Multifactor Experimental Designs with more than 2 factors

Two Factor Linear Model (Sample Model)

\[ Y_{ijk} = \bar{Y} + a_j + b_k + (ab)_{jk} + e_{ijk} \]

Score on Y for the \( i \)th individual in the \( j \)th treatment of factor A and the \( k \)th treatment of factor B.

Effect offset for level \( j \) of Factor A.

Effect offset for level \( k \) of Factor B.

Effect offset for combined effect of Factors in treatment \( jk \).

Error

Three Factor Linear Model (Sample Model)

\[ Y_{ijkl} = \bar{Y} + a_j + b_k + c_l + (ab)_{jk} + (ac)_{jl} + (bc)_{kl} + (abc)_{jkl} + e_{ijkl} \]

Score on Y for the \( i \)th individual in the \( j \)th treatment of factor A, the \( k \)th treatment of factor B and the \( l \)th treatment of factor C.

Error
Three Factor Linear Model
(Sample Model)

\[ Y_{ijkl} = \bar{Y} + a_j + b_k + c_l + (ab)_{jk} + (ac)_{jl} + (bc)_{kl} + (abc)_{jkl} + e_{ijkl} \]

Score on \( Y \) for the \( i \)th individual in the \( j \)th treatment of factor A, the \( k \)th treatment of factor B and the \( l \)th treatment of factor C = Grand Mean +

Main Effects of Factor A
Factor B
Factor C

Two-Way Interactions
A x B
A x C
B x C

Three-Way Interaction
A x B x C
Three Factor Linear Model
(Sample Model)

\[ Y_{ijkl} = \bar{Y} + a_j + b_k + c_l + (ab)_{jk} + (ac)_{jl} + (bc)_{kl} + (abc)_{kl} + e_{ijkl} \]

Score on Y for the ith individual in the jth treatment of factor A, the kth treatment of factor B and the lth treatment of factor C

Tests of Effects for Three Factors

- Overall Effect of Factor A
  \[ F_A = \frac{MS_A}{MS_{error}} \]

- Overall Effect of Factor B
  \[ F_B = \frac{MS_B}{MS_{error}} \]

- Overall Effect of Factor C
  \[ F_C = \frac{MS_C}{MS_{error}} \]

- Effects of Two-Way Combinations:
  \[ F_{AB} = \frac{MS_{AB}}{MS_{error}} \]

- Effects of Three-Way Combinations:
  \[ F_{ABC} = \frac{MS_{ABC}}{MS_{error}} \]
Tests of Effects for Three Factors

- Overall Effect of Factor A:
  \[ F_A = \frac{MS_A}{MS_{error}} \]

- Overall Effect of Factor B:
  \[ F_B = \frac{MS_B}{MS_{error}} \]

- Overall Effect of Factor C:
  \[ F_C = \frac{MS_C}{MS_{error}} \]

- Effects of Two-Way Combinations:
  \[ F_{AB} = \frac{MS_{AB}}{MS_{error}} \quad F_{AC} = \frac{MS_{AC}}{MS_{error}} \]

- Effects of Three-Way Combinations:
  \[ F_{ABC} = \frac{MS_{ABC}}{MS_{error}} \]

Multifactor Experimental Designs

with 4 factors
Full Model

Reduced Model

The General Linear Test

\[ F = \frac{\text{Reduction in Error}}{\text{# Added Parameters}} \]

\[
\frac{\text{Baseline Error}}{}
\]
### The General Linear Test

\[ F = \frac{SS_{\text{error}}(R) - SS_{\text{error}}(F)}{SS_{\text{error}}(F) / df_{\text{error}}(F)} \]

### Analysis of Variance

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS of Error</th>
<th>Mean Square</th>
<th>F Ratio</th>
<th>( p ) Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error</td>
<td></td>
<td>1183.438</td>
<td>22.646</td>
<td>1.155</td>
<td>0.290</td>
</tr>
<tr>
<td>C Total</td>
<td>15</td>
<td>2987.443</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>21123.92</strong></td>
<td><strong>52.13</strong></td>
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### Reduced Model

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### Summary

- The General Linear Test is used to compare two regression models.
- The F statistic is calculated as the difference between the sum of squares for the reduced model and the full model, divided by their respective degrees of freedom and mean squares.
- The F statistic is then compared to the critical value from the F-distribution table to determine whether the difference between the models is statistically significant.
\[
F = \frac{SS_{error}(R) - SS_{error}(F)}{MS_{error}(F)}
\]

1275.15 - 

\[
F = - - -
\]

\[
F = \frac{df_{error}(R) - df_{error}(F)}{MS_{error}(F)}
\]

1275.15 - 1169.5

\[
F = - - -
\]
### Full Model

#### Analysis of Variance

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<td>Model</td>
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<td>244.545</td>
<td>8.4506</td>
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\[ F = \frac{SS_{model}(R) - SS_{model}(F)}{MS_{error}(F)} \]

\[ F = \frac{1275.15 - 1169.5}{8.122} = 12.45 \]

\[ F > F_{0.05, 7, 15} = 3.74 \]

**Critical Region:** Rejected

---

### Reduced Model

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\[ F > F_{0.05, 7, 15} = 3.74 \]

**Critical Region:** Rejected
**Full Model**

F = \frac{SS_{error}(R) - SS_{error}(F)}{MS_{error}(F)}

\[
F = \frac{1275.15 - 1169.5}{152 - 144} = \frac{105.65}{8.122} = \frac{8}{8.122} = 1.626
\]

**Reduced Model**

F = \frac{SS_{error}(R) - SS_{error}(F)}{MS_{error}(F)}

\[
F = \frac{1275.15 - 1169.5}{152 - 144} = \frac{105.65}{8.122} = \frac{8}{8.122} = 1.626
\]

\[\text{df}_{\text{numerator}} = 8\]
\[\text{df}_{\text{numerator}} = 144\]