Reduction of Sampling Inspection Using Variables Data in the Presence of Batch-to-Batch Variability

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Attributes Sampling

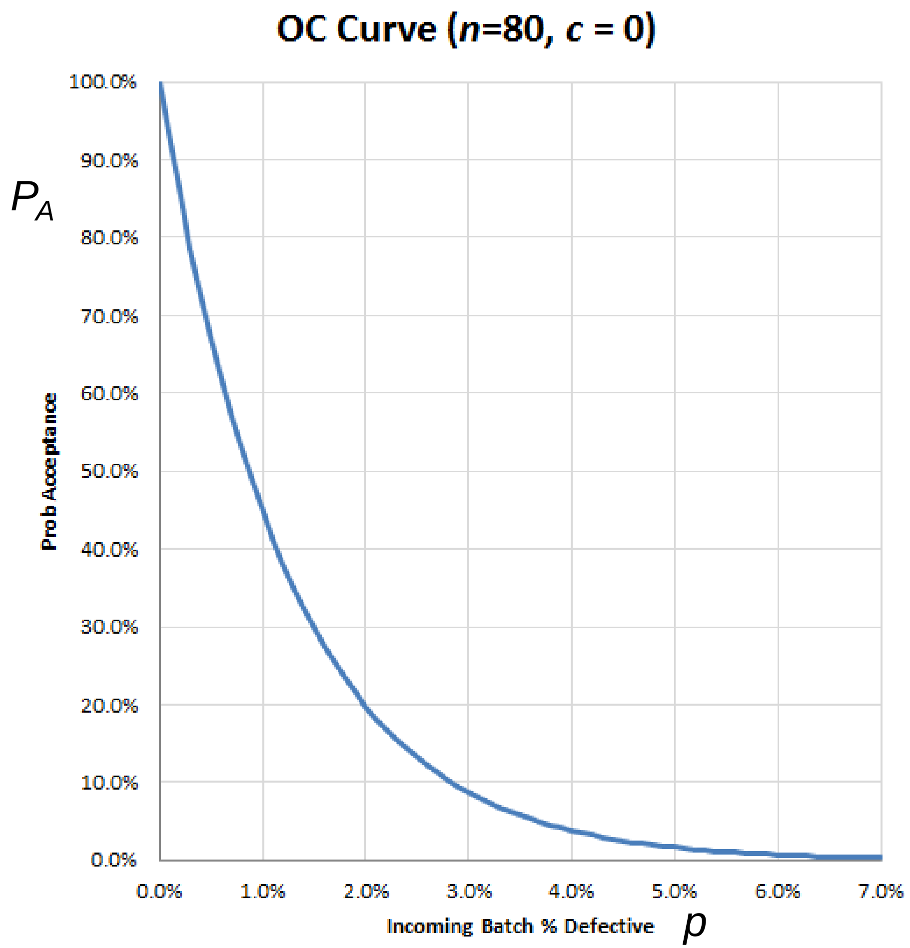
- In attributes sampling, a specified sample size is randomly selected from a batch, and the CTQs (Critical To Quality) parameters are measured and compared to specification limits. Samples not meeting these limits are counted as defective.
- Incoming Quality Control (IQC) had been inspecting batches using attributes sampling plans, measuring many CTQs on typically n = 80 parts. The batch would be accepted for c = 0 defectives and rejected for c = 1 or more defectives.
- This sampling plan corresponds to acceptable quality levels (AQL's) and rejectable quality levels (RQL's) of the proportion defective shown on the next slide.

Operating Characteristic (OC) Curve for IQC Sampling Plan

Acceptance Probabilities P_A

$n = 80$, $c = 0$

- P_A = 95% (AQL): p_A = 0.064%
- P_A = 50% (Mid): p_M = 0.86%
- P_A = 10% (RQL): p_R = 2.84%

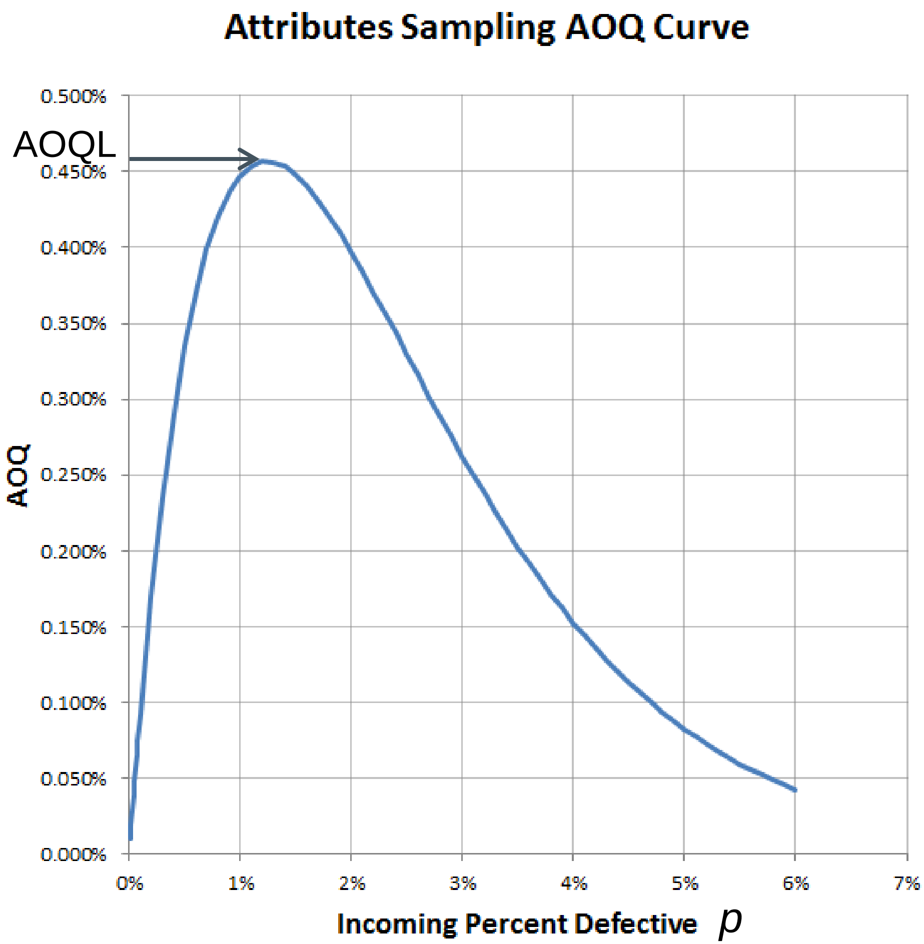


AOQL Curve for IQC Sampling Plans

The average outgoing quality (AOQ) is ~$P_A \cdot p$

Assuming 100% inspection of rejected lots, the average outgoing quality limit (AOQL) is:

N=80: 0.46%



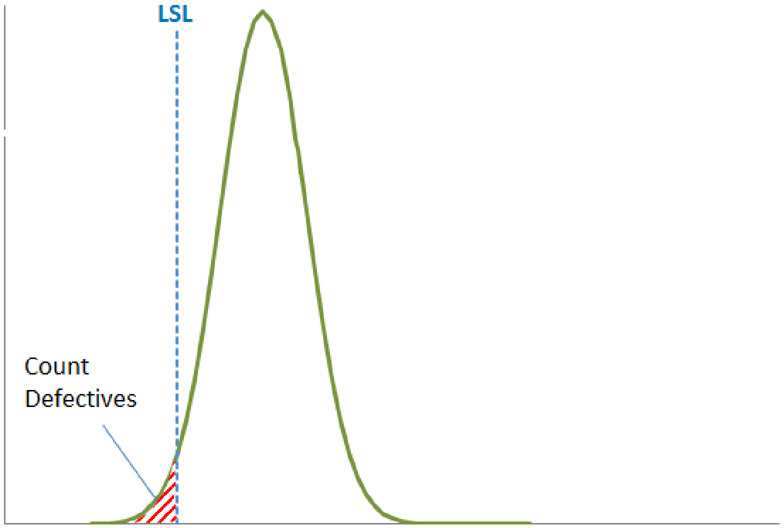
Proposal

The proposal was to employ ***variables*** sampling plans that would result in a significant reduction in the sample size inspection requirements compared to the attributes sampling plans while providing equivalent protection against bad batches.

Attributes Vs. Variables Sampling Plans

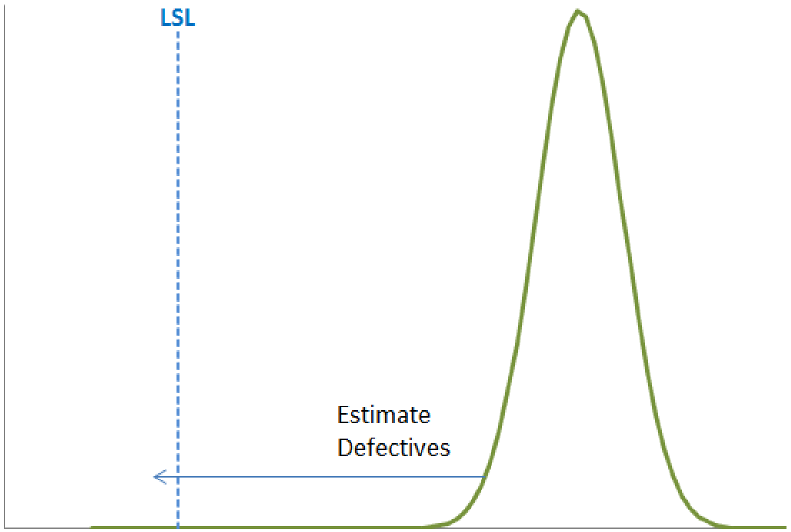
Actual measurements of a continuous CTQ parameter provide more information than simple counts of defective parts. From the mean and σ estimates of the assumed distribution, we can estimate the % defective relative to the specification limit.

Large Sample



Attributes Plans: Defective Counts

Small Sample



Variables Plans for Proportion Defective

Variables Sampling Plans

- It is possible to obtain equivalent or better discrimination power with smaller sample sizes when using variables data than when using attributes (count) data (pass/fail, go/no go, in-spec/out-of-spec, etc.).

- Variables sampling plans require knowledge of the type of distribution (generally assumed normal) and the standard deviation, which can be estimated from a specified number (e.g., 20) of the most recent lots.

Acceptance Sampling by Variables

- **Variables Sampling Plans:** For each CTQ, based on the estimated (and assumed known) standard deviation, the specifications limits, the AQL and RQL proportions defective and associated α (producer's) and β (consumer's) risks, we determine a **sample size *n*** and an **acceptance mean *K***.
- **Single-Sided Limits:** For a **LSL**, if the average of the *n* units is equal to or above *K*, the CTQ passes. For an **USL**, if the average of the *n* units is equal to or below *K*, the CTQ passes. If the average of the sample mean is below *K* for a LSL or above *K* for an USL, the batch fails for this CTQ.

Acceptance Sampling by Variables

- **Two-Sided Limits:** If the average of the *n* units is between a lower mean K_L and an upper mean K_U, the CTQ passes. If the average of the *n* units is outside the limits [K_U , K_L], the CTQ fails.

- If the batch fails for any CTQ based on variables sampling, one option is to apply the associated attributes sampling plan before choosing to screen the rejected batch.

Steps in Variables Sampling: Single Sided LSL

One Way Protection on LSL, σ Known

1. Estimate **pooled** standard deviation from 20 recent lots.
2. Determine the **desirable AQL mean** μ_A corresponding to a specified **acceptable proportion** defective level p_A. We will use the p_A of the current attributes plan $p_A = 0.064\%$.
3. Determine the **undesirable RQL mean** μ_R corresponding to a specified **rejectable proportion** defective level p_R. We will use the p_R of the current attributes plan $p_R = 2.84\%$.

Steps in Variables Sampling: Single Sided LSL

One Way Protection on LSL, σ Known

4. Specify the risk α (producer's risk) of rejecting a batch if the true mean is the desirable mean. Typically α = 5%.
5. Specify the risk β (consumer's risk) of accepting a batch if the true mean is the undesirable mean. Typically β = 10%.

Steps in Variables Sampling: Single Sided LSL

One Way Protection on LSL, σ Known

6. Determine the sample size n from the formula[1] (rounding up to integer)

$$n = \left(\frac{z_\alpha + z_\beta}{z_R - z_A} \right)^2$$

7. Determine K by the formula[1]

$$K = LSL + \left(\frac{z_\alpha z_R + z_\beta z_A}{z_\alpha + z_\beta} \right) \sigma$$

8. Plan: If the mean of n items $\bar{y} < K$, reject the batch. If the mean of n items $\bar{y} \geq K$, accept the batch.

9. If the batch is rejected, then the original attributes sampling plan can be used.

Measurement Error Analysis

- It is important to periodically verify that a **measurement system analysis** (gauge R&R) has been performed on the tools used to measure CTQs.
- The measurement error should be small enough to detect part to part variation.
- A measurement system analysis (**MSA**) can be performed in JMP based on the **EMP** (Evaluating the Measurement Process) analysis approach of Don Wheeler[2].

Example X-Dim: JMP MSA

- A Measurement System Analysis was done and showed that the measurement system was First Class.

System	Classification
Current (with bias)	First Class
Current (with bias and interactions)	First Class
Potential (no bias)	First Class

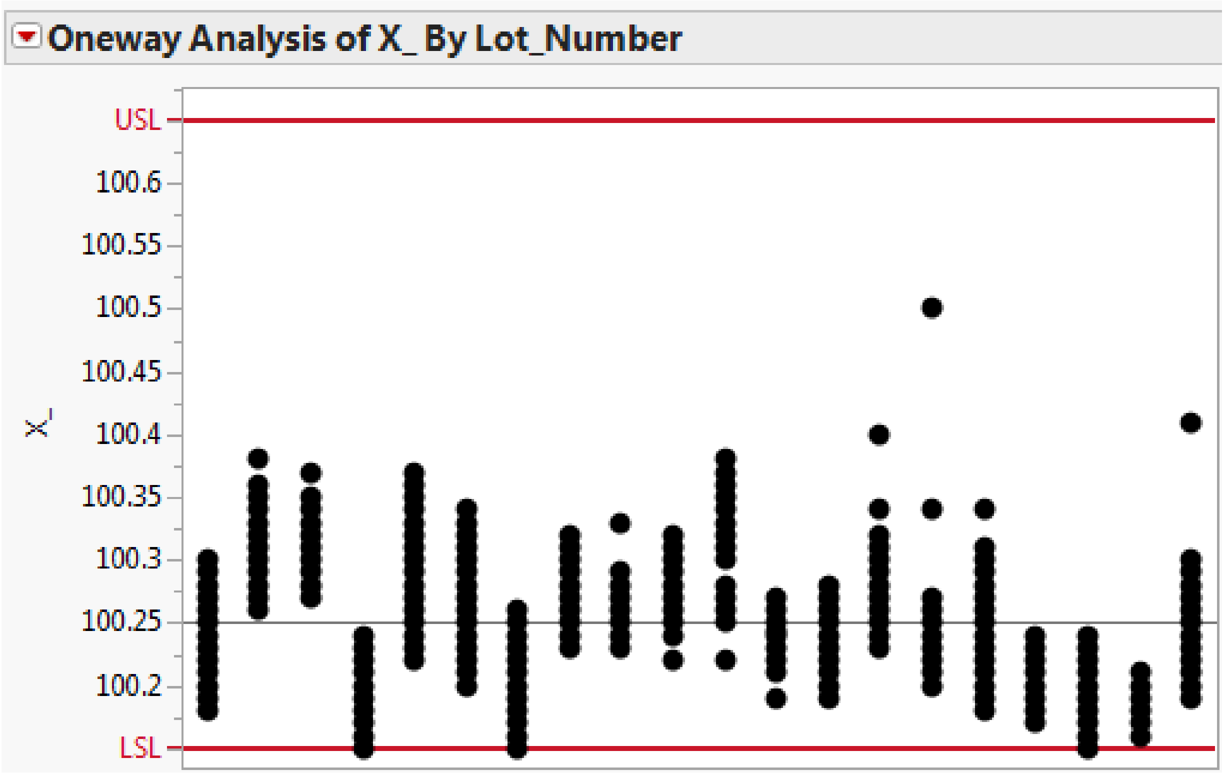
- The Gauge R&R showed that 99.3% of the total variation was product variation, with only ~0.7% of variation attributable to Gauge R&R.

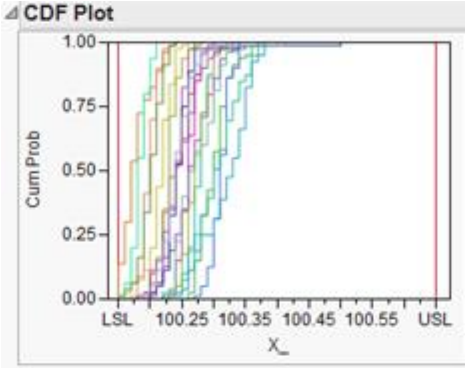
EMP Gauge R&R Results

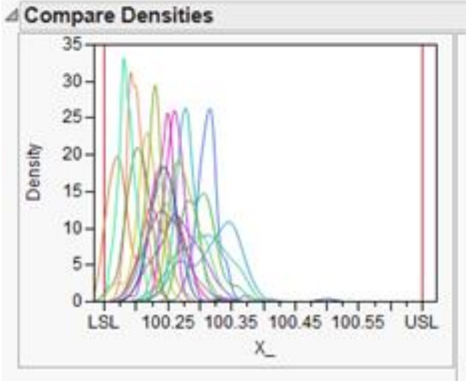
Component	Std Dev	Variance Component	% of Total	20 40 60 80
Gauge R&R	0.01259871	0.00015873	0.6867	
Repeatability	0.01156796	0.00013382	0.5789	
Reproducibility	0.00499097	0.00002491	0.1078	
Product Variation	0.15151001	0.02295528	99.3	
Interaction Variation	0.00000000	0.00000000	0.0	
Total Variation	0.15203292	0.02311401	100.0	

Example X-Dim: JMP Fit Y by X Report

Data is shown for 20 recent IQC batches. 80 samples were measured per batch. Four batches are close to LSL = 100.15.

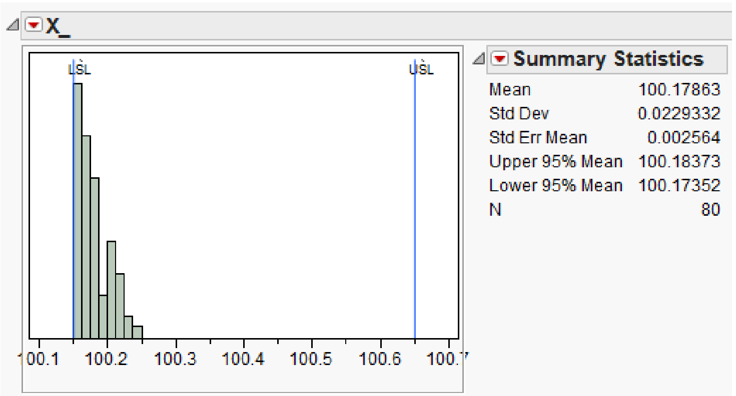


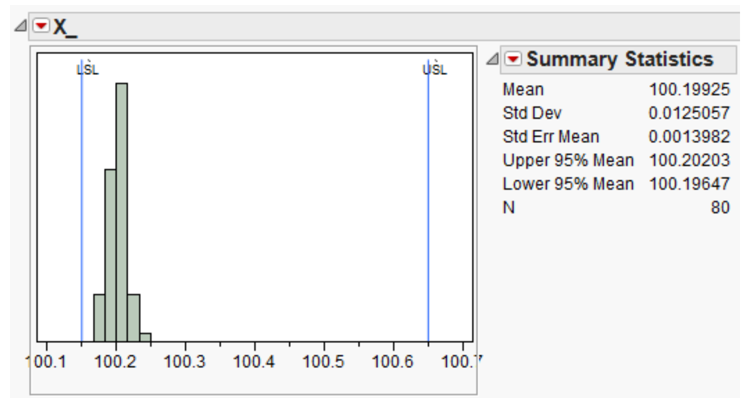


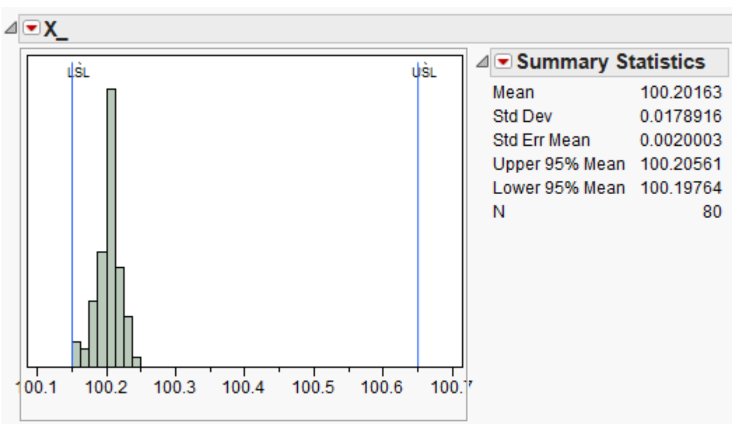


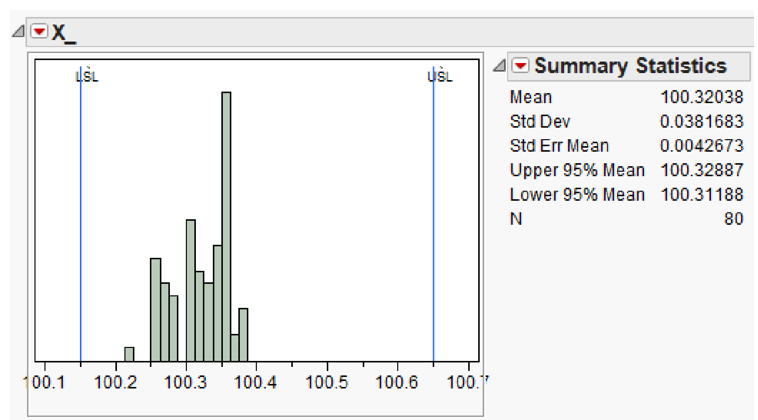
Example X-Dim: JMP Distribution Analysis

Recent 20 Batches: We observe that three X-dim batches closest to the LSL have standard deviations that are smaller than batches furthest from the LSL, indicating possible bias (truncation?) in the measurements.









Example X-Dim: Estimating Sigma

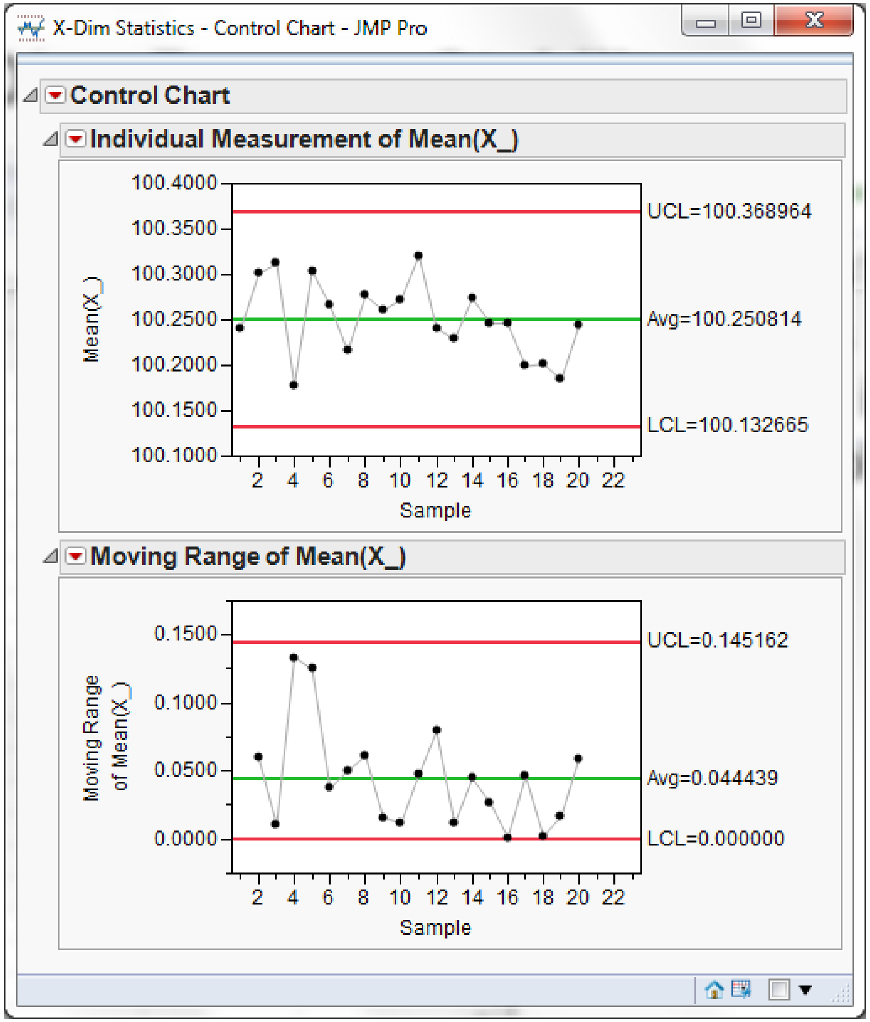
To estimated sigma, we use the data from the most recent 20 batches. However, we need to verify that the batch-to-batch means reflect a stable process, with no out-of-control signals in the **individuals** (IR) control chart of batch means. Then, we can estimate a pooled σ_p sigma from the individual batch sigmas σ_i using the formula:

$$\hat{\sigma}_p = \sqrt{\frac{\hat{\sigma}_1^2 + \hat{\sigma}_2^2 + \cdots \hat{\sigma}_{20}^2}{20}}$$

Example X-Dim: Batch-to Batch Process Stability Using JMP IR Control Charts

Individuals control chart plots of the last 20 process ***means*** show no out-of-control signals.

The X-dimension sigma (pooled standard deviation) estimated from these 20 IQC batches (n = 80) is 0.0252.

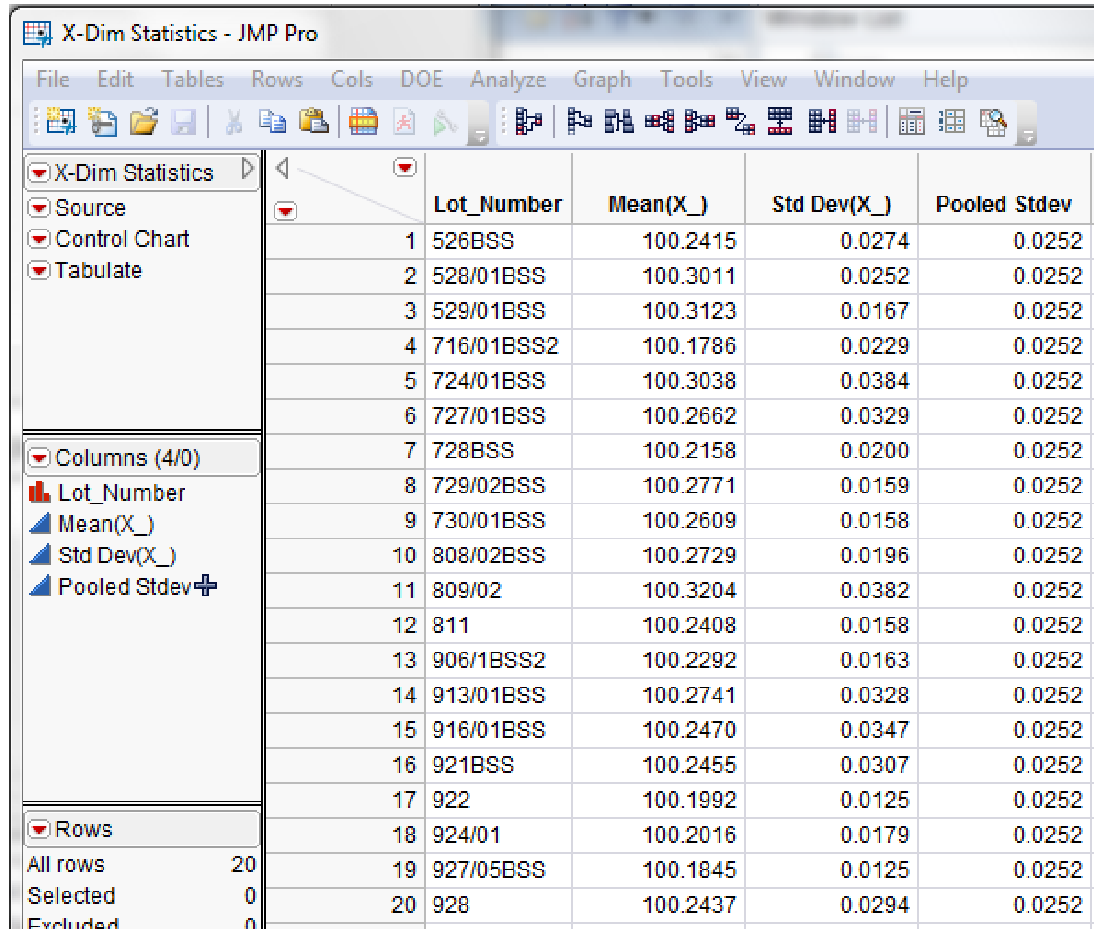


Example X-Dim: Estimating Sigma in JMP

The X-dimension sigma (pooled standard deviation) determined from the most recent 20 IQC lots (n=80) is 0.0252.

The JMP formula used for the Pooled Stdev column is

$$\sqrt[2]{\frac{\text{Col Sum}\left(\mathit{Std\ Dev(X\_)}^{2}\right)}{20}}$$



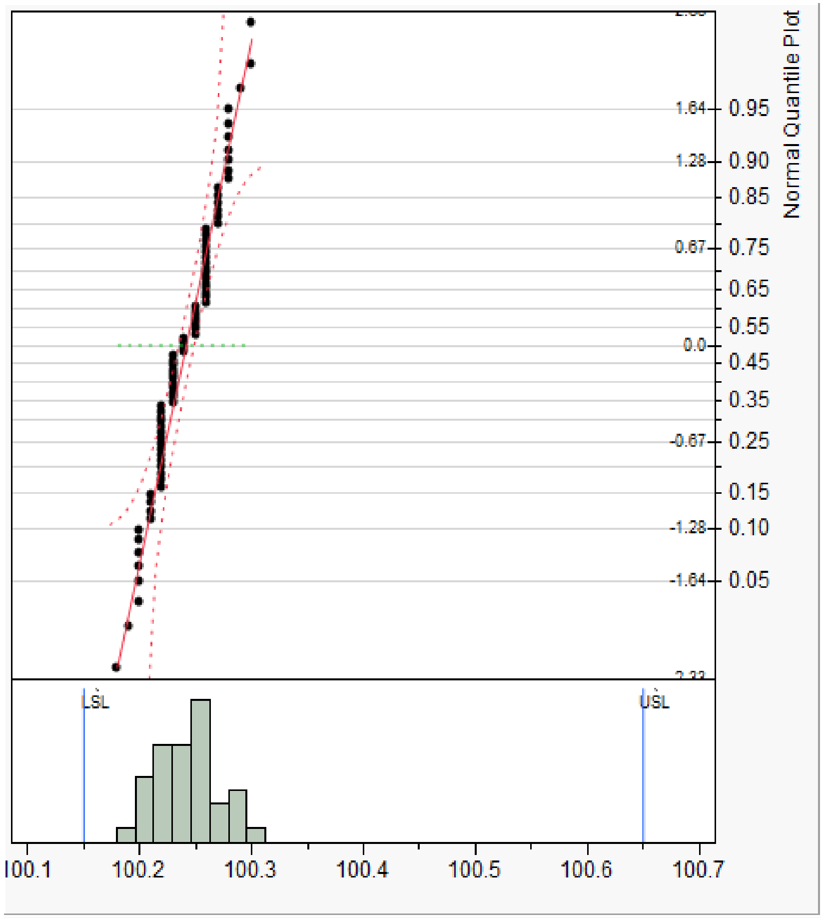
	Lot_Number	Mean(X_)	Std Dev(X_)	Pooled Stdev
1	526BSS	100.2415	0.0274	0.0252
2	528/01BSS	100.3011	0.0252	0.0252
3	529/01BSS	100.3123	0.0167	0.0252
4	716/01BSS2	100.1786	0.0229	0.0252
5	724/01BSS	100.3038	0.0384	0.0252
6	727/01BSS	100.2662	0.0329	0.0252
7	728BSS	100.2158	0.0200	0.0252
8	729/02BSS	100.2771	0.0159	0.0252
9	730/01BSS	100.2609	0.0158	0.0252
10	808/02BSS	100.2729	0.0196	0.0252
11	809/02	100.3204	0.0382	0.0252
12	811	100.2408	0.0158	0.0252
13	906/1BSS2	100.2292	0.0163	0.0252
14	913/01BSS	100.2741	0.0328	0.0252
15	916/01BSS	100.2470	0.0347	0.0252
16	921BSS	100.2455	0.0307	0.0252
17	922	100.1992	0.0125	0.0252
18	924/01	100.2016	0.0179	0.0252
19	927/05BSS	100.1845	0.0125	0.0252
20	928	100.2437	0.0294	0.0252

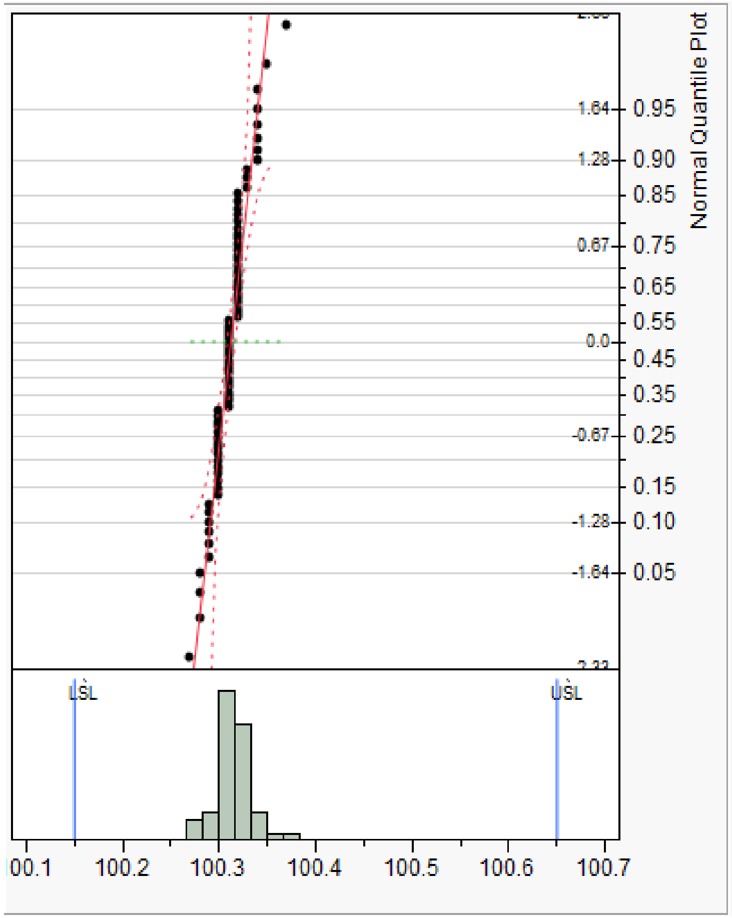
Example X-Dim: Checking Normality

- The data should also be checked to verify that the normal distribution assumption is reasonable.
- There are several reports in JMP that can be used as a check for normality.
- A simple analysis can be done in JMP’s **Distribution** platform by selecting **Normal Quantile Plot** for the CTQ measured. If the data points fall close to a straight line, the normal distribution assumption is considered reasonable.

Example X-Dim: Normal Quantile Plots

Note that these two batches with means some distance from the LSL appear to have normally distributed data values.





Example X-Dim: Normal (?) Quantile Plot

In this example, the mean of the distribution (100.179) is close to the LSL, and the histogram appears right skewed. A normal distribution with the estimated mean and sigma predicts 10.6% of the 80 readings, or 8 to 9 below the LSL. The actual number was **zero**! The probability of 0 occurrences out of 80 is 0.00013, indicating an extremely rare event. Possibly the distribution was truncated below the LSL or the measurements were adjusted upward at inspection to avoid part rejection.

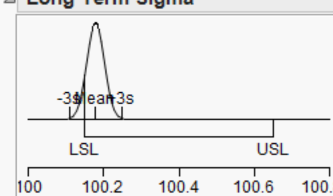
Summary Statistics

Mean	100.17863
Std Dev	0.0229332
Std Err Mean	0.002564
Upper 95% Mean	100.18373
Lower 95% Mean	100.17352
N	80

Capability Analysis

Specification	Value	Portion	% Actual
Lower Spec Limit	100.15	Below LSL	0.0000
Spec Target	.	Above USL	0.0000
Upper Spec Limit	100.65	Total Outside	0.0000

Long Term Sigma



Sigma = 0.02293

Capability	Index	Lower CI	Upper CI
CP	3.634	3.068	4.199
CPK	0.416	0.318	0.514
CPM	.	.	.
CPL	0.416	0.318	0.513
CPU	6.851	5.782	7.919

Portion	Percent	PPM	Sigma Quality
Below LSL	10.5981	105981.08	2.748
Above USL	0.0000	0.0000	.
Total Outside	10.5981	105981.08	2.748

Example: X-Dim, LSL, AQL Mean

One Way Protection on LSL, σ Known

1. The **LSL** on the X-Dim is 100.15.
2. The AQL percent defective p_A in the attributes sampling plan for n = 80, c = 0 is 0.064%, which equates to a normal distribution mean μ_A = 100.23, based on σ = 0.0252. The formula to determine the AQL mean μ_A can be estimated in two steps:
   a) Determine the standard normal variate z_A for the specified AQL percent p_A using the EXCEL function

      =-NORMSINV(0.00064) = 3.220.

   b) The AQL mean is $\mu_A = LSL + z_A\sigma$ = 100.15+3.220(0.0252) = 100.23

Thus, a normal distribution with mean 100.23 and sigma 0.0252 will have 0.064% of the values below the LSL.

Example: X-Dim, LSL, RQL Mean

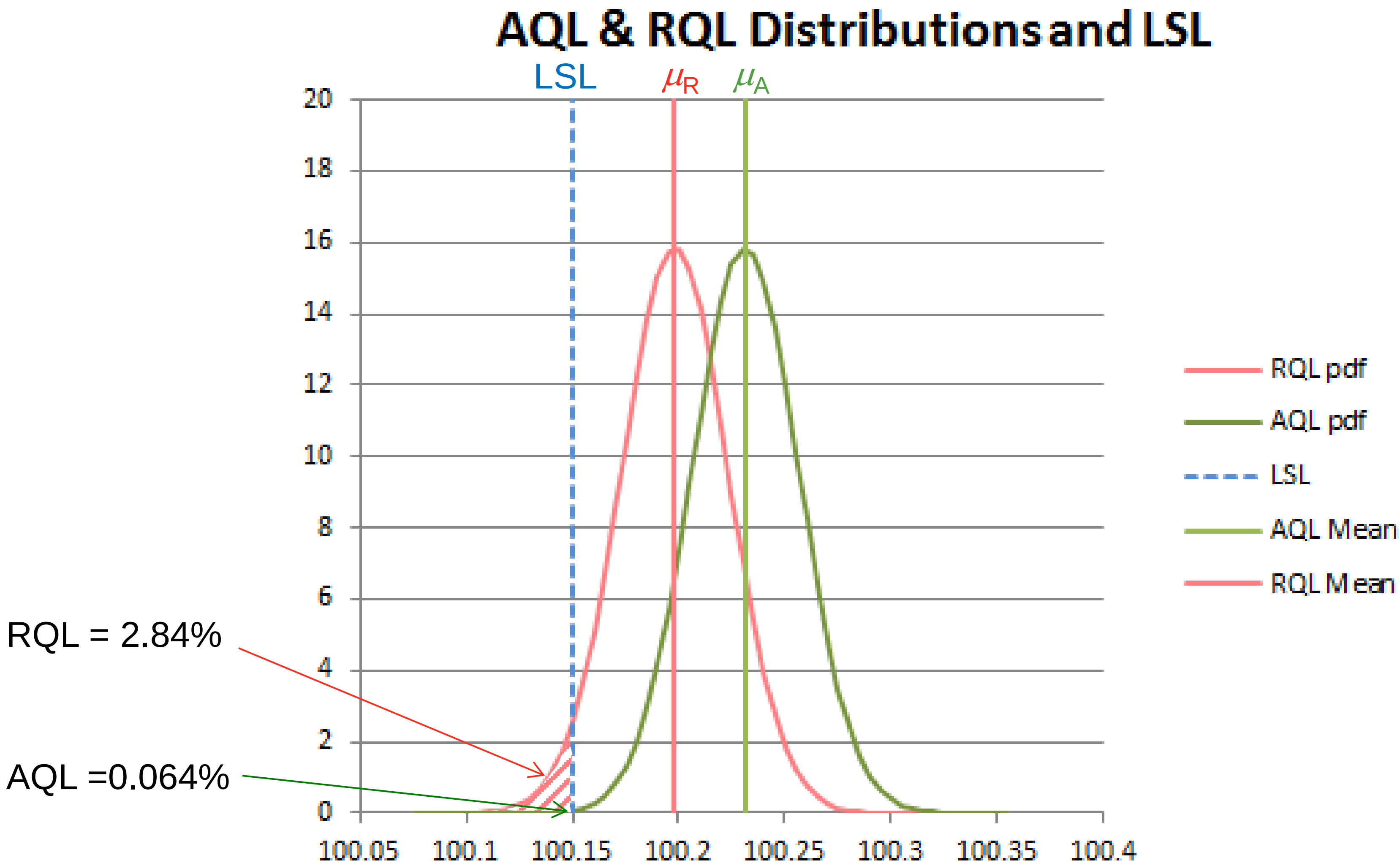
One Way Protection on LSL, σ Known

3. The RQL percent defective p_R in the attributes sampling plan for n = 80, c = 0 is 2.84%, which equates to a normal distribution mean μ_R = 100.198, for sigma of 0.0252. To determine the corresponding mean μ_R we find:
   a) The standard normal variate z_R for the specified RQL percent p_R using the EXCEL function

      =-NORMSINV(0.0284) = 1.905.
   b) The RQL mean is $\mu_R = LSL + z_R\sigma$ = 100.15+1.905(0.0252) = 100.198

Thus, a normal distribution with mean 100.198 and sigma 0.0252 will have 2.84% of the values below the LSL.

Example: X-Dim One Way Protection on LSL



Steps in Variables Sampling

One Way Protection on LSL, σ Known

4. Determine the sample size from the formula[1]

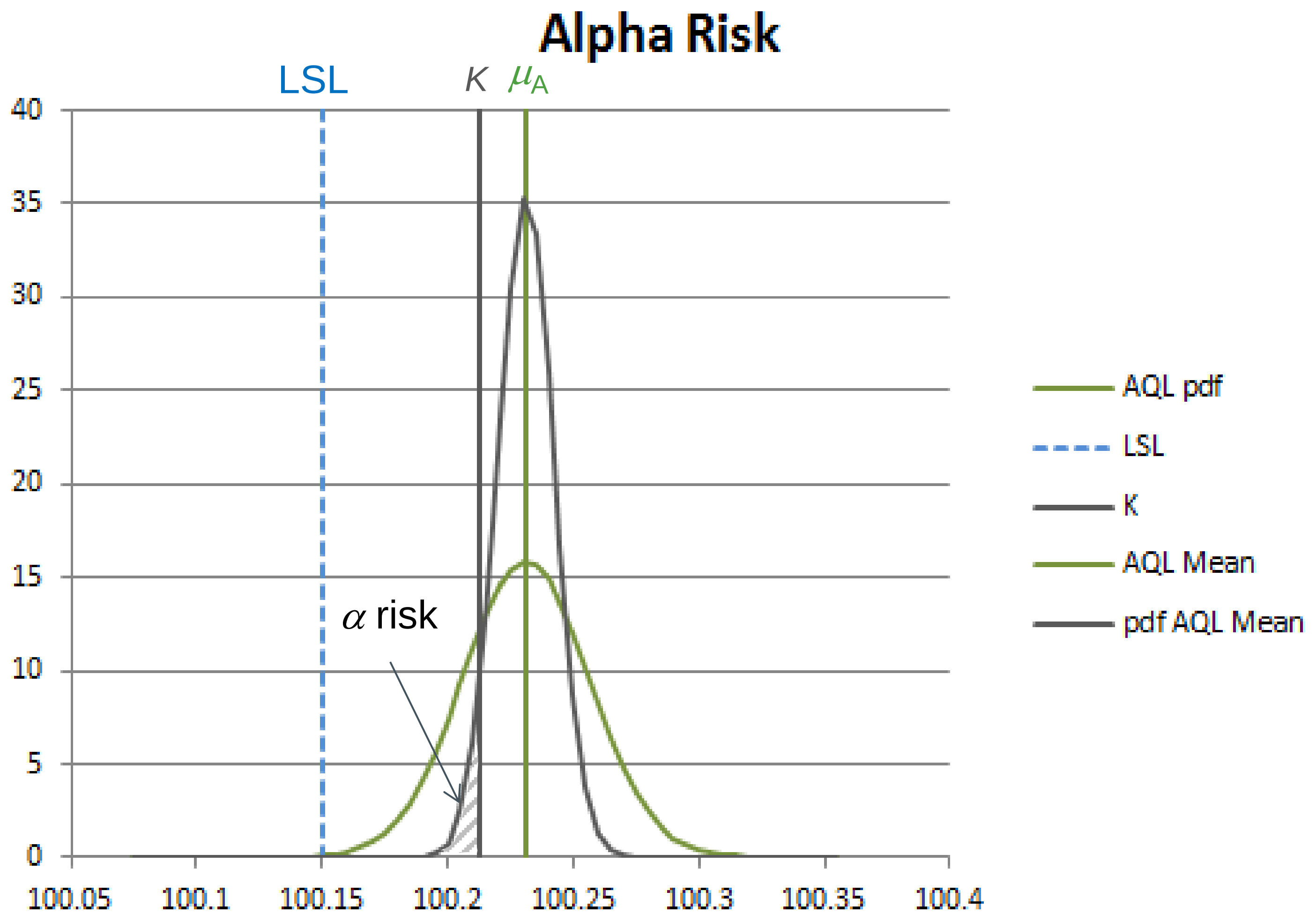
$$n = \left(\frac{z_\alpha + z_\beta}{z_R - z_A} \right)^2 = \left(\frac{1.645 + 1.282}{1.905 - 3.220} \right)^2 \sim 5$$

5. Determine *K*

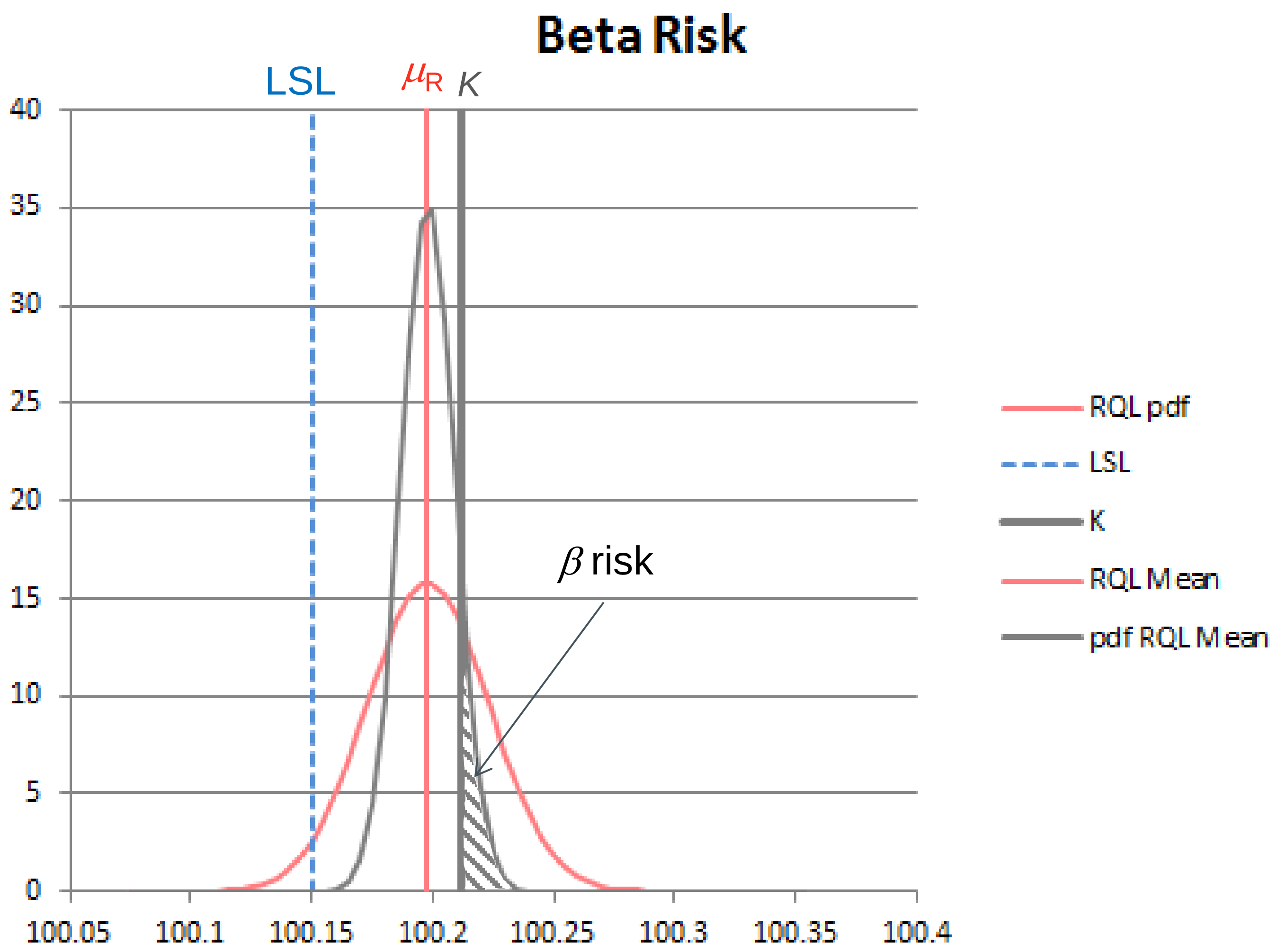
$$K = LSL + \left(\frac{z_\alpha z_R + z_\beta z_A}{z_\alpha + z_\beta} \right) \sigma = 100.15 + \left(\frac{1.645 \times 1.905 + 1.282 \times 3.220}{1.645 + 1.282} \right) 0.0252 = 100.2125$$

6. Plan: If the mean of 5 items is <100.2125, reject the batch. If the mean of 5 items is > or = 100.2125, accept the batch.
7. If the batch is rejected, the original attributes sampling plan can be used.

Example X-Dim: Alpha Risk (n = 5, K = 100.2125)

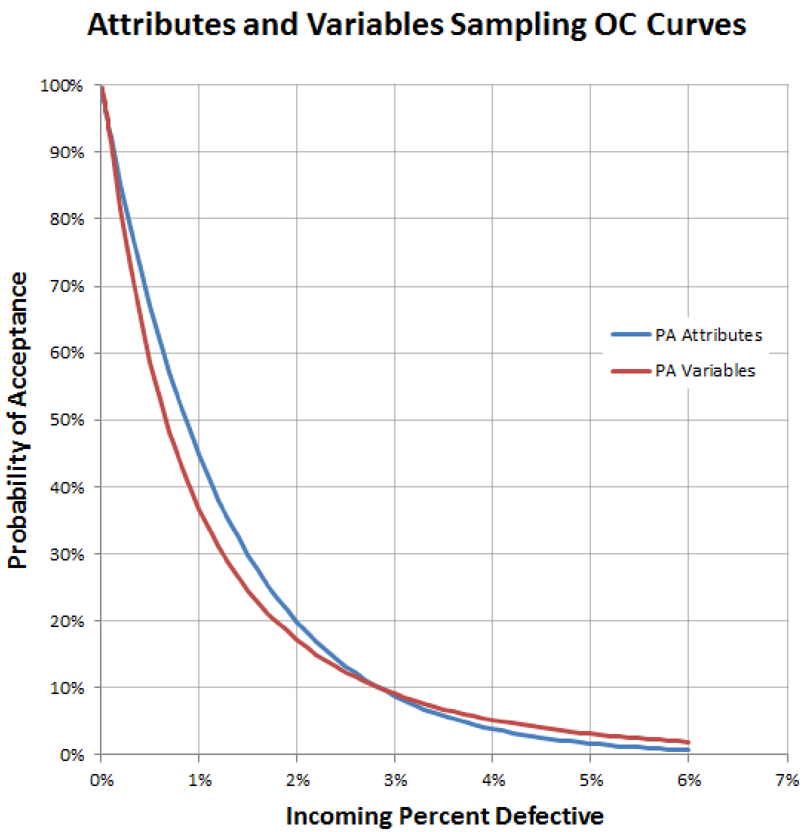


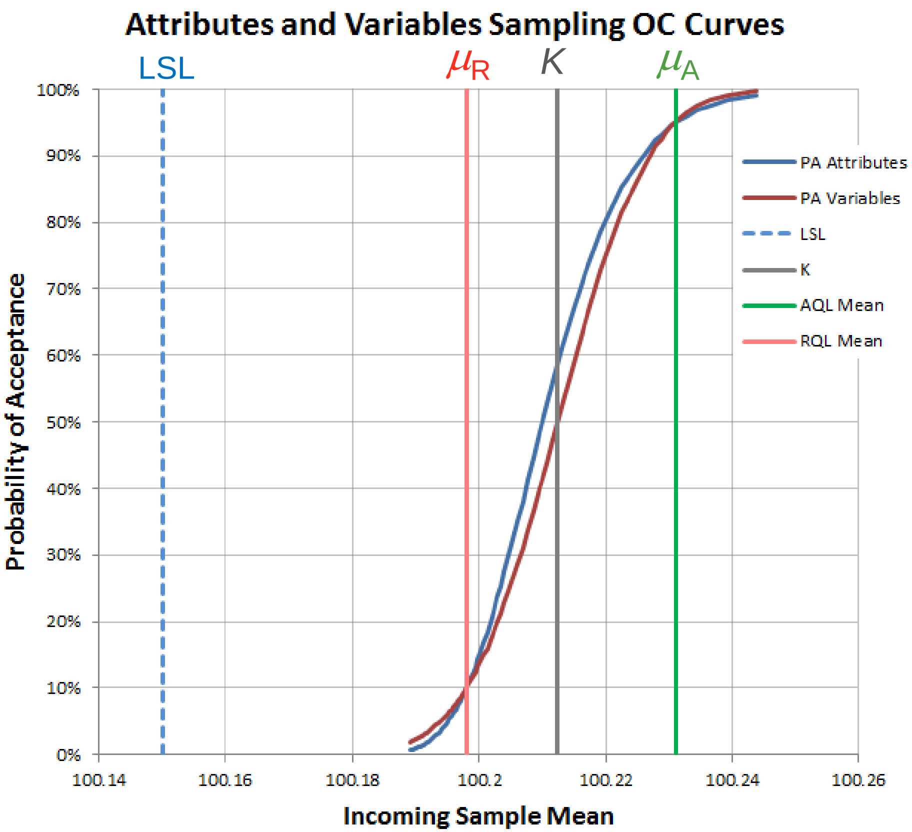
Example X-Dim: Beta Risk (*n* = 5, *K* = 100.2125)



Example X-Dim: OC Curves Compared

Both curves have same P_A at AQL (95%) and RQL (10%) points. % defective and sample mean differ at P_A = 50%, being lower % defective and a higher mean for variables sampling.





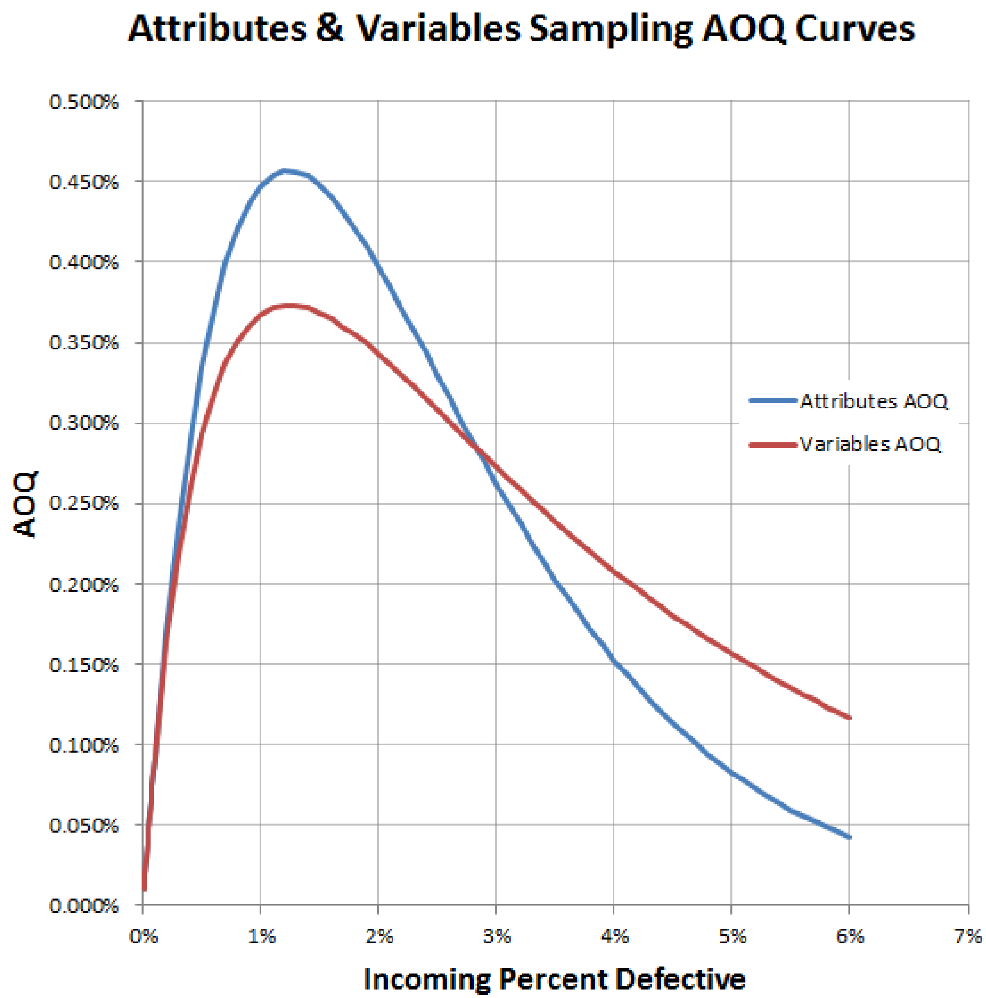
P_A = 50%: 0.86% Attributes, 0.66% Variables

P_A = 50%: 100.2100 Attributes, 100.2125 Variables

Example X-Dim: AOQ Curves Compared

Assuming 100% inspection of rejected lots, the AOQLs are:

- Attributes (N=80) 0.46%
- Variables (N=5) 0.37%



Example X-Dim: Application to Recent 20 Batches

- As an application of the variables approach, we will use the first five measurements of the 20 recent batches, which have 80 measurements per batch.
- There are four means that fall below $K = 100.2125$.
- Thus, 4 batches out of 20 would require the typical IQC attributes sampling of 80 measurements, a reduction of 80% in **batches** needing extensive inspection.
- The 16 batches with means above K=100.2125 would be accepted based on a sample size of 5, a reduction in **samples** measured of ~94%.

	Batch	Mean(X_)
1	1	100.2415
2	2	100.3011
3	3	100.3123
4	4	100.1786
5	5	100.3038
6	6	100.2662
7	7	100.2157
8	8	100.2771
9	9	100.2609
10	10	100.2729
11	11	100.3204
12	12	100.2408
13	13	100.2292
14	14	100.2741
15	15	100.2470
16	16	100.2455
17	17	100.1992
18	18	100.2016
19	19	100.1845
20	20	100.2437

Steps in Variables Sampling: Two-Sided Limits

Protection on LSL and USL, σ Known

1. Estimate pooled standard deviation from 20 recent lots.
2. Determine the desirable AQL means μ_{AL} and μ_{AU} corresponding to a specified lower and upper acceptable percent defect level p_A applicable to both LSL and USL. We will use the p_A of the current attributes plan.
3. Determine the undesirable RQL means μ_{RL} and μ_{RU} corresponding to a specified lower and upper rejectable percent defect level p_R applicable to both LSL and USL. We will use the p_R of the current attributes plan.

Steps in Variables Sampling: Two-Sided Limits

Protection on LSL and USL, σ Known

4. Specify the risk α (producer's risk) of rejecting a lot if either the upper or lower AQL means is a desirable mean. Typically α = 5%. The α risk will be divided in half for the USL and LSL.
5. Specify the risk β (consumer's risk) of accepting a lot if either the upper or lower RQL means is an undesirable mean. Typically β = 10%.

Steps in Variables Sampling: Two Sided Limits (Continued)

Protection on LSL and USL, σ Known

6. Determine the sample size n from the formula[1] (rounding up to integer)

$$n = \left(\frac{z_{\alpha/2} + z_{\beta}}{z_R - z_A} \right)^2$$

7. Determine K_L by the formula for LSL.

$$K_L = LSL + \left(\frac{z_{\alpha/2} z_R + z_{\beta} z_A}{z_{\alpha/2} + z_{\beta}} \right) \sigma$$

8. Determine K_U by the formula for USL.

$$K_U = USL - \left(\frac{z_{\alpha/2} z_R + z_{\beta} z_A}{z_{\alpha/2} + z_{\beta}} \right) \sigma$$

Steps in Variables Sampling: Two Sided Limits

Protection on LSL and USL, σ Known

9. Plan: If the mean $\bar{y}$ of n items is $K_L \leq \bar{y} \leq K_U$, the batch passes for this CTQ. If the mean $\bar{y}$ of n items is $\bar{y} > K_U$ or $\bar{y} < K_L$ the batch fails for this CTQ.
10. If the batch is rejected, an option is to revert to the original attributes sampling plan before choosing to screen the entire batch.

Example: X-Dim, LSL & USL, AQL Mean

Protection on LSL and USL, σ Known

1. The **LSL** on the X-Dim is 100.15.
2. The AQL percent p_{AL} as defined by the attributes sampling plan for n=80, c = 0 is 0.064%, which implies a normal distribution lower mean μ_{AL} = 100.231, for a sigma of 0.0252.

For a normal distribution, the lower mean μ_{AL}, is found in two steps:

First determine the standard normal variate z_A for the specified AQL percent p_{AL} using the EXCEL function =-NORMSINV(0.00064) = 3.220.

The lower AQL mean is μ_{AL} = LSL+$z_A\sigma$ = 100.15+3.220(0.0252) = 100.231

Thus, a normal distribution with mean 100.231 and sigma 0.0252 will have 0.064% of the values below the LSL = 100.15.

Example: X-Dim, LSL & USL, AQL Mean

Protection on LSL and USL, σ Known

The **USL** on the X-Dim is 100.65.

Similarly, for a normal distribution, the upper AQL mean is μ_{AU} = 100.569, based on a sigma of 0.0252. The formula for the upper AQL mean is

$$\mu_{AU} = USL - z_A\sigma = 100.65-3.220(0.0252) = 100.569.$$

Example: X-Dim, LSL and USL, RQL Mean

Protection on LSL and USL, σ Known

3. The RQL percent defective p_R as defined by the attributes sampling plan n=80, c = 0 is 2.84% for either the LSL or the USL, which implies a lower mean μ_{RL} = 100.198, for a sigma of 0.0252.

For a normal distribution, the mean μ_{RL} is found in two steps:

a) First determine the standard normal variate z_R for the specified RQL percent p_{RL} using the EXCEL function =-NORMSINV(0.0284) = 1.905.

b) The lower RQL mean is $\mu_{RL} = LSL + z_R\sigma$ = 100.15+1.905(0.0252) = 100.1980.

Thus, a normal distribution with mean 100.198 and sigma 0.0252 will have 2.84% of the values below the LSL.

Example: X-Dim, LSL and USL, RQL Mean

Protection on LSL and USL, σ Known

The USL on the X-Dim is 100.65.

Similarly, for a normal distribution, the upper RQL mean is

μ_{RU} = 100.602, for a sigma of 0.0252.

The formula for the upper RQL mean is

$\mu_{RU} = USL - z_R\sigma = 100.65 - 1.905(0.0252) = 100.602.$

Thus, a normal distribution with mean 100.602 and sigma 0.0252 will have 2.84% of the values above the USL = 100.65.

Steps in Variables Sampling

Protection on LSL and USL, σ Known

5. Determine the sample size from the formula

$$n = \left(\frac{z_{\alpha/2} + z_{\beta}}{z_R - z_A} \right)^2 = \left(\frac{1.960 + 1.282}{1.905 - 3.220} \right)^2 \sim 7$$

6. We next find K_L and K_U

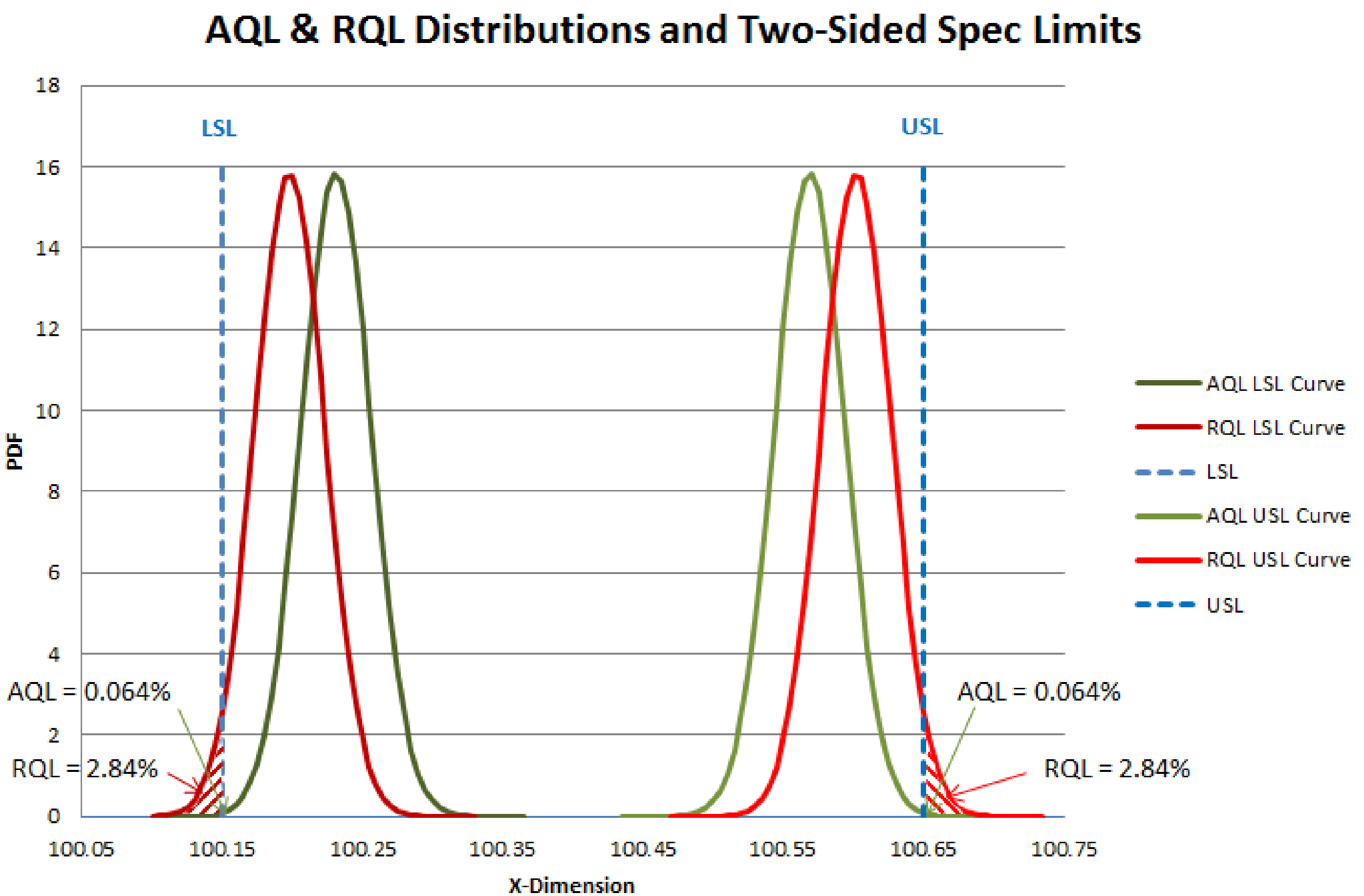
$$K_L = LSL + \left(\frac{z_{\alpha/2} z_R + z_{\beta} z_A}{z_{\alpha/2} + z_{\beta}} \right) \sigma = 100.15 + \left(\frac{1.96 \times 1.905 + 1.282 \times 3.220}{1.96 + 1.282} \right) 0.0252 = 100.211$$

$$K_U = USL - \left(\frac{z_{\alpha/2} z_R + z_{\beta} z_A}{z_{\alpha/2} + z_{\beta}} \right) \sigma = 100.65 - \left(\frac{1.96 \times 1.905 + 1.282 \times 3.220}{1.96 + 1.282} \right) 0.0252 = 100.589$$

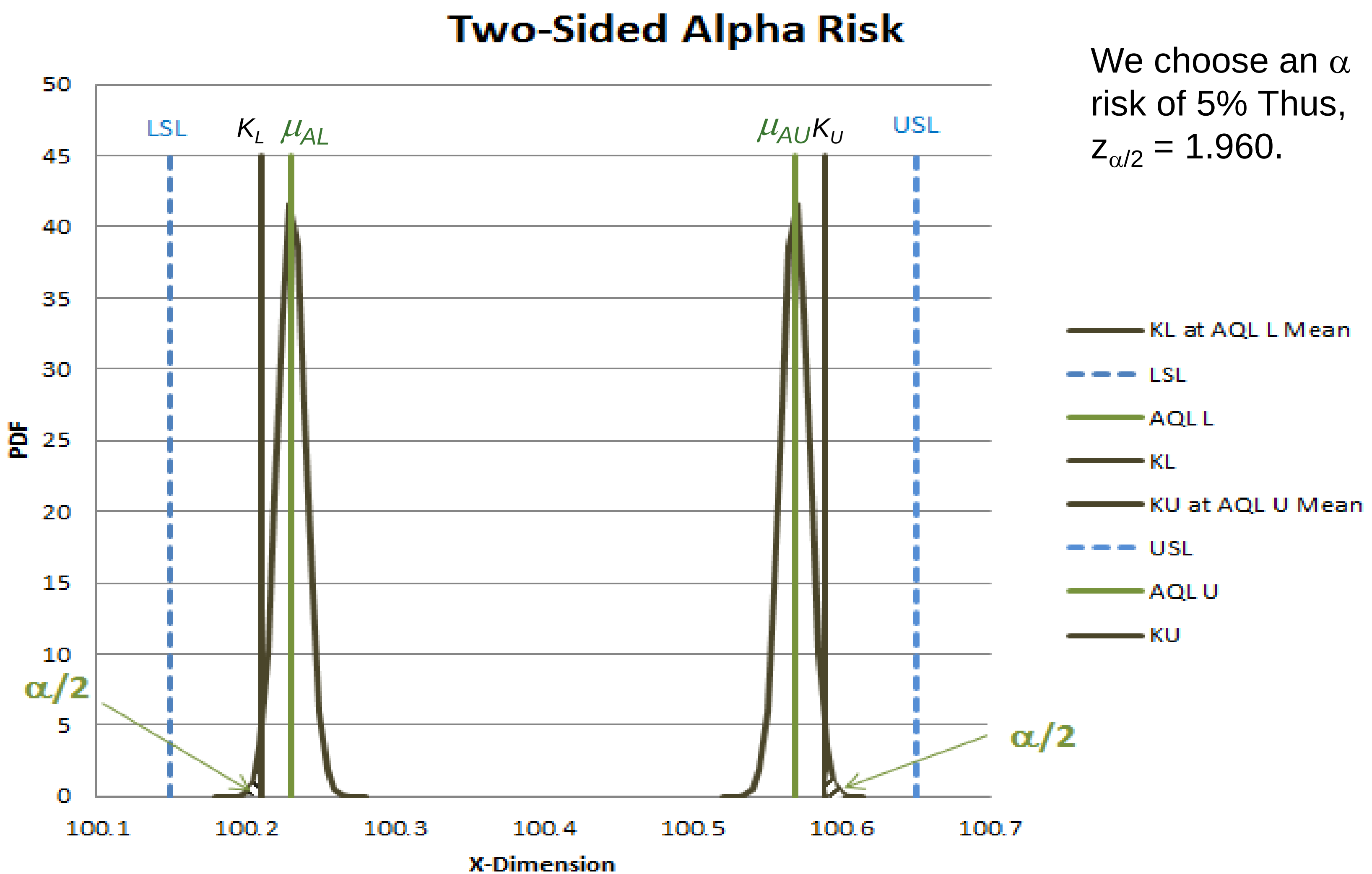
7. Plan: If the mean of 7 items is between 100.211 and 100.589, accept the batch. If the mean is outside these limits, reject the batch.
8. If the batch is rejected, the original attributes sampling plan applies.

Example: X-Dim, LSL & USL, AQL & RQL

Two Way Protection on LSL and USL, σ Known

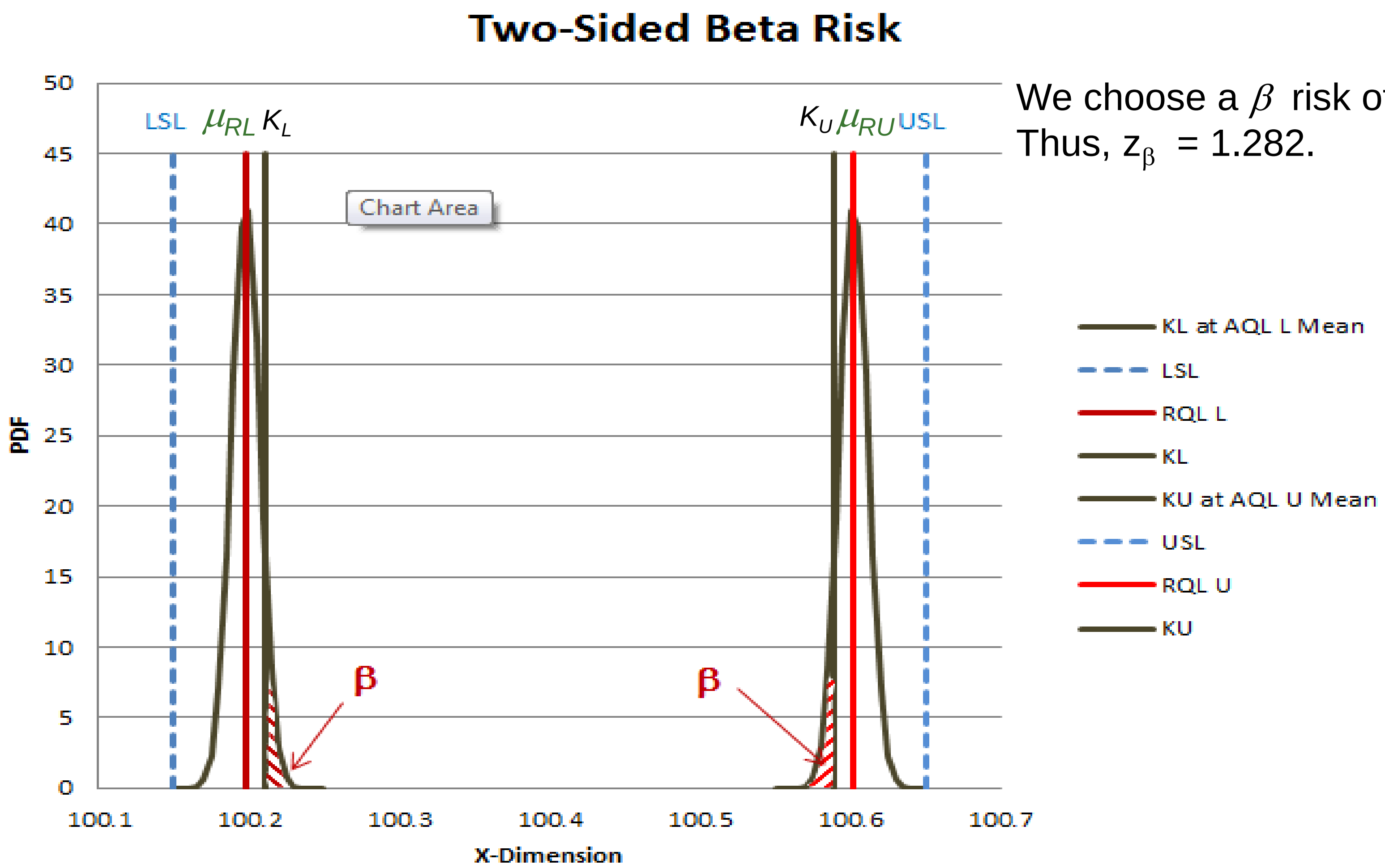


Example X-Dim: Alpha Risk (n = 7)



We choose an α risk of 5% Thus, $z_{\alpha/2}$ = 1.960.

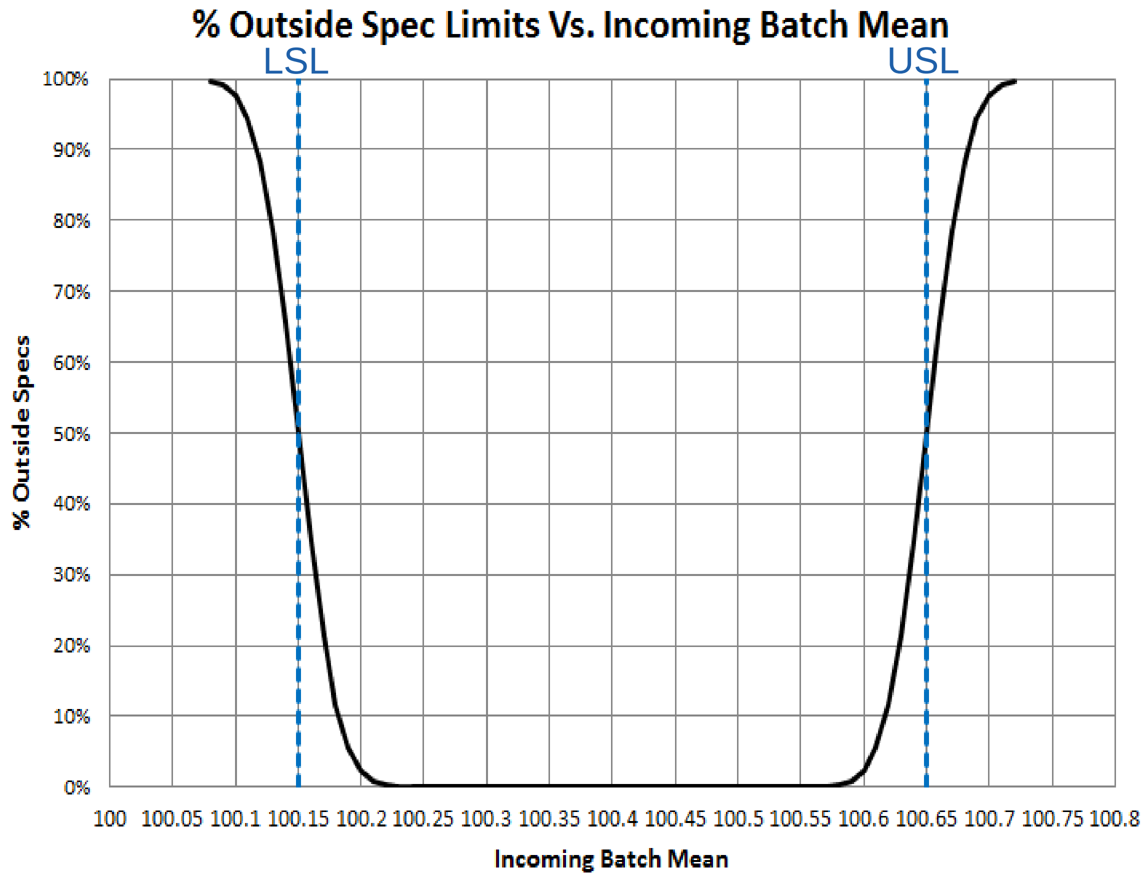
Example X-Dim: Beta Risk (n = 7)



We choose a β risk of 10%. Thus, z_β = 1.282.

Example X-Dim: LSL and USL, % Defective Vs. Incoming Batch Means Curve

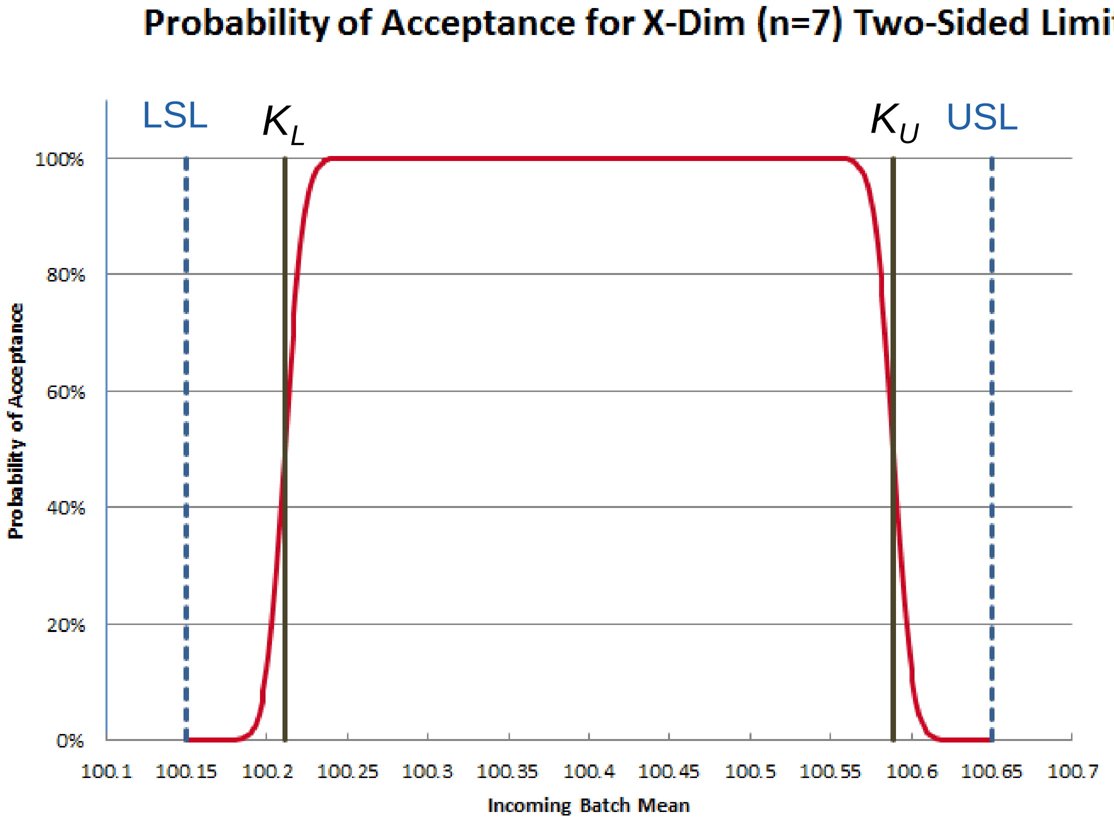
The graph shows the percent defective in a batch for various incoming batch means for LSL = 100.15 and USL = 100.65 .



σ =0.0252

Example X-Dim: LSL and USL, OC Curve

The graph shows the probability of accepting batches for the variables plan with n = 7 and K_L = 100.211 for a LSL = 100.15 and K_U = 100.589 for a USL = 100.65 .



Example X-Dim: Application to Recent 20 Lots

- As an application of the variables approach, we will use the first 7 measurements of 20 recent lots, which have 80 measurements per lot.
- There are four means that fall below K_L = 100.211. No lots are above K_U.
- Thus, 4 lots out of 20 would require the typical IQC attributes sampling of 80 measurements, a reduction of 80% in lots needing extensive inspection.
- The 16 lots with means between K_L and K_U would be accepted based on a sample size of 7, a reduction in samples measured of 91.25%.

	Lot_Number	Mean(X_)
1	526BSS	100.2629
2	528/01BSS	100.3157
3	529/01BSS	100.3071
4	716/01BSS2	100.1714
5	724/01BSS	100.3186
6	727/01BSS	100.2700
7	728BSS	100.2171
8	729/02BSS	100.2743
9	730/01BSS	100.2657
10	808/02BSS	100.2643
11	809/02	100.2986
12	811	100.2329
13	906/1BSS2	100.2200
14	913/01BSS	100.2700
15	916/01BSS	100.2543
16	921BSS	100.2243
17	922	100.1886
18	924/01	100.1943
19	927/05BSS	100.1900
20	928	100.2671

Summary

Variables sampling plans:

- are based on the measurements rather than simple count data of the number of rejectable parts
- can provide equivalent or better protection compared to attributes sampling plans
- are based on assumptions that must be verified for each CTQ
- can significantly reduce samples sizes, time, and costs for IQC inspection

References

1. *Acceptance Sampling in Quality Control*, 2nd ed., Edward G. Shilling, Dean V. Neubauer, 2009, CRC Press, Taylor & Francis Group, Boca Raton, FL
2. *EMPIII Evaluating the Measurement Process & Using Imperfect Data*, Donald J. Wheeler, 2006, SPC Press, 5908 Toole Drive, Suite C, Knoxville, TB 37919