

Pure Quadratic Curvature Test Add-In

v1.0

Minimum JMP Version: JMP 16

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What does this add-in do?

This add-in conducts a **hypothesis test for pure quadratic curvature in a factorial experiment with replicated center points**. It is useful for determining if one or more factors exhibit curvature and therefore axial points should be added to the design.

The add-in implements a t-test procedure outlined in Montgomery's *Design and Analysis of Experiments* (8e) which compares the mean of the center points to the mean of the factorial points. Because response variability at the factorial points includes both treatment effects and error, the test uses the center point replicates to estimate error. This distinguishes the test from a typical pooled or independent variances t-test available under Fit Y by X, creating the need for this add-in. (See *Statistical Details* below for further detail.)

The add-in comes bundled with two sample data sets in the add-in directory:

- *Mongtomery Example 6-7.jmp* is the data from the relevant example in Montgomery's book.
- *Reactor with Center Points.jmp* is a version of the *Reactor 32 Runs.jmp* data from the Sample Data Library that has been augmented with 5 center points.

What kind of experimental designs is this add-in appropriate for?

Use this for **balanced factorial designs only**. This analysis is not appropriate for unbalanced designs. If using with a fractional factorial design and certain higher-order interactions are active, the test could yield misleading results.

This analysis requires **at least two center points** in the design. If the add-in does not detect at least two center points, the add-in will terminate with an informative error message.

The analysis takes only continuous factors into account. Categorical factors may be present in the design but are ignored. If the design includes categorical factors, **ensure that center point replicates are balanced across all possible combinations of categorical factor levels**. Failure to do so may yield misleading results.

How to use the add-in and interpret its outputs

After installing the add-in, choose "Pure Quadratic Curvature Test" under the Add-Ins menu. If you do not have a data table open, you will be prompted to open one.

The launch dialog has three roles:

- **Y, Response:** A single continuous column of the response data
- **X, Factors:** Any number of continuous, ordinal, or nominal columns of the factor settings (ordinal and nominal factors are ignored in the analysis)
- **Center Point:** An optional column indicating which rows correspond to center points. The column **must contain a numerical value of 1 for each center point and 0 for each factorial point**. If the column has a continuous modeling type, it will be changed to nominal. If a Center Point column is not provided, the add-in will attempt to identify center points automatically and will add a new Center Point column to your data table.

Test for quadratic effects in a balanced factorial design with center points

Select Columns	Cast Selected Columns into Roles	Action
Temp	Y, Response: Filtration Rate	OK
Pressure	X, Factors: Temp, Pressure, Concentration, Stir Rate	Cancel
Concentration	X, Factors: optional	Remove
Stir Rate	X, Factors: optional	Help
Filtration Rate	Y, Response: Filtration Rate	
My CP Col	Center Point: optional	

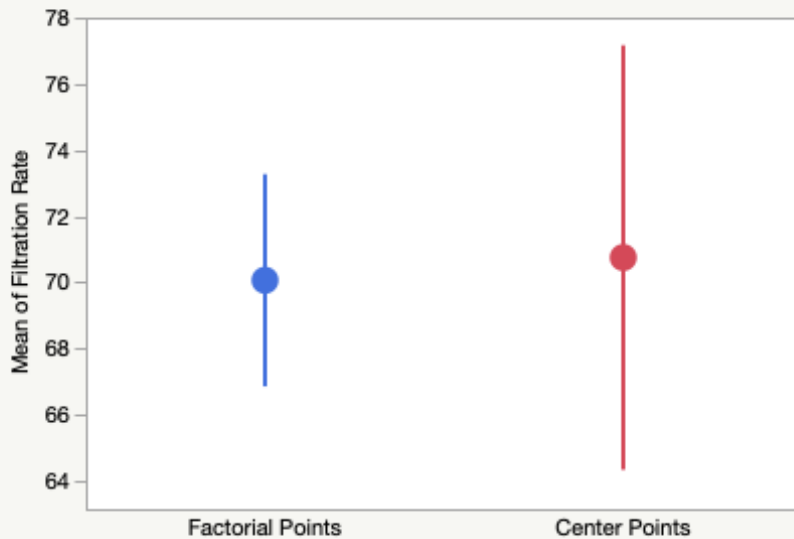
The analysis report has three sections. The top includes a plot of the mean response and 95% confidence interval for the factorial points and center points. The values plotted correspond to the means and confidence intervals in the Summary Stats section.

The Summary Stats section includes sample size (n), mean, standard deviation, and 95% confidence intervals for the two groups. Note that the standard deviation of the factorial points includes both treatment effects and error variation, while the center point standard deviation is an estimate of pure error. The center point standard deviation is therefore used in the calculation of the confidence intervals.

The t-Test section shows the results of the t-test and mirrors the content of the Pooled t Test results in Fit Y by X with the exception of the graph to the right, which is of the sampling distribution of the t-statistic. The red line corresponds to the t-statistic obtained, and the shaded portion of the distribution represents the two-tailed p-value. The standard deviation of the center points is used as the error estimate for the purpose of the test. P-values follow the conditional formatting set in the JMP Preferences (default is red for $p < 0.05$, orange for $p < 0.01$).

A statistically significant result is considered evidence that curvature is present in one or more factors.

Pure Quadratic Curvature Test



Each error bar is constructed from 95% CI Lower to 95% CI Upper.

Summary Stats

	n	Mean	St Dev	95% CI Lower	95% CI Upper
Factorial	16	70.0625	19.5464	66.8553	73.2697
Center	4	70.75	4.03113	64.3356	77.1644

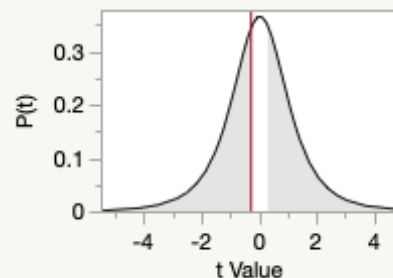
*Note that factorial st dev includes variation due to both error and treatment. Confidence intervals calculated using center point variance as error estimate.

t-Test

Difference = Factorial - Center

Error variance estimated using the 4 center points

Difference	-0.6875	t Value	-0.3051
Std Err Dif	2.25347	DF	3
Upper CL Dif	6.48405	Prob > t	0.78024
Lower CL Dif	-7.859	Prob > t	0.60988
Confidence	0.95	Prob < t	0.39012



The statistical details

For full details, see Montgomery's *Design and Analysis of Experiments* (8e), Section 6-8 in both the main text and Supplemental Materials. This is a brief summary.

In a balanced full-factorial design in which no curvature is present, the true mean response across the factorial points will equal the true mean response at the global center point. If curvature is present, this will not be the case. This analysis thus compares the observed mean responses at the factorial and center points, with a statistically significant difference suggesting the presence of curvature.

The null and alternative hypotheses specifically are:

$$H_0: \beta_{11} + \beta_{22} + \dots + \beta_{kk} = 0$$

$$H_1: \beta_{11} + \beta_{22} + \dots + \beta_{kk} \neq 0$$

The test is conducted using the t-statistic:

$$t_0 = \frac{\bar{y}_F - \bar{y}_C}{\sqrt{\hat{\sigma}^2 \left(\frac{1}{n_F} + \frac{1}{n_C} \right)}}$$

with $n_C - 1$ degrees of freedom, and where:

- n_F is the number of factorial points
- n_C is the number of center points
- \bar{y}_F is the mean of the factorial points
- \bar{y}_C is the mean of the center points
- $\hat{\sigma}^2$ is the variance of the center points