.00$
Marginal	$10\% - 30\%$	$0.91 - 0.99$
Unacceptable	$30\% - 100\%$	$0.00 - 0.91$

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  - Three Factors Staggered Nested Design (highly unbalanced) with 120 measurements - Performance was very bad!

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- **AIAG classifications**:
  - Were generally pretty good for two factors crossed but very bad for two factors nested designs, especially for acceptable systems.
  - We hoped that increasing sample size would help with these classifications.
  - Worst case was 100% incorrect classification by the Bayesian method for the two factors nested design with an acceptable system.

Current Research - Sample Size

How does sample size affect our ability to estimate the variance components and classify systems with the EMP and AIAG methods?

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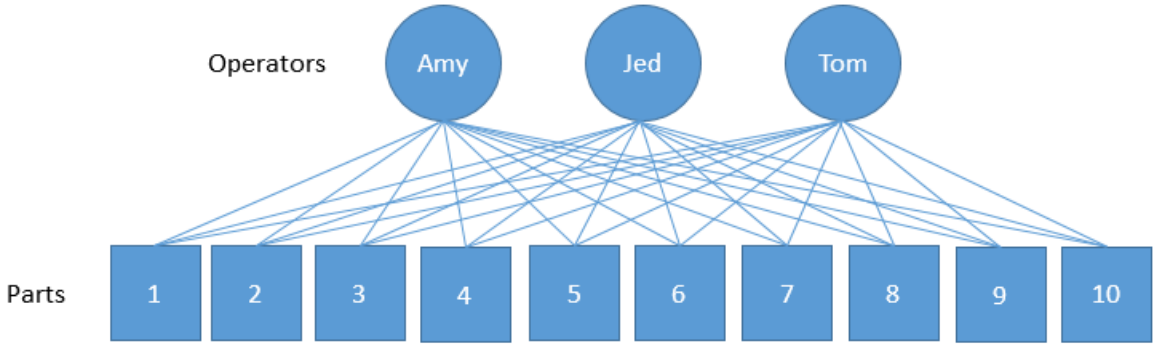
- Called the following estimation methods in the Variability platform:
  - REML
  - Bayesian (Portnoy-Sahai)
  - Hybrid (JMP default setting)
    If zeroed variance components $\Rightarrow$ *Bayesian estimates.*
    Otherwise $\Rightarrow$ *REML estimates.*

Two Factors Crossed Design



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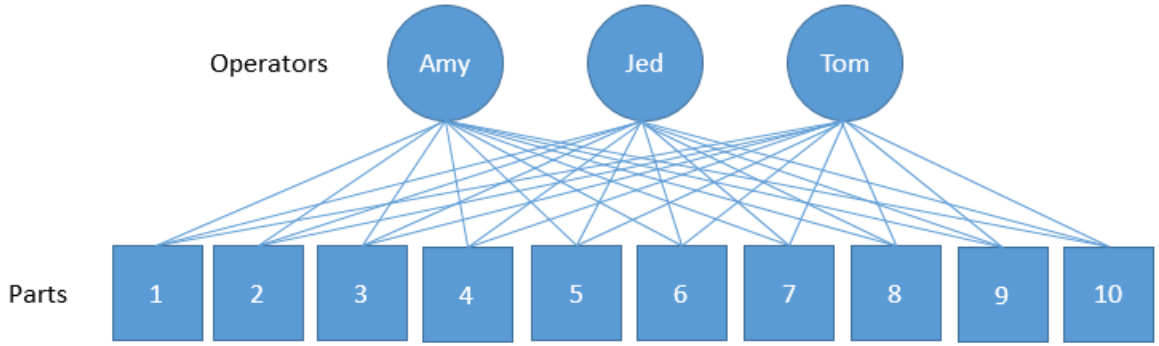

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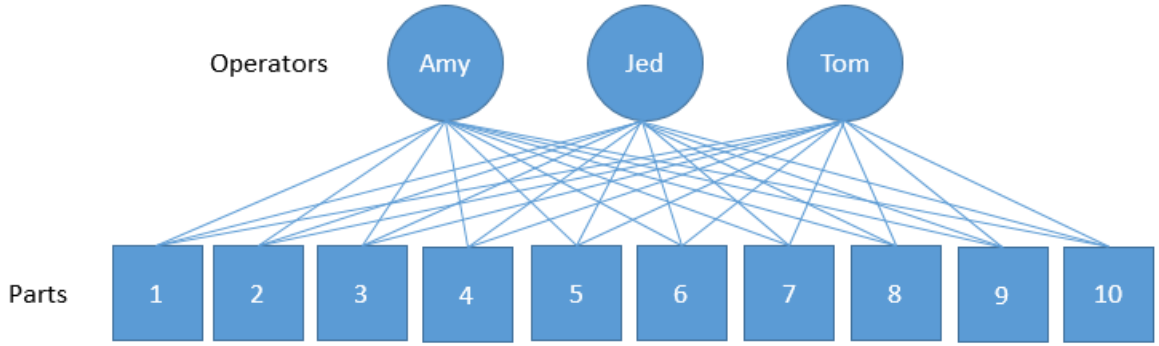

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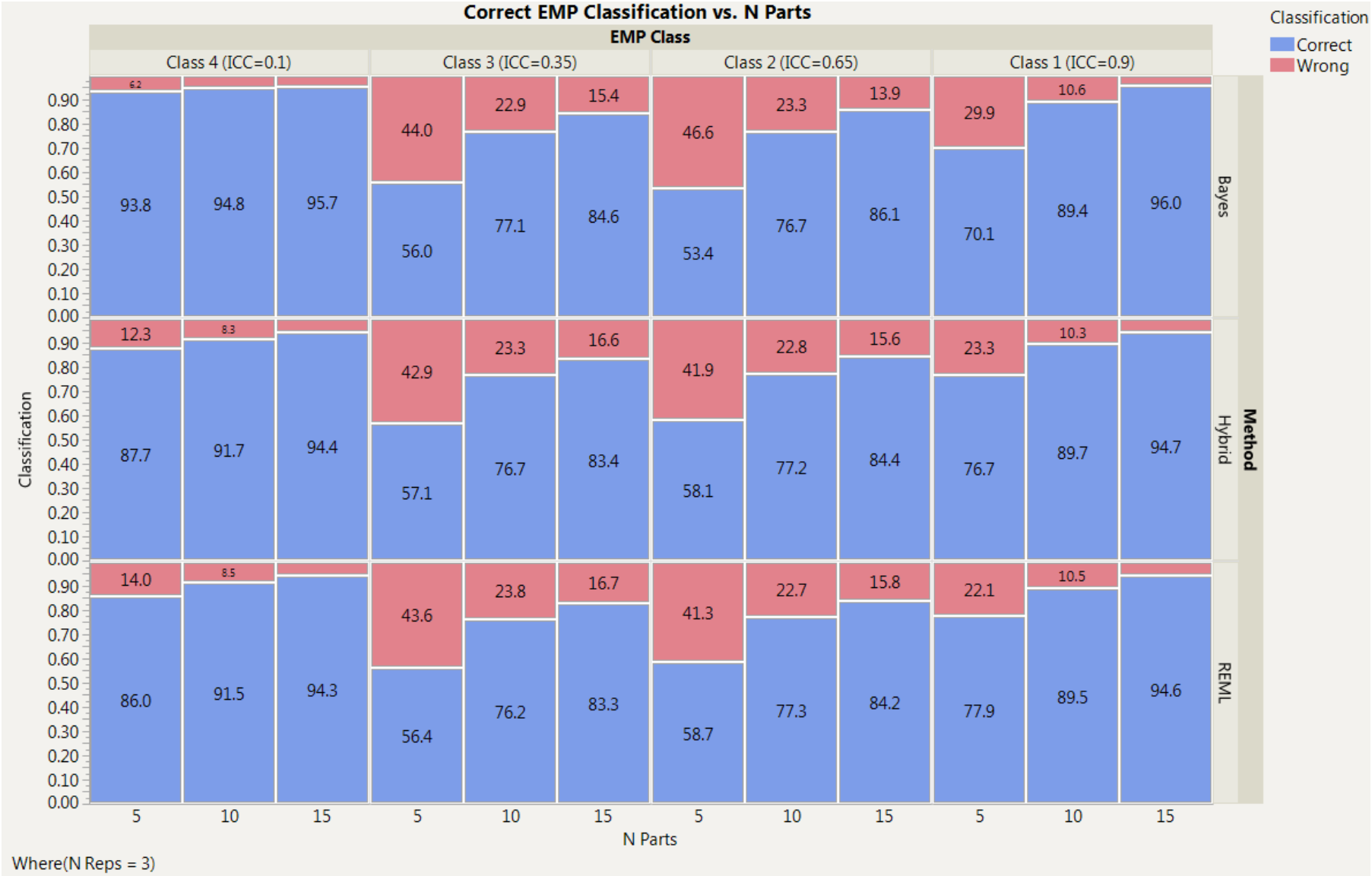

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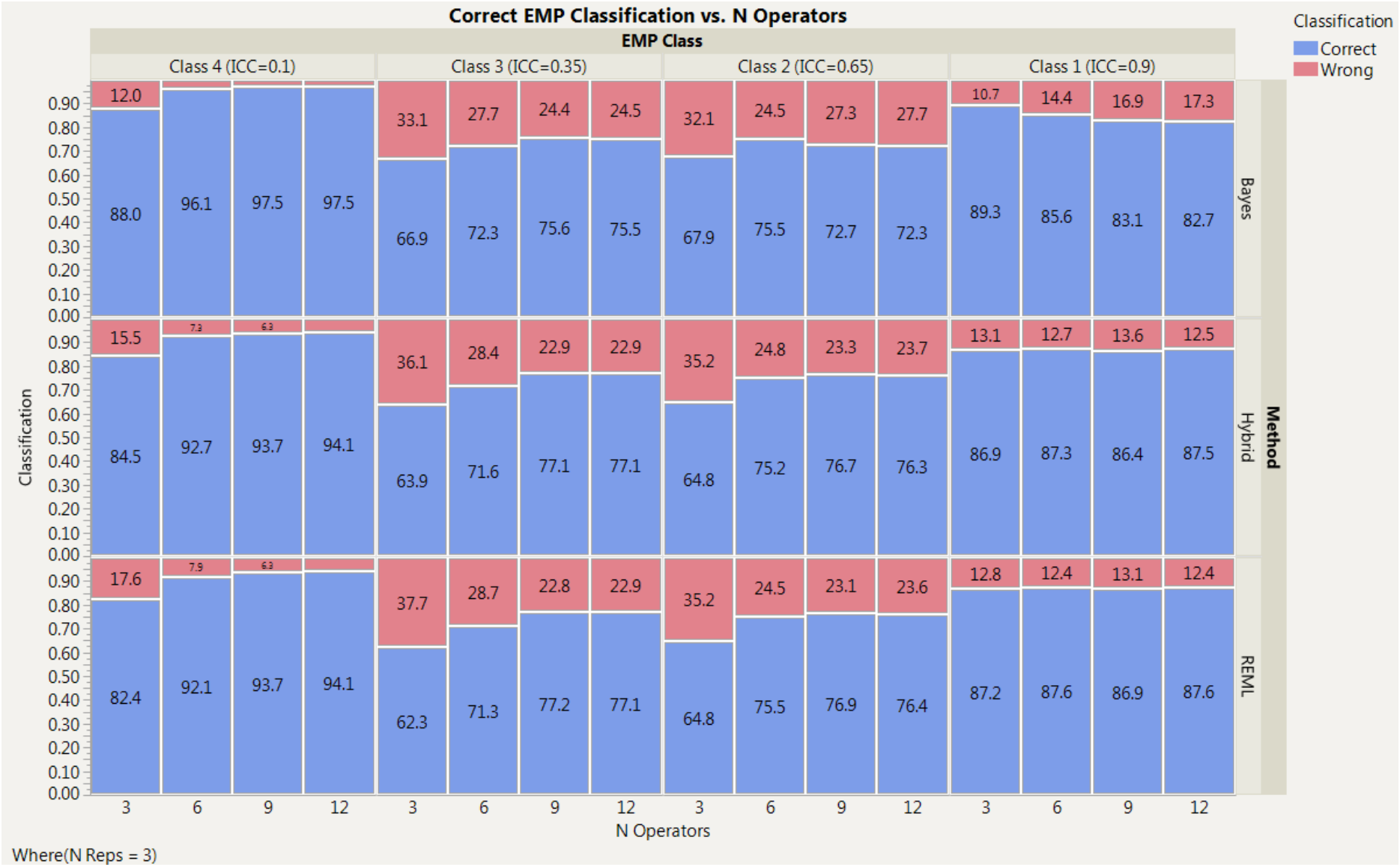

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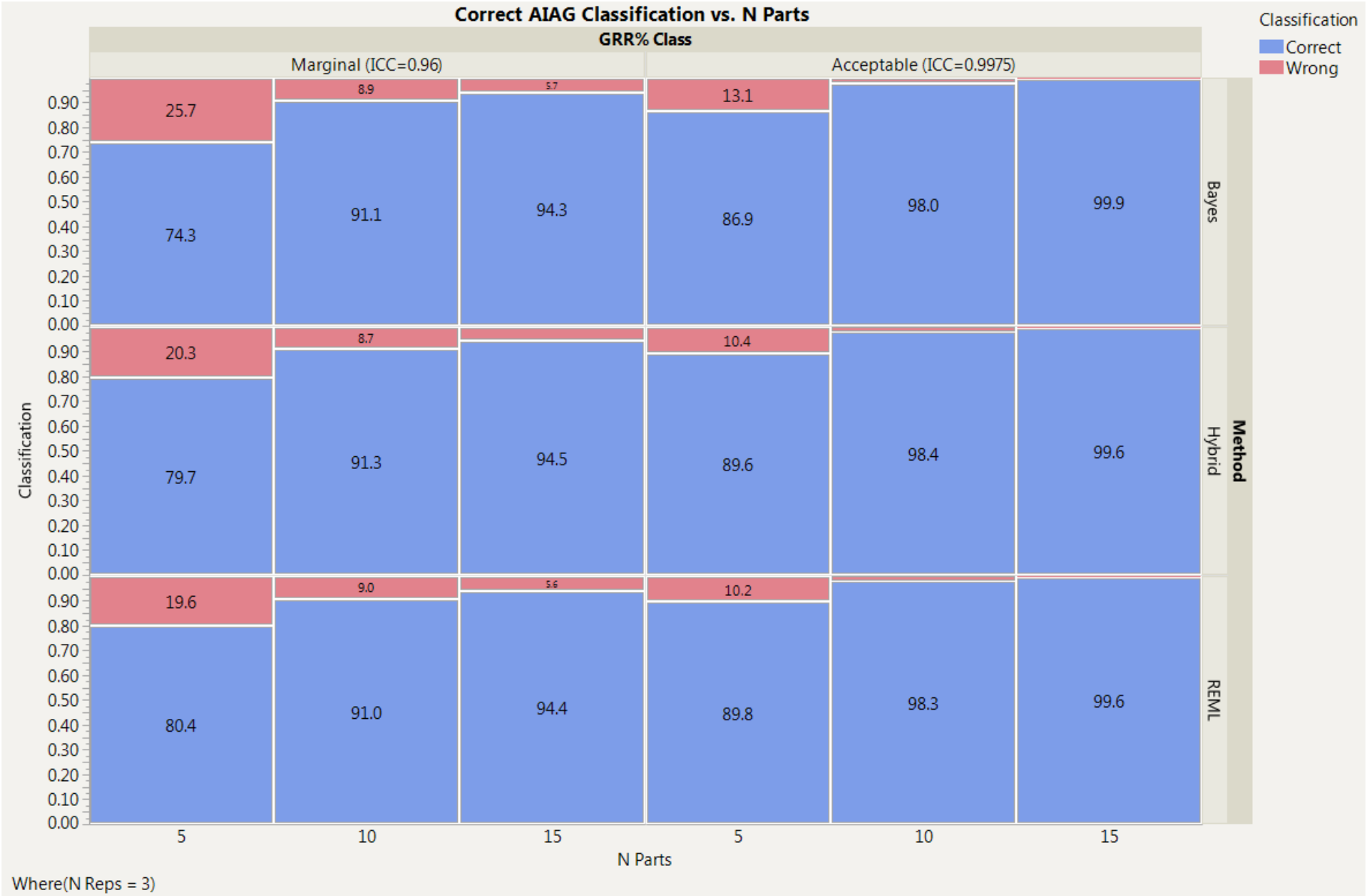

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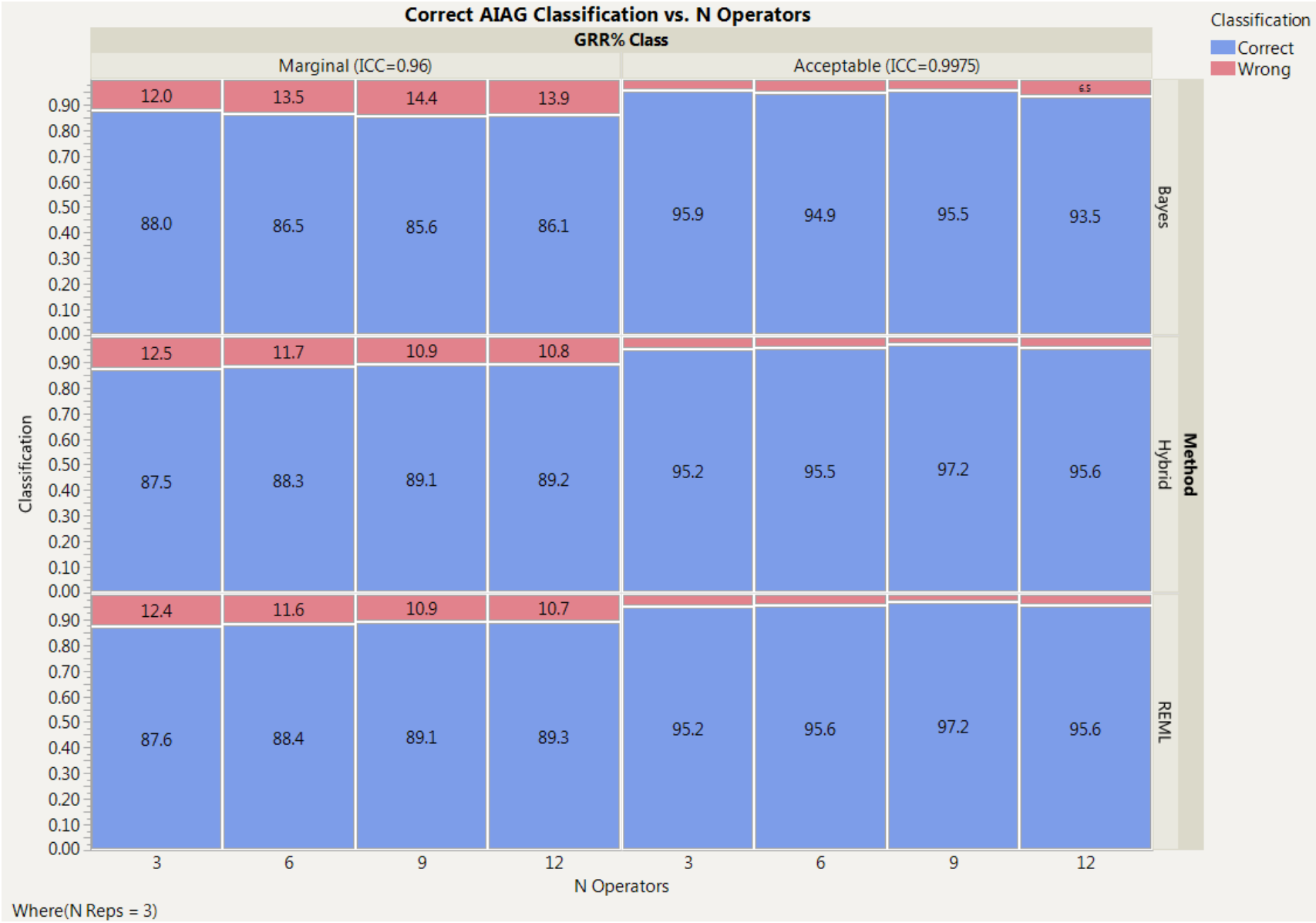

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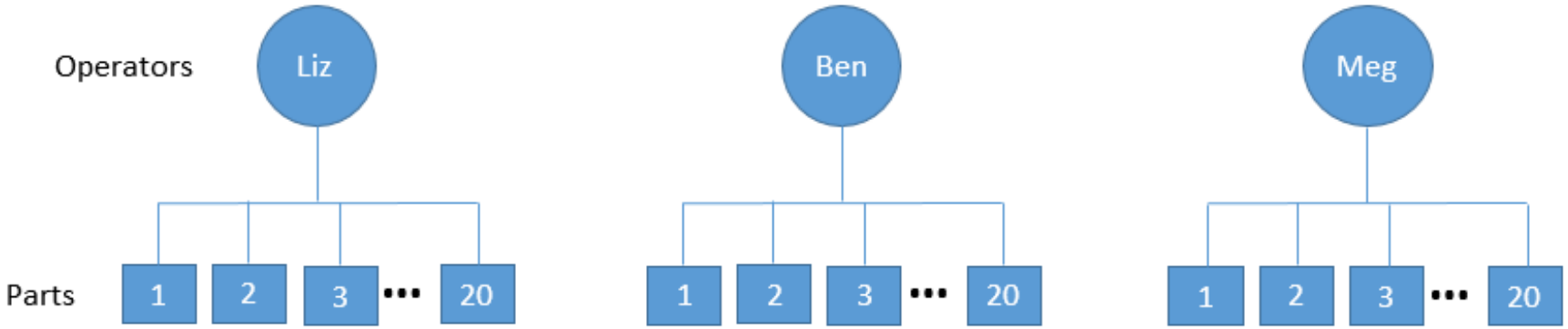

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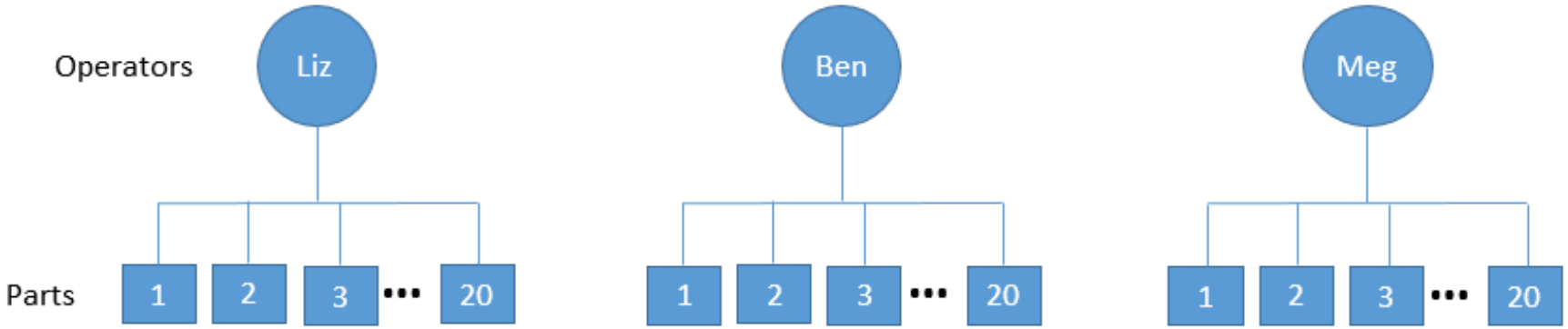

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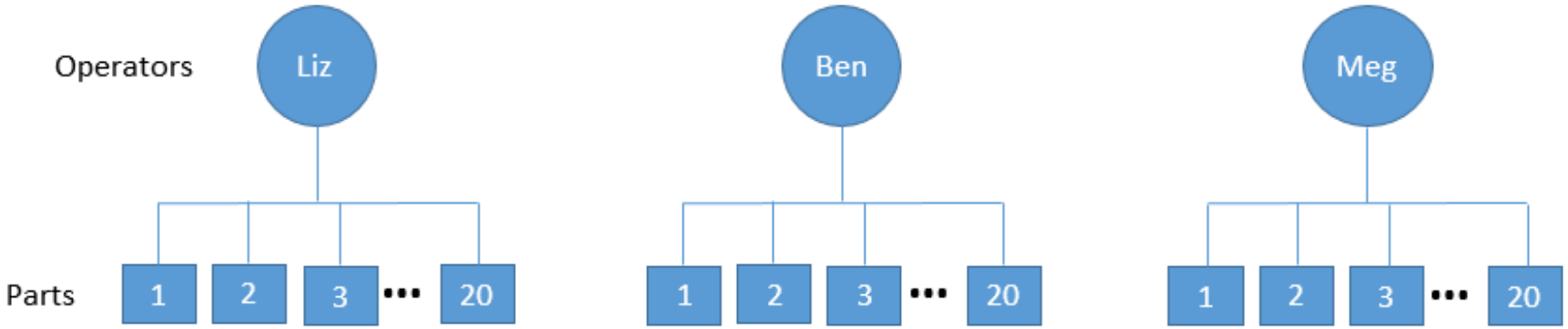

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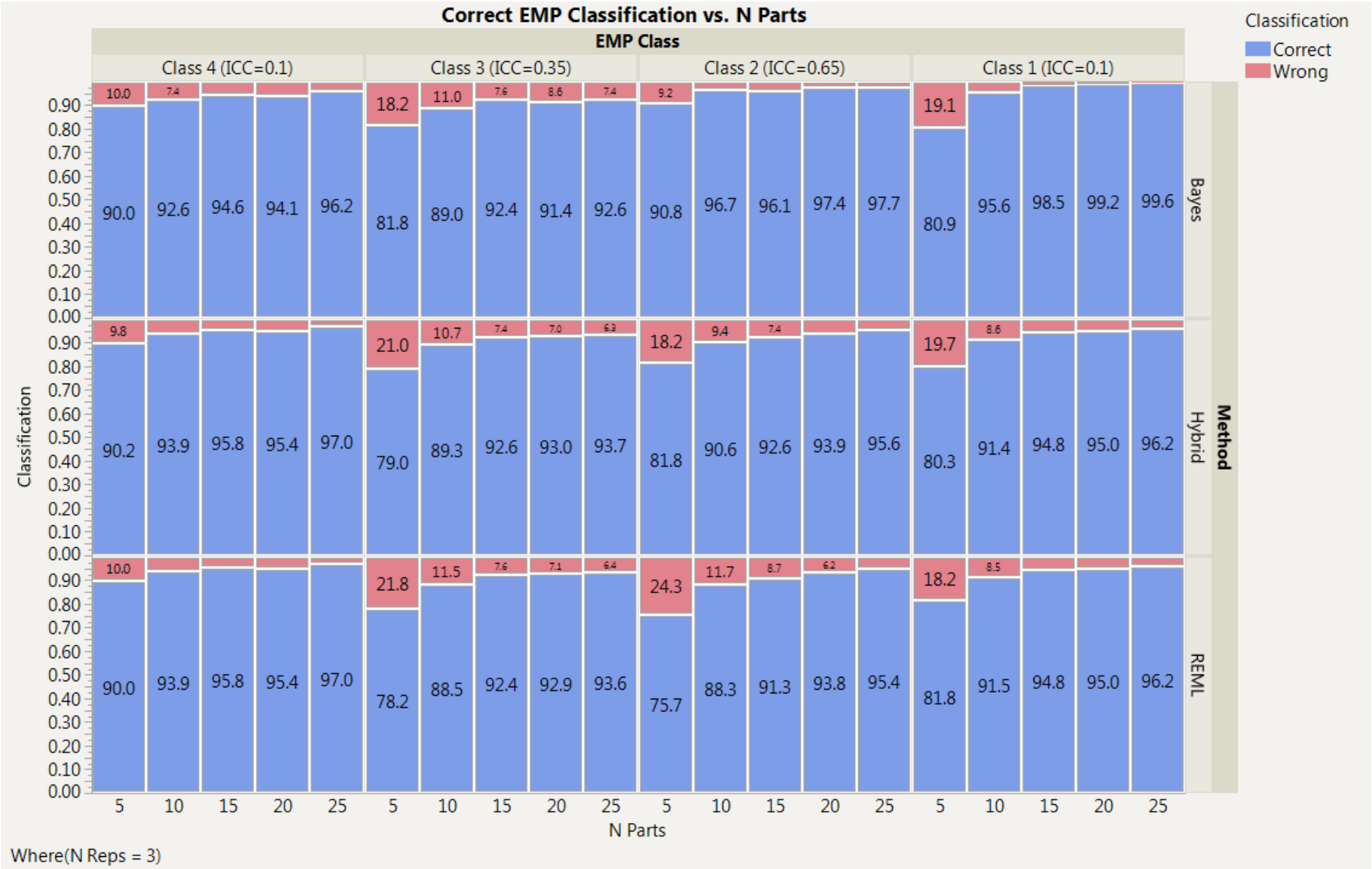

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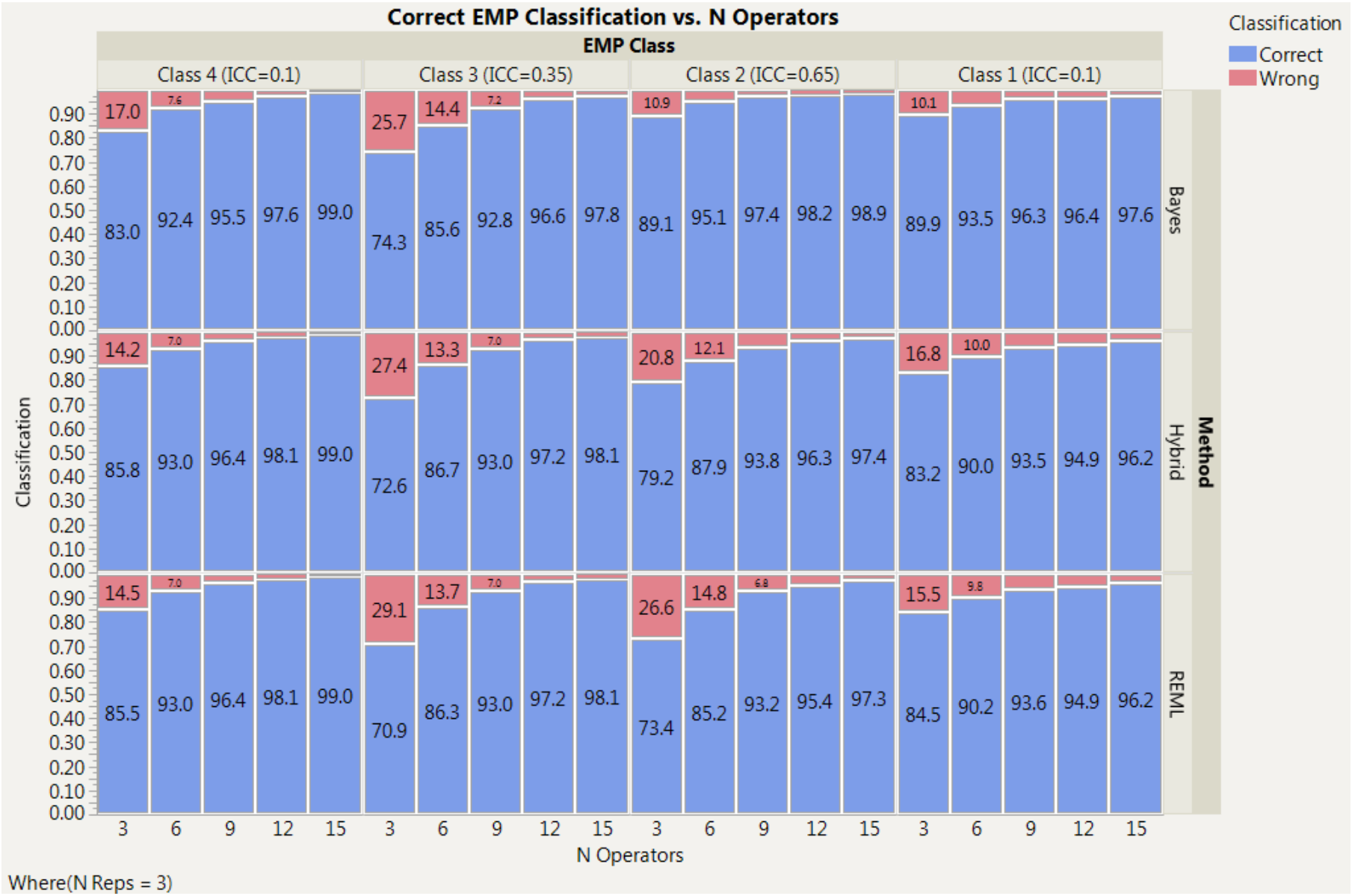

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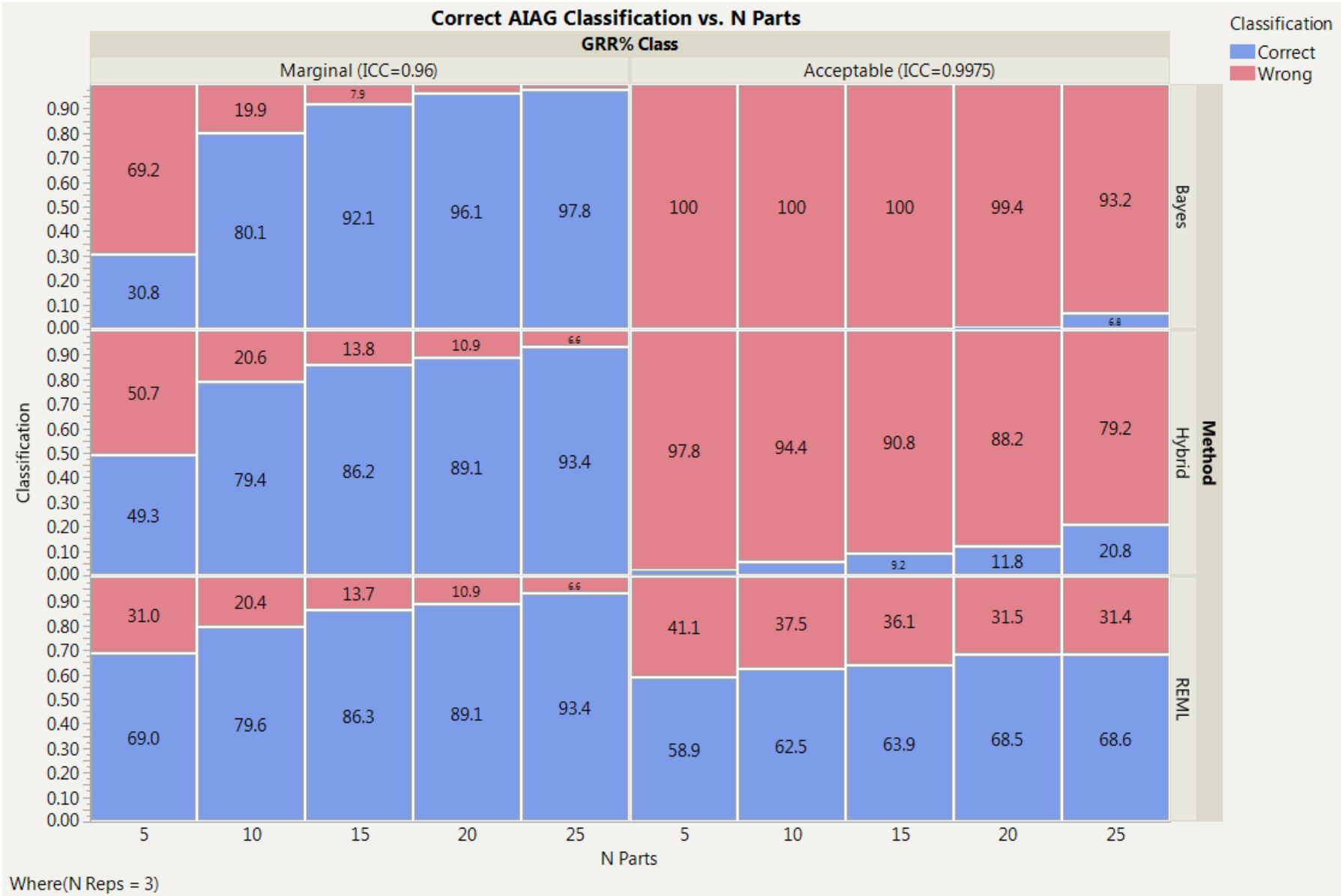

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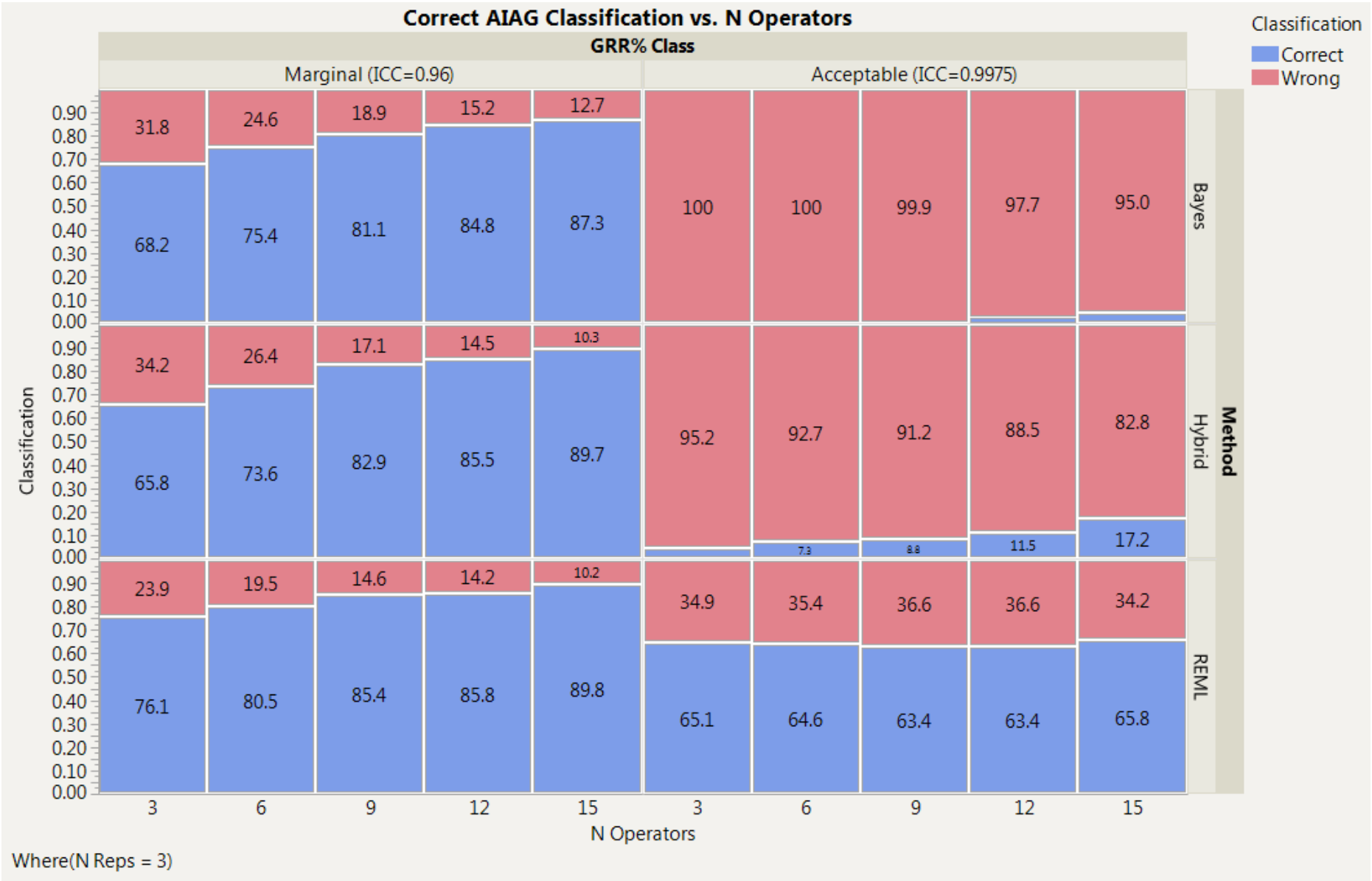
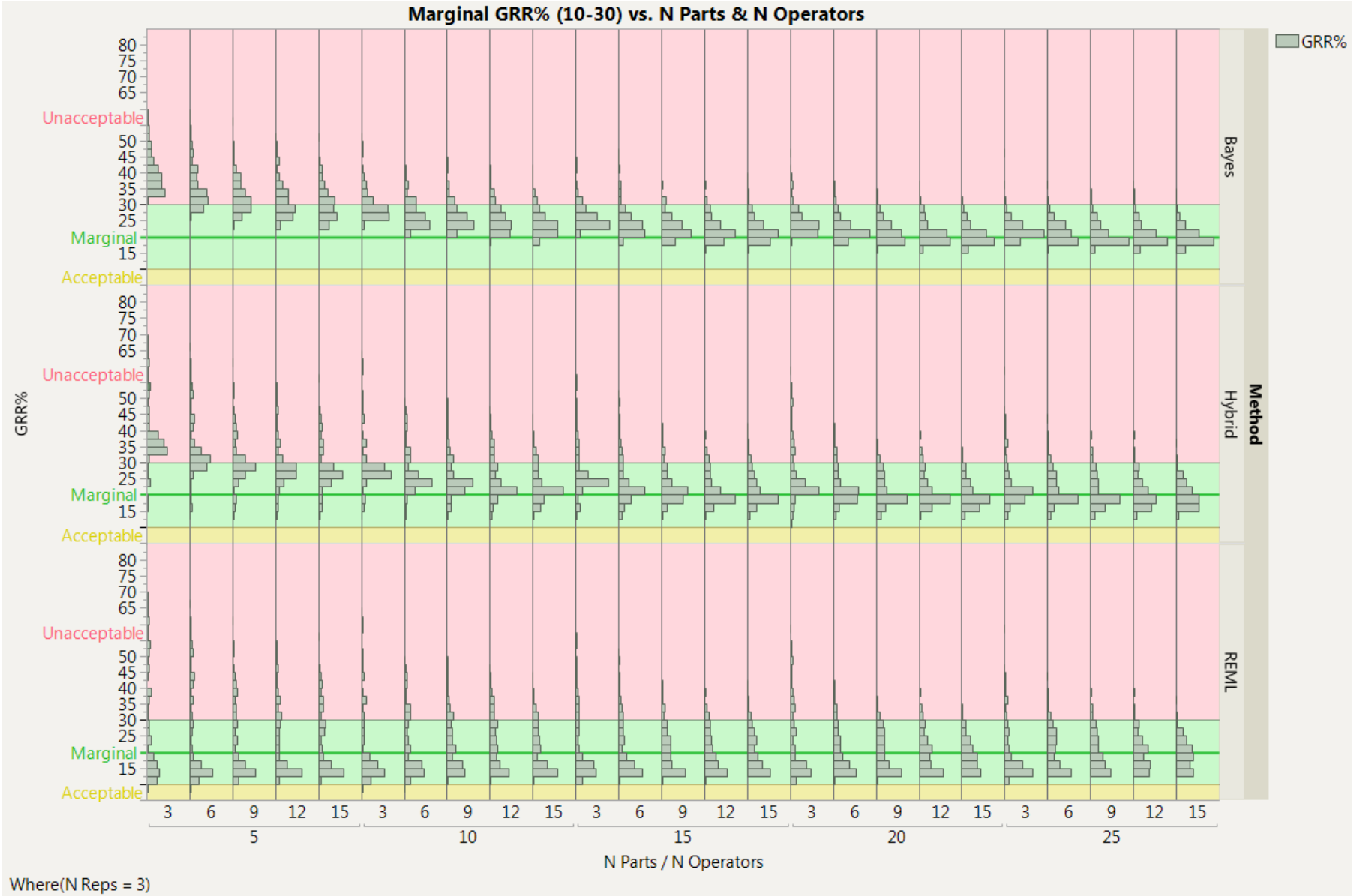
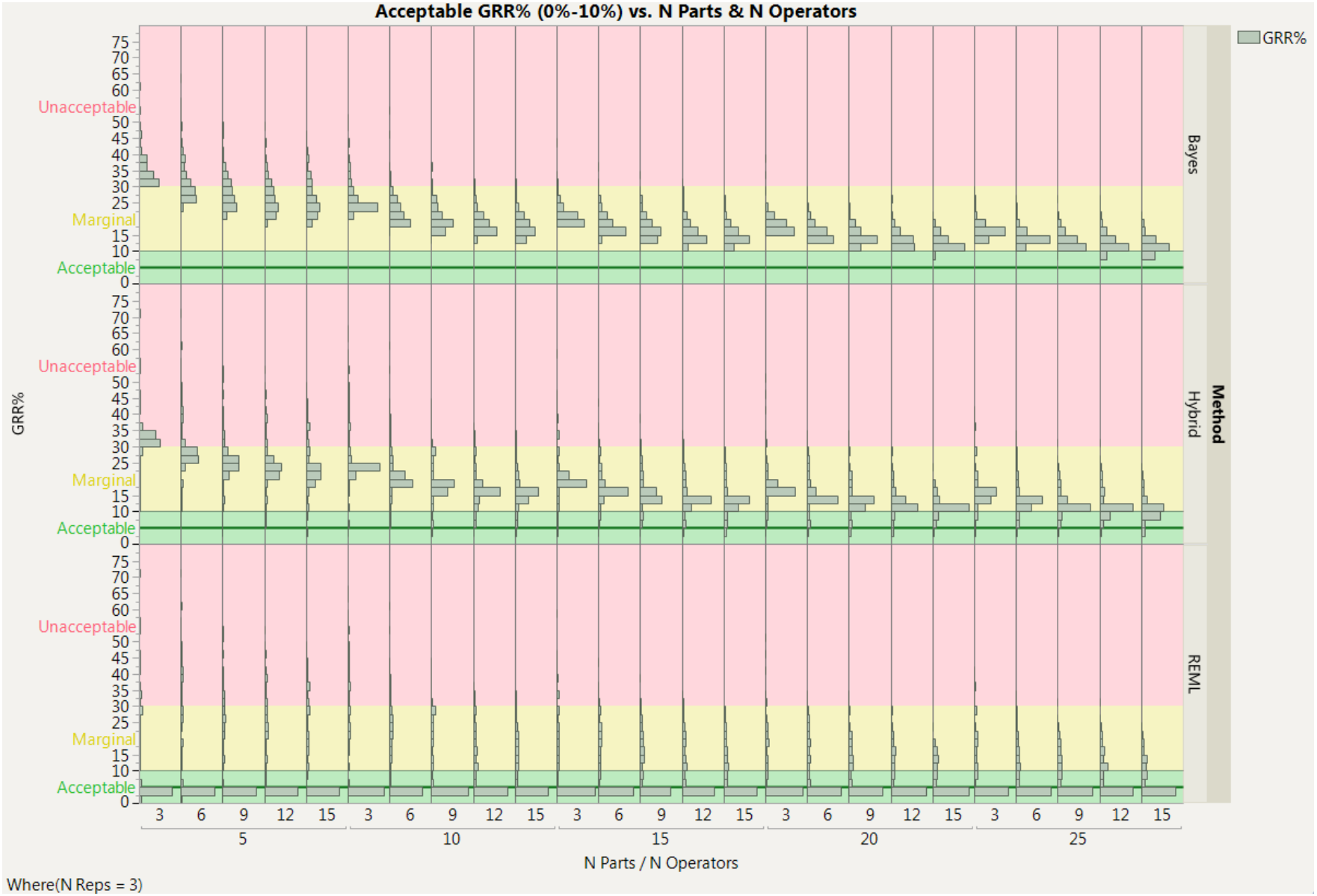


Two Factors Nested - Marginal AIAG Classifications



Two Factors Nested - Acceptable AIAG Classifications



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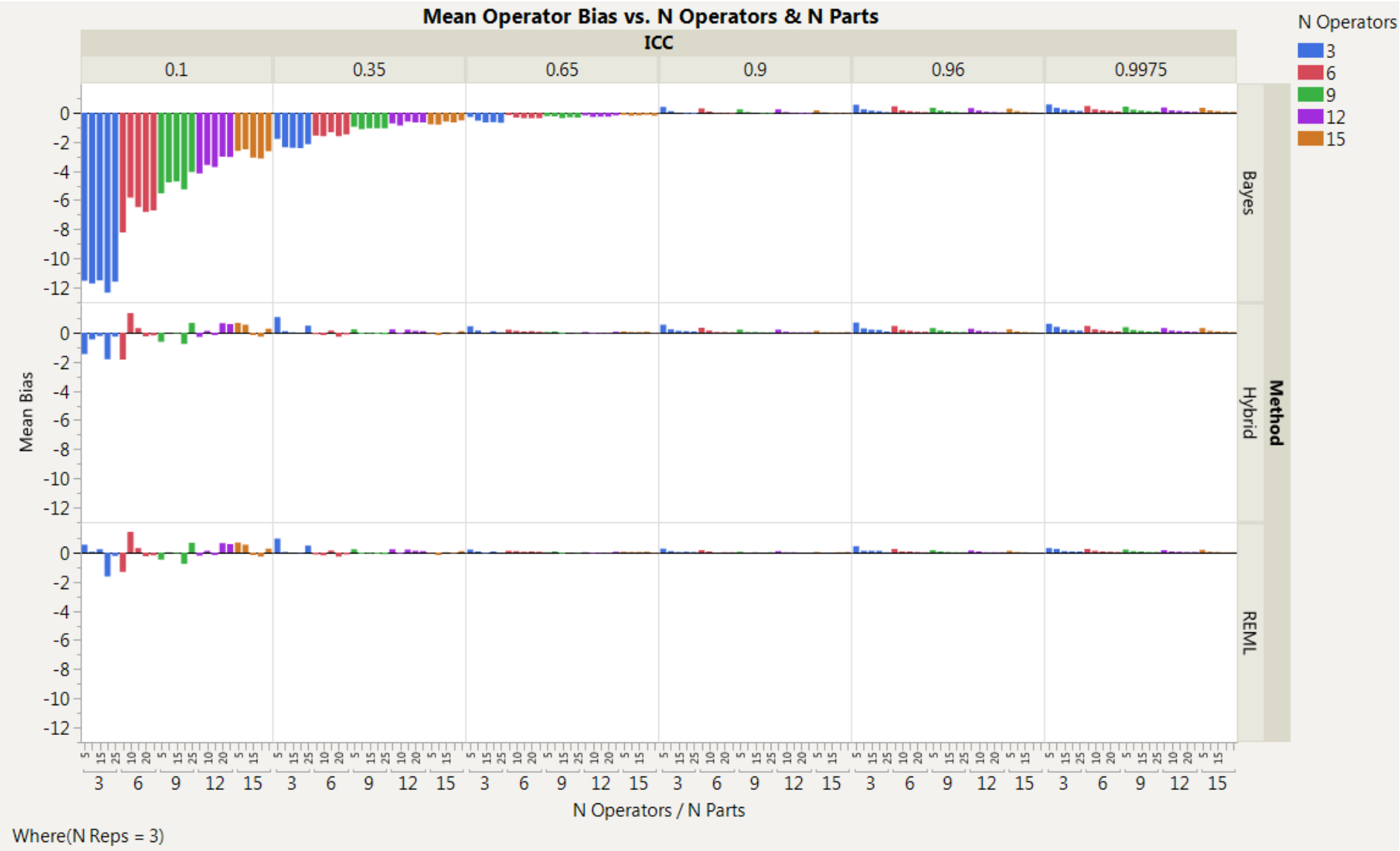

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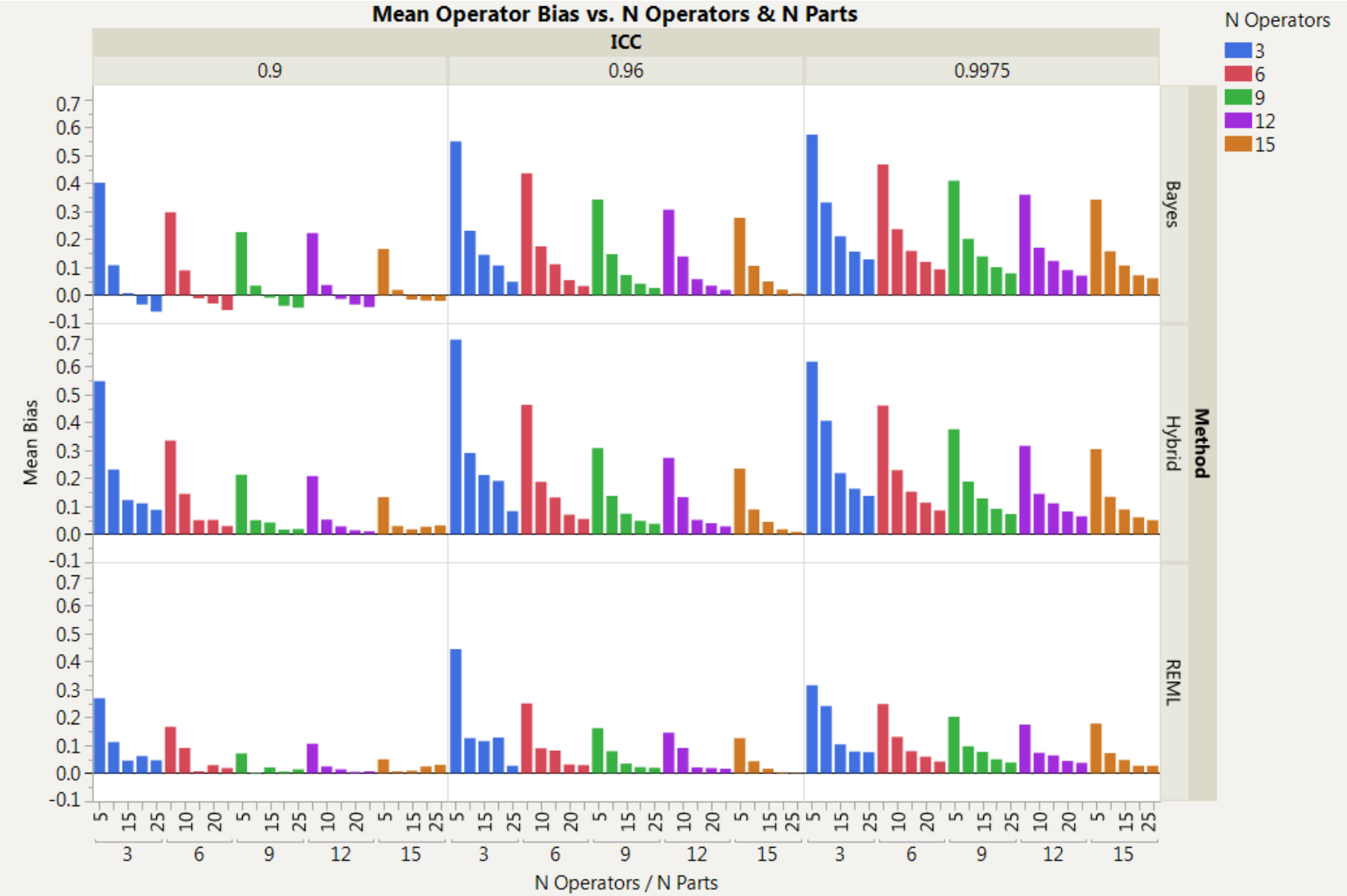
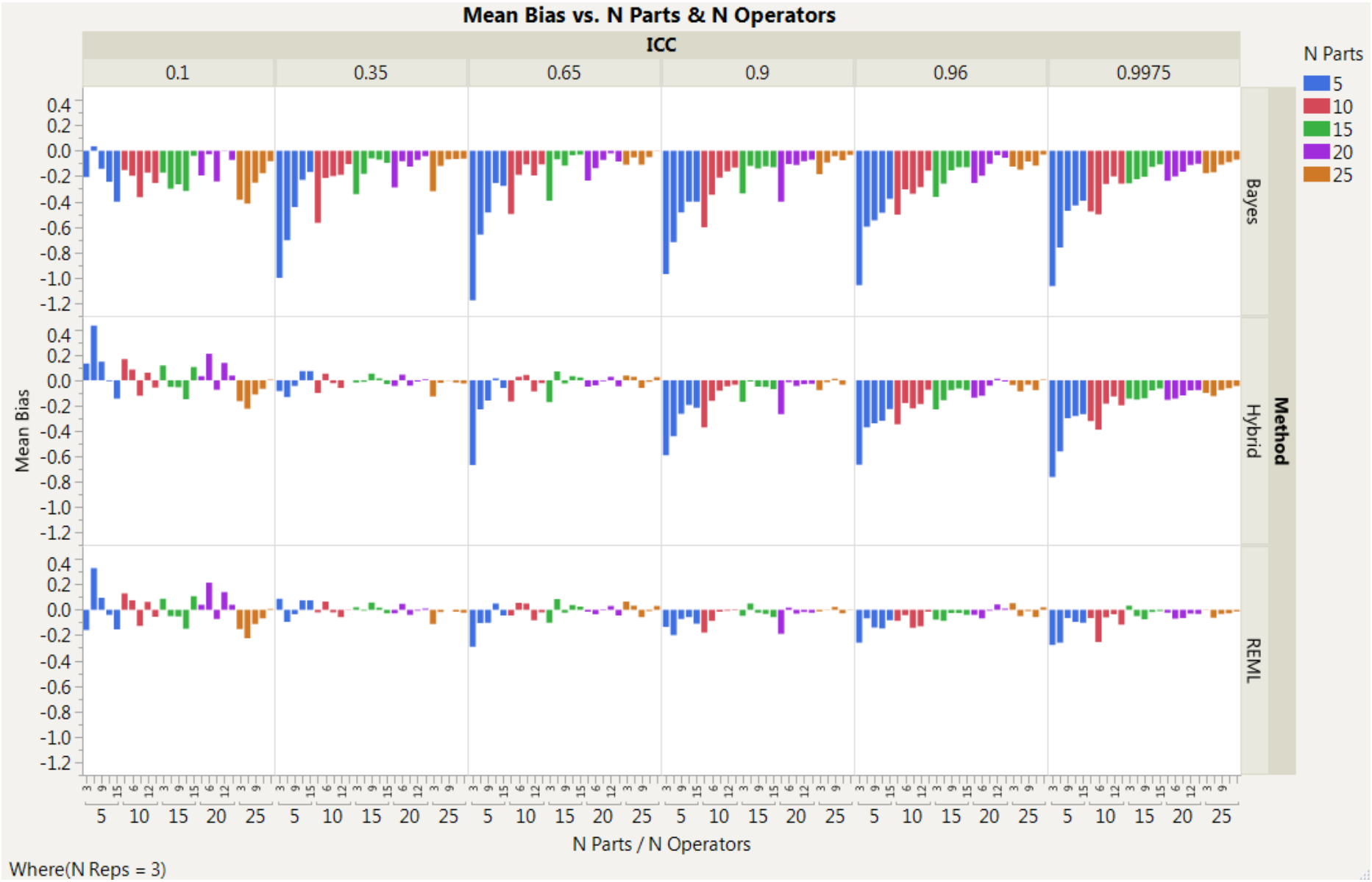


Two Factors Nested - Mean Part Variance Bias



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  - **Recommendation: Use REML, especially if you think your system is acceptable! Sample sizes with more than 3 operators and at least 10 parts are best. Caution: REML was still only correct 73.2% with 15 operators and 25 parts.**

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- Try different breakdowns of error variance.

References

- Automotive Industry Action Group (2002), *Measurement Systems Analysis Reference Manual, 3rd Edition*.
- Portnoy (1971), “Formal Bayes Estimation With Application To a Random Effect Model,” *Annals Of Mathematical Statistics*, 42, 1379-1402.
- Sahai (1974), “Some Formal Bayes Estimators of Variance Components in Balanced Three-Stage Nested Random Effects Model,” *Communications in Statistics*, 3, 233-242.

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